LINEAR STABILITY ANALYSIS ON THE ONSET OF DDC IN A DPM SATURATED WITH CSF WITH INTERNAL HEATING

Kamble Shravan S.

Assistant Professor, Dept. of Mathematics, Sri Shivalingeshwara Govt. First Grade College, Madanhipparga, Dist. Kalaburagi, Karnataka-India-585236

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Abstract *- The effect of rotation on the onset of double diffusive convection (DDC) in a horizontal couple stress fluid saturated porous layer with an internal heat source is investigated using linear stability analysis. The linear stability analysis is based on the classical normal mode technique. The extended Darcy model which includes the time derivative term and Coriolis term has been employed in the momentum equation. The expressions for stationary and oscillatory Rayleigh number are obtained as a function of governing parameters such as internal Rayleigh number, couple stress parameter, Taylor number, normalized porosity and Lewis number and their effects on the stability of the system are shown graphically*

Key Words*: *Rotation, couple stress fluid (CSF), Darcy Porous medium (DPM), Double diffusive convection (DDC), Internal Heat Source.

Nomenclature

a	Wavenumber, $\sqrt{l^2+m^2}$
c	Specific heat of solid
c_p	Specific heat of fluid at constant pressure
C	Couple stress parameter, $\mu_c/\mu d^2$
d	Height of the porous layer
Da	Darcy number, $\left(K/d^2\right)$
$\mathbf{g}$	Gravitational acceleration, $(0,0,-g)$
$\mathbf{i}$	Unit normal vector in the x-direction
$\mathbf{j}$	Unit normal vector in the y-direction
K	Permeability
$\mathbf{k}$	Unit normal vector in the z-direction
l,m	Horizontal wavenumbers
Le	Lewis number, κ_T/κ_S
p	Pressure
Pr	Prandtl number, ν/κ_T
Pr_D	Darcy-Prandtl number, $\varepsilon\gamma Pr/Da$
q	Velocity vector, (u,v,w)
Q	Internal heat source
Ra_S	Solute Rayleigh number, $\beta_S g\Delta SKd/\nu\kappa_T$
Ra_T	Thermal Rayleigh number, $\beta_T g\Delta TKd/\nu\kappa_T$
R_i	Internal Rayleigh number, Qd^2/κ_T
Ra_S	Solute Rayleigh Number, $\left(\beta_S g\Delta SKd/\nu\kappa_T\right)$
Ra_T	Thermal Rayleigh Number, $\left(\beta_T g\Delta TKd/\nu\kappa_T\right)$
S	Solute concentration
ΔS	Salinity difference between the walls
t	Time
T	Temperature
Ta	Taylor number, $\left(2\mathbf{\Omega}K_z/\varepsilon\nu\right)^2$
ΔT	Temperature difference between the walls
x,y,z	Space coordinates

Greek symbols

a	Wave number, $\sqrt{l^2+m^2}$
β_S	Solute coefficient of expansion
β_T	Thermal coefficient of expansion
γ	Ratio of specific heats, $\left[\varepsilon\left(\rho c_p\right)_f+(1-\varepsilon)(\rho c)_s\right]/\left(\rho c_p\right)_f$
ε	Porosity
χ	Normalized porosity, ε/γ
η	Thermal anisotropy parameter, κ_{Tx}/κ_{Tz}
Θ	Dimensionless amplitude of temperature perturbation

κ_S	Solute diffusivity
κ_T	Thermal diffusivity
μ	Dynamic viscosity
μ_c	Couple stress viscosity
ν	Kinematic viscosity, μ/ρ_0
ρ	Fluid density
ρ_0	Reference density
σ	Growth rate
Φ	Dimensionless amplitude of concentration perturbation
$\mathbf{\Omega}$	Angular velocity, $(0,\ 0,\ \Omega)$

Other symbols

D	$\dfrac{d}{dz}$
∇_1^2	$\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$
∇^2	$\nabla_1^2+\dfrac{\partial^2}{\partial z^2}$

Subscripts

b	Basic state
c	Critical
f	Fluid
h	Horizontal
m	Porous medium
0	Reference
s	Solid

Superscripts

*	Dimensionless quantity
′	Perturbed quantity
Osc	Oscillatory
St	Stationary

1. INTRODUCTION

Double diffusive convection (DDC) in porous media has been intensively studied because of its applications in different branches of science and engineering, such as underground disposal of nuclear wastes, groundwater pollution, contaminant transport in fluid saturated soils, liquid gas storage, and food processing [see [11] & [13]]. Double diffusive convection (DDC) in porous media has attracted many authors like [2] & [14] during the last several decades.

The effect of internal heat source is important in several applications that include reactor safety analysis, metal waste form development for nuclear fuel, fire and combustion studies, and storage of radioactive materials. The onset of convection due to internal heat source has become an interesting problem in various areas of geophysics and engineering under the situations of radioactive decay or a week exothermic reaction within the porous material.

Earlier studies of convective flows in porous media within rectangular enclosures, without the local heat source effects. A very little attention has been devoted to this problem with non-Newtonian fluids. The corresponding problem in the case of porous medium has also not received much attention until recently. With growing importance of non-Newtonian fluids with suspended particles in modern technology and industries, the investigations of such fluids are desirable. The studies of such fluids have applications in number of processes that occur in industry, such as the extrusion of polymer fluids, solidifications of liquid crystals, cooling of metallic plate in a bath, exotic lubrication and colloidal and suspension solutions. In the category of non-Newtonian fluids couple stress fluids have distinct features, such as polar effects.

The main aim feature of couple stresses will be to introduce a size dependent effect that is not present in the classical viscous fluids. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. The constitutive equations for couple stress fluids were given by [1].

Anisotropy is generally a consequence of preferential orientation of symmetric geometry of porous matrix or fibers and is in fact encountered in numerous systems in industry and nature. Also artificial porous matrix anisotropy can be made deliberately according to applications. [3] studied the combine defect of horizontal and vertical heterogeneity and anisotropy on the onset of convection in a porous medium. [4] performed linear and nonlinear stability analysis of double diffusive convection in anisotropic porous media including Soret effect and reported that the effect of mechanical anisotropic parameters is to destabilize and of thermal anisotropic parameters is to stabilize the system.

There are large number of practical situations in which convection is driven by internal heat source in the porous media. The wide applications of such convections occur in nuclear reactions, nuclear heat cores, nuclear energy, nuclear waste disposals, oil extractions, and crystal growth. [5] investigated linear stability analysis for the onset of natural convection in a fluid saturated porous medium with uniform internal heat source and density maximum in an local thermal nonequilibrium model and predicted that internal heat source parameter advances the onset convection, [6] studied the onset of stationary convection in a low Prandtl number with internal heat source and found that effect of internal heat source parameter is destabilization. Recently, [7-10] have studied the problem of thermal instability in porous media with internal heat source. Few authors have studied on rotation with couple stress fluid in porous media [see [19], [20], [21], [22] & [23]].

Although few literatures on DDC in a porous medium saturated by ordinary fluid with or without an internal heat source is available, no attention has been devoted to the study of DDC in a porous layer saturated by a couple stress fluids in the presence of an internal heat and rotation. Therefore in the present work we intend to investigate the onset double diffusive convection in a rotating couple stress fluid saturated porous layer with an internal heat source employing a modified Darcy model using linear stability analyses. Our objective is to study how the onset criterion for stationary and oscillatory convection are affected by the Lewis number, solute Rayleigh number, Taylor number, Couple stress parameter, internal heat source and normalized porosity.

2. GOVERNING EQUATIONS

We consider an infinite horizontal couple stress fluid saturated porous layer confined between the planes $z=0$ and $z=d$ with the vertically downward gravity force $\mathbf{g}$ acting on it. A constant temperature $\Delta T+T_0$ and T_0 with stabilizing concentrations $\Delta S+S_0$ and S_0 respectively are maintained between the lower and upper surfaces. A Cartesian frame of reference is chosen with the origin in the lower boundary and z-axis vertically upwards. The porous layer rotates uniformly about the z-axis with a constant angular velocity $\Omega=(0,0,\Omega)$. The modified Darcy model, which includes the time derivative and the Coriolis term is employed as a momentum equation [see [17]]. The basic governing equations are

$$\Delta.q=0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}=-\nabla p+\rho_0(\beta_T T-\beta_S S)g-\frac{2\rho_0}{\varepsilon}\Omega\times q -\frac{1}{K}(\mu-\mu_c\nabla^2)q \quad , \tag{2}$$

$$\gamma\frac{\partial T}{\partial t}+(q.\nabla)T=\kappa_T\nabla^2 T+Q(T-T_0)\ , \tag{3}$$

$$\varepsilon\frac{\partial T}{\partial t}+(q.\nabla)S=\kappa_S\nabla^2 S\ , \tag{4}$$

where, the variables and constants have their usual meaning, as given in the Nomenclature. Further $\gamma=(\rho c)_m/(\rho c_p)_f, (\rho c)_m=(1-\varepsilon)(\rho c)_s+\varepsilon(\rho c_p)_f$, c is the specific heat of the solid and c_p is the specific heat of the fluid at constant pressure respectively.

1.1 BASIC STATE

The basic state of the fluid is assumed to be quiescent and is given

$$q_b=(0,0,0), P=P_b(z), T=T_b(z), \quad S=S_b(z), \rho=\rho_b(z), \rho=\rho_b(z) \quad , \tag{5}$$

The temperature $T_b(z)$, solute concentration $S_b(z)$, pressure $P_b(z)$ and density $\rho_b(z)$, satisfy the following equations

$$\frac{dP_b}{dz}=-\rho_b g, \kappa_T\frac{d^2T_b}{dz^2}+Q(T_b-T_0)=0, \frac{d^2S_b}{dz^2}=0, \quad \rho_b=\rho_0[1-\beta_T(T_b-T_0)+\beta_S(S_b-S_0)] \quad , \tag{6}$$

Then the conduction state temperature and concentration are given by

$$T_b(z)=\frac{\Delta T Sin(\sqrt{R_i}(1-z/d))}{Sin\sqrt{R_i}}+T_0, \quad S_b(z)=\Delta S(1-z/d)+S_0\ . \tag{7}$$

1.2 PERTURBED STATE

On the basic state we superpose infinitesimal perturbations in the form

$$q=q_b+q', T=T_b+T', S=S_b+S', \quad P=P_b+P', \rho=\rho_b+\rho' \quad , \tag{8}$$

where , primes indicate perturbations. Substituting Eq. (5) into Eqs. (1)- (4) and using Eqs. (5)- (7), the perturbed equations are given by

$$\nabla.q'=0\ , \tag{9}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial q'}{\partial t}=-\nabla p'+\rho_0(\beta_T T'-\beta_S S')g -\frac{2\rho_0}{\varepsilon}\Omega\times q'-\frac{1}{K}(\mu-\mu_c\nabla^2)q' \quad , \tag{10}$$

$$\gamma\frac{\partial T'}{\partial t}+(q'.\nabla)T'-\frac{\Delta T'}{d}w'=\kappa_T\nabla^2T'+QT' \quad , \qquad (11)$$

$$\varepsilon\frac{\partial S'}{\partial t}+(q'.\nabla)S'-\frac{\Delta S'}{d}w'=\kappa_S\nabla^2S' \,, \qquad (12)$$

By operating curl twice-on equation (10), we eliminate p' from it and then render the resulting equation and the Eqs. (11) and (12) dimensionless using the following transformations.

$$(x',y',z')=(x^*,y^*,z^*)d,(u',v',w')=(\kappa_T/d)(u^*,v^*,w^*),$$
$$t=t^*(\gamma d^2/\kappa_T),T'=(\Delta T)T^*,S'=(\Delta S)S^* \,.$$
(13)

To obtain non-dimensional equations as (on dropping the asterisks for simplicity),

$$\begin{aligned}&[(\frac{1}{Pr_D}\frac{\partial}{\partial t}+1-C\nabla^2)^2\nabla^2+Ta\frac{\partial^2}{\partial z^2}]w\\&=(\frac{1}{Pr_D}\frac{\partial}{\partial t}+1-C\nabla^2)\,\nabla_1^2(Ra_T-Ra_S)\end{aligned} \quad , \qquad (14)$$

$$[\frac{\partial}{\partial t}-\nabla^2-R_i+(q.\nabla)]T-w=0 \quad , \qquad (15)$$

$$\chi\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2+(q.\nabla)S-w=0\,, \qquad (16)$$

where, $Ta=(\frac{2\Omega K}{\nu\varepsilon})^2$ is the Taylor number and $R_i=Qd^2/\kappa_T$ is the internal Raylegh number, and all the other non-dimensional parameters are as defined in the Nomenclature.

The boundary conditions are assumed to be stress free, isothermal and isohaline, the Eqs. (14)-(16) are to be solved for the boundary conditions

$$w=\frac{\partial^2w}{\partial z^2}=T=S=0 \quad \text{at} \quad z=0,1\,. \qquad (17)$$

2. LINEAR STABILITY ANALYSIS

We predict the thresholds of both marginal and oscillatory convections using linear theory. The Eigenvalue problem defined by Eqs. (14)- (16) subject to the boundary conditions (17) is solved using the time-dependent periodic disturbances in a horizontal plane. Assuming that the amplitudes of the perturbations are very small, we write

$$(w,T,S)=(W(z),\Theta(z),\Phi(z))exp[i(lx+my)+\sigma t]$$
(18)

Where l,m are horizontal wavenumbers and σ is the growth rate. Infinitesimal perturbations of the rest state may either damped or grow depending on the value of the parameter σ. Substituting Eq. (18) into the linearized version of Eqs. (14)- (16), we obtain

$$\begin{aligned}&[(\frac{\sigma}{Pr_D}-C(D^2-a^2)+1)^2(D^2-a^2)+TaD^2]W\\&+a^2Ra_T(\frac{\sigma}{Pr_D}-C(D^2-a^2)+1)\Theta\\&-a^2Ra_S(\frac{\sigma}{Pr_D}-C(D^2-a^2)+1)\Phi=0\end{aligned} \quad , \qquad (19)$$

$$[\sigma-(D^2-a^2)-R_i]\Theta-W=0 \quad , \qquad (20)$$

$$[\chi\sigma-\frac{1}{Le}(D^2-a^2)]\Phi=0 \quad , \qquad (21)$$

Whrere, $D\equiv d/dz$ and $a^2=l^2+m^2$. The boundary conditions (17) now becomes

$$W=\frac{\partial^2W}{\partial z^2}=\Theta=\Phi=0 \qquad \text{at } z=0,1. \qquad (22)$$

We assume the solutions of Eqs. (19)-(21) satisfying the boundary conditions (22) in the form

$$(W(z),\Theta(z),\Phi(z))=(W_0,\Theta_0,\Phi_0)Sin(n\pi z),$$
$$(n=1,2,3,......)\,. \qquad (23)$$

The most unstable mode corresponds to $n=1$ (fundamental mode). Therefore, substituting Eq. (23) with $n=1$ into Eqs. (19)- (21), we obtain a matrix equation

$$\begin{bmatrix}-(\frac{\sigma}{Pr_D}+\eta)^2\delta^2-\pi^2Ta & a^2Ra_T(\frac{\sigma}{Pr_D}+\eta) & -a^2Ra_S(\frac{\sigma}{Pr_D}+\eta)\\ -1 & (\sigma+\delta^2-R_i) & 0\\ -1 & 0 & \chi\sigma+\frac{1}{Le}\delta^2\end{bmatrix}\begin{pmatrix}W_0\\ \Theta_0\\ \Phi_0\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$
(24)

where, $\delta^2=\pi^2+a^2$.

The condition of a non-trivial solution of the above system of homogeneous linear Eq. (24) yields the expression of thermal Rayleigh number in the form

$$Ra_T = \frac{Le\,Pr_D(\delta^2+\sigma-R_i)[-(\frac{\delta^2}{Le}+\chi\sigma)\times(-\pi^2 Ta-\delta^2(\eta+\frac{\sigma}{Pr_D})^2)+a^2(\eta+\frac{\sigma}{Pr_D})Ra_S]}{a^2(\delta^2+Le\chi\sigma)(\sigma+\eta Pr_D)}, \tag{25}$$

2.1 STATIONAR STATE

For the validity of the principal of exchange of stabilities (i.e., steady oscillatory), we have $\sigma=0$ (i.e. $\sigma_r=\sigma_i=0$) at the marginal of stability. Then the Rayleigh number at which the marginally stable steady mode exists becomes

$$Ra_T^{St} = \frac{(\delta^2-R_i)(\pi^2 Ta\delta^2+\delta^4\eta^2+a^2 Le\eta Ra_S)}{a^2\delta^2\eta}, \tag{26}$$

In the absence of heat source $(i.e., R_i=0)$, Eq. (26) reduces to

$$Ra_T^{St} = \frac{(\pi+a^2)^2(1+C(\pi+a^2))}{a^2}+\frac{Ta\pi^2(\pi+a^2)}{a^2(1+C(\pi+a^2))}+LeRa_S, \tag{27}$$

This result exactly coincides with the one given by [17].

It is important to note that the critical wavenumber a_c^{St} depends on the couple stress parameter and Taylor number. In the absence of Taylor number $(i.e., Ta=0)$, Eq. (27) gives

$$Ra_T^{St} = \frac{1}{a^2}(\pi^2+a^2)^2[1+C(\pi^2+a^2)]+LeRa_S, \tag{28}$$

Which is the result given by [12]. For single component fluid $Ra_S=0$, in Eq. (27) gives

$$Ra_T^{St} = \frac{(\pi^2+a^2)^2(1+C(\pi^2+a^2))}{a^2}+\frac{Ta\pi^2(\pi^2+a^2)}{a^2(1+C(\pi^2+a^2))}, \tag{29}$$

Which is the one obtained by [18]. When $C=0$ (i.e., Newtonian fluid case), Eq. (29) reduces to

$$Ra_T^{St} = \frac{(\pi^2+a^2)^2}{a^2}+\frac{Ta\pi^2(\pi^2+a^2)}{a^2}. \tag{30}$$

This coincides with the results of [24]. Further $Ta=0$, Eq.(30) gives

$$Ra_T^{St} = \frac{(\pi^2+a^2)^2}{a^2}, \tag{31}$$

which has the critical value $Ra_T^{St}=4\pi^2$ for $a_c^{St}=\pi^2$ obtained by [15] and [16].

2.2 OSCILLATORY STATE

We now set $\sigma=i\sigma_i$ in Eq. (25) and clear the complex quantities from the denominator, to obtain

$$Ra_T = \Delta_1 + i\sigma_i\Delta_2, \tag{32}$$

Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (32) it follows that either $\sigma_i=0$ or $\Delta_2=0$ ($\sigma_i\neq 0$, oscillatory onset). For oscillatory onset, setting $\Delta_2=0$ $(\sigma_i\neq 0)$ gives an expression for frequency of oscillations in the form (on dropping the subscript i)

$$w^2 = \frac{Pr_D((\delta^2-R_i)(2\delta^4\eta+a^2 LeRa_S)}{\delta^4+Le\delta\,(\delta^2+2\eta Pr_D-R_i)\chi} + \frac{Pr_D(a^2 Le\eta Ra_S+\pi^2 Ta+\delta^2\eta^2)(\delta^2+Le(\delta^2-R_i)\chi)))}{\delta^4+Le\delta\,(\delta^2+2\eta Pr_D-R_i)\chi} \tag{33}$$

Now Eq. (32) with $\Delta_2=0$, gives

$$Ra_T^{Osc} = Pr_D(\delta^2-R_i)(\pi^2 Ta\delta^2+\delta^4\eta^2+a^2 Le\eta Ra_S) + \frac{Le\delta^2\chi}{Pr_D}w^4 + [(Le(\pi^2 Ta+\delta^2\eta^2)Pr_D^2\chi-\delta^6-\delta^4 R_i) + Pr_D(a^2 LeRa_S+2\delta^2\eta(\delta^2+Le(\delta^2-R_i)\chi))]w^2 \tag{34}$$

The analytical expression for the oscillatory Rayleigh number given by Eq. (34) is minimized with respect to the wavenumber numerically, after substituting for $w^2(>0)$ from Eq. (33), for various values of physical parameters in order to know their effects on the onset of stationary and oscillatory convection.

3. RESULT AND DISCUSSION

The effect of rotation on the onset of double diffusive convection (DDC) in a horizontal couple stress fluid saturated porous layer with an internal heat source is investigated using linear stability analysis. The linear stability analysis is based on the classical normal mode technique. Only the linear part has considered in this paper.

The neutral stability curves in the $Ra_T - a$ plane for various parameter values are as shown in Figs.1-6. We fixed the values for the parameters except the varying parameter. From these figures it is clear that the neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number Ra_{Tc}, below which the system is stable and unstable above.

In Fig. 1 the marginal stability curves for different values of couple stress parameter C are drawn. It is observed that with the increase of C the values of Rayleigh number and the corresponding wavenumber for oscillatory mode decreases while those for stationary mode increases. Therefore, the effect of C is to advance the onset of oscillatory convection while its effect is to inhibit the stationary convection.

Fig. 2 depicts the effect of Taylor number Ta on the neutral stability curves. We find that the effect of increasing Ta is to increase the value of the Rayleigh number for stationary and oscillatory modes and the corresponding wavenumber. Thus the Taylor number Ta has a stabilizing effect on the double diffusive convection in a horizontal couple stress fluid saturated porous layer with an internal heat source.

Fig. 3 indicates the effect of internal Rayleigh number R_i on the neutral stability curves for the fixed values of other parameters. It is observed that the value of the Rayleigh number for stationary and oscillatory mode increases with increasing R_i, indicating that the effect of R_i is to inhibit the onset of stationary and oscillatory convection.

In Fig. 4 the marginal stability curves for different values of Lewis number Le are drawn. It is observed that with the increase of Le the values of Rayleigh number and the corresponding wavenumber for oscillatory mode decreases while those for stationary mode increases. Therefore, the effect of Le is to advance the onset of oscillatory convection while its effect is to inhibit the stationary convection.

Fig. 5 depicts the effect of solute Rayleigh number Ra_S on the neutral stability curves for stationary and oscillatory modes. We find that the effect of increasing Ra_S is to increase the critical value of the Rayleigh number for stationary and oscillatory modes and the corresponding wavenumber. Thus the solute Rayleigh number Ra_S has a stabilizing effect on the double diffusive convection in a horizontal couple stress fluid saturated porous layer with an internal heat source.

The effect of normalized porosity parameter χ is depicted in the Fig. 6. We find that an increase in χ decreases the minimum of the Rayleigh number for oscillatory mode, indicating that the effect of increasing χ is to advance the onset of oscillatory convection.

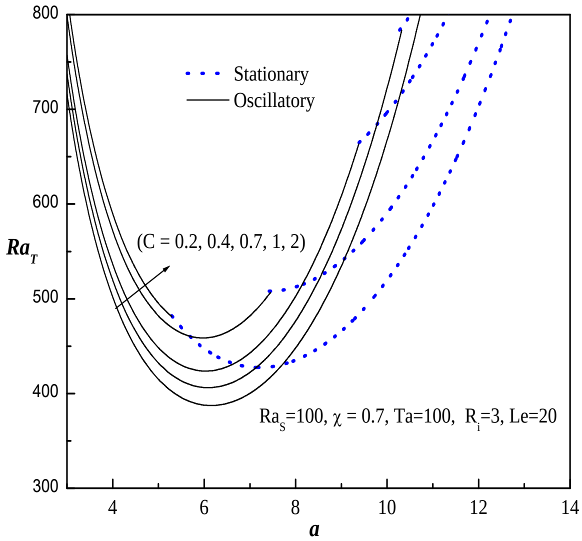

Fig. 1. Neutral stability curves for differents values of C.

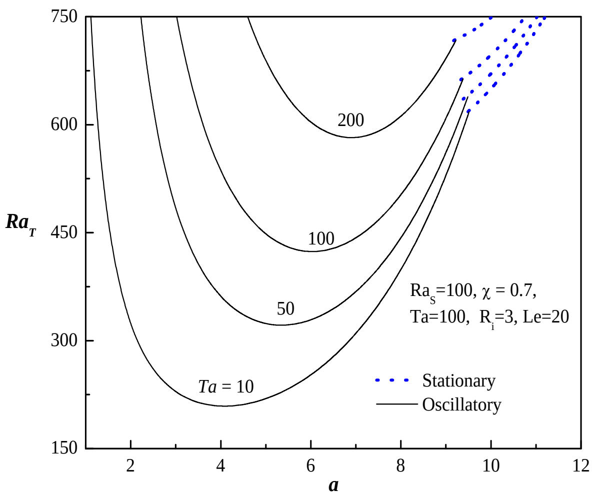

Fig. 2. Neutral stability curves for differents values of *Ta*.

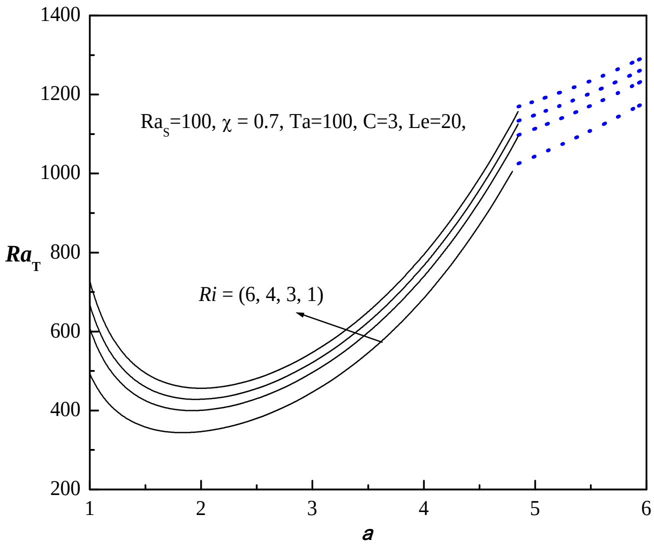

Fig.3. Neutral stability curves for different values of internal Rayleigh number *Ri*.

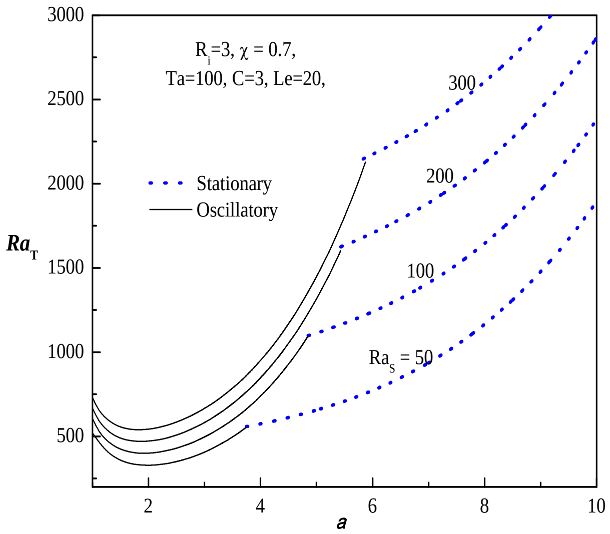

Fig. 5. Neutral stability curves for different values of solute Rayleigh number Ra_S.

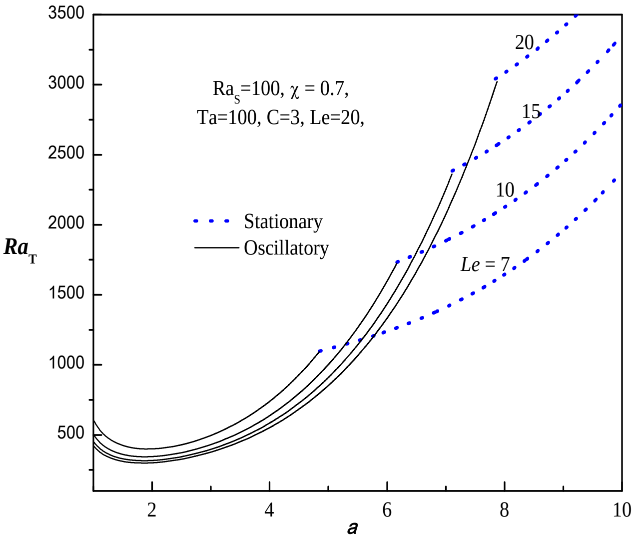

Fig. 4. Neutral stability curves for different values of Lewis number *Le*.

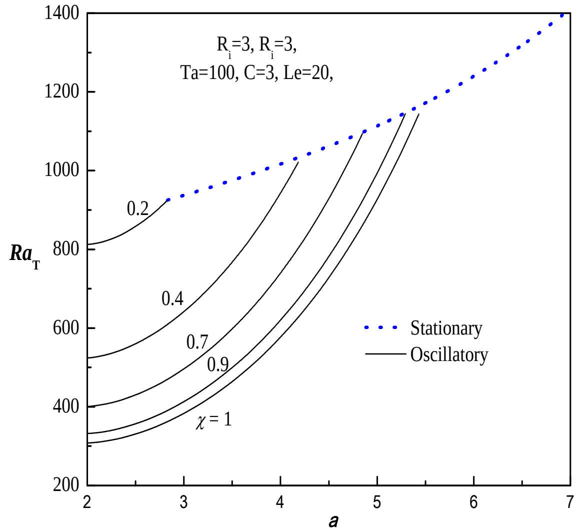

Fig.6. Neutral stability curves for different values of normalized porosity λ.

5. CONCLUSION

The effect of rotation on the onset of double diffusive convection (DDC) in a horizontal couple stress fluid saturated porous layer with an internal heat source is investigated using linear stability analysis. The linear stability analysis is based on the classical normal mode technique. The following conclusions are drawn: The Taylor number Ta has a stabilizing effect on the double diffusive convection in a horizontal couple stress fluid saturated porous layer with an internal heat source. The effect of solute Rayleigh number Ra_S is to delay both stationary and oscillatory convection. And the effect of Lewis number Le is to delay the onset of stationary convection while it advances the oscillatory convection. The internal Rayleigh number R_i has a destabilizing effect on the double diffusive convection in a porous medium. The effect of couple stress parameter C is to advance the onset of oscillatory convection whereas its effect is to inhibit the stationary onset. The normalized porosity parameter χ has a destabilizing effect in the case of oscillatory mode.

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