

ULTRA NOVEL FOURIER INNOVATIVE OPTIMIZATION TECHNIQUE FOR ANALYSIS OF FERMATEAN FUZZY OPERATORS WITH SOLID TRANSPORTATION PROBLEM

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Abstract - Fermatean fuzzy set is an effective and immense of exploration. The Fermatean fuzzy set acquire more fortunate optimal solution than Intuitionistic fuzzy set and Pythagorean fuzzy set. It is manipulated extensively in numerous area of expertise. They are contemplated as efficacious apparatus for assess the specified information. Hamacher Operator is one of the trenchant operators. It is prominent to shrink the influence of refusal particulars. It contribute more authentic outcome. We solicit FFIWG, FFEIWG operators on Fermatean fuzzy sets to acquire satisfactory choice. The scope of research solid transportation problem is an extensiveness of conventional transportation problem. In this paper, Fermatean fuzzy solid transportation problem is build on the basis of Fermatean fuzzy number. The two prominent operators in particular Fermatean Fuzzy Iterative Weighted Geometric [FFIWG] operator and Fermatean Fuzzy Einstein Iterative Weighted Geometric [FFEIWG] operator are implemented to acquire the rebuild Fermatean fuzzy conventional transportation problem. A novel defuzzification function for Fermatean fuzzy number is elucidated and utilized to acquire crisp transportation problem. A new ultra-novel Fourier Elimination method harnessing FFIWG, FFEIWG operators is proposed. The proposed technique is described with numerical values.

Key Words: Fermatean fuzzy set, Fermatean fuzzy solid transportation problem, Fourier Elimination method, FFIWG, FFEIWG operators.

1. INTRODUCTION

In 1986, Atanassov [1] introduced the approach of Intuitionistic fuzzy sets in which the acceptance grade and non-acceptance grade is described and gratifying the limitation that the sum of acceptance grade and non- acceptance grade is confine to unity. In 2019 Zhuang [12] ; 2021 Rani [9]; 2021 Hao [7]; based on this solitary utility IFS has performed as particular beneficial technique for indicating unreliability and ambiguity of actual issue. In 2014, Yagar [11] determined the modernized form of IFS, characterized by the acceptance grade and non- acceptance grade and non- acceptance grade and satisfied the restriction that the square sum of acceptance grade and non- acceptance grade limited to unity, is known as the approach of Pythagorean Fuzzy. In 2020, Guleria and Bajaj [6] newly suggested a contemporary (R,S) norm report established mixture decision-making framework for PFS and further used it for unravelling an MCDM issue. In 2019, Senapati and Yagar [11] presented the concept of Fermatean fuzzy set, to characterize this consequence. They conferred fundamental functioning regulations for Fermatean fuzzy sets and extended score and accuracy procedures for Fermatean fuzzy numbers. They also offered a fiction decision-making framework underneath Fermatean fuzzy sets circumstances and also generated some weighted averaging and weighted geometric operators for Fermatean fuzzy sets, for their application in MCDM situations. In 2020, Aydemir and Gunduz [3] expanded the Dombi Aggregation operators for Fermatean fuzzy sets. In a current investigation in 2020, Akram [2] proposed a new Aggregation operators established Fermatean fuzzy based decision making framework for assessing the numerous appropriate sanitizer to diminish COVID-19 condition. Later, In 2020, Garg [4] suggested a series of Fermatean fuzzy aggregation operators and further utilized them to solve the COVID-19 establishment evaluation issue. In 2020, Ghorabae [5] suggested a Fermatean fuzzy established judgment approval technique for assessing and determining the most acceptable feasible structure supplier.

2. PRELIMINARIES

Definition 2.1: Let χ be a set of universe. Let ω be the Fermatean fuzzy set contained in χ i.e, $\omega \leq \chi$. Then the non-empty Fermatean fuzzy set is characterized by

$$\omega = \{ \langle y, \sigma_{\omega}(y), \delta_{\omega}(y) \rangle \}$$
(1)

Where $\sigma_\omega(y): \chi \rightarrow [0,1]$, $\delta_\omega(y): \chi \rightarrow [0,1]$ and $\varphi_\omega(y) = \sqrt[3]{1 - (\sigma_\omega(y))^3 - (\delta_\omega(y))^3}$ specify the acceptance degree, non-acceptance degree and hesitance degree respectively.

Definition 2. 2: The proficient function for a Fermatean fuzzy set $\omega = (\sigma_\Omega(y), \delta_\Omega(y))$ is given by

$$P(\Omega) = (\sigma_\Omega)^3 - (\delta_\Omega)^3, P(\Omega) \in [-1,1] \tag{2}$$

and the Exactness function for a Fermatean fuzzy set is given by

$$E(\Omega) = (\sigma_\Omega)^3 + (\delta_\Omega)^3, E(\Omega) \in [0,1] \tag{3}$$

Definition 2.3: Let $\Omega_r = \langle \sigma_r, \delta_r \rangle, r = 1, 2, \dots, n$ be a collection of Fermatean Fuzzy sets and $w = (w_1 w_2 \dots w_n)^T$ represents the weight vector such that each $w_r > 0$ and $\sum_{r=1}^n w_r = 1$, then

$FFHIWG(\Omega_1, \Omega_2, \dots, \Omega_n) = (w_1 \Omega_1 \otimes w_2 \Omega_2 \otimes \dots \otimes w_n \Omega_n)$ is defined as,

$$FFHIWG(\Omega_1, \Omega_2, \dots, \Omega_n) = \left\langle \sqrt[3]{\frac{u \left\{ \prod_{r=1}^n (1 - \sigma_r^3)^{w_r} - \prod_{r=1}^n (1 - \delta_r^3 - \sigma_r^3)^{w_r} \right\}}{\prod_{r=1}^n (1 + (u-1)\delta_r^3)^{w_r} + (u-1) \prod_{r=1}^n (1 - \delta_r^3)^{w_r}}}, \sqrt[3]{\frac{\prod_{r=1}^n (1 + (u-1)\delta_r^3)^{w_r} - \prod_{r=1}^n (1 - \delta_r^3)^{w_r}}{\prod_{r=1}^n (1 + (u-1)\delta_r^3)^{w_r} + (u-1) \prod_{r=1}^n (1 - \delta_r^3)^{w_r}}} \right\rangle$$

Case (i): For $u = 1$, FFHIWG operator is turn into FFIWG operator characterized by,

$$FFIWG(\Omega_1, \Omega_2, \dots, \Omega_n) = \left\langle \sqrt[3]{\prod_{r=1}^n (1 - \delta_r^3)^{w_r} - \prod_{r=1}^n (1 - \delta_r^3 - \sigma_r^3)^{w_r}}, \sqrt[3]{1 - \prod_{r=1}^n (1 - \delta_r^3)^{w_r}} \right\rangle$$

Case (ii): For $u = 2$, FFHIWG operator is turn into FFEIWG operator characterized by,

$$FFEIWG(\Omega_1, \Omega_2, \dots, \Omega_n) = \left\langle \sqrt[3]{\frac{2 \left\{ \prod_{r=1}^n (1 - \sigma_r^3)^{w_r} - \prod_{r=1}^n (1 - \delta_r^3 - \sigma_r^3)^{w_r} \right\}}{\prod_{r=1}^n (1 + \delta_r^3)^{w_r} + \prod_{r=1}^n (1 - \delta_r^3)^{w_r}}}, \sqrt[3]{\frac{\prod_{r=1}^n (1 + \delta_r^3)^{w_r} - \prod_{r=1}^n (1 - \delta_r^3)^{w_r}}{\prod_{r=1}^n (1 + \delta_r^3)^{w_r} + \prod_{r=1}^n (1 - \delta_r^3)^{w_r}}} \right\rangle$$

Definition 2.4: A typical Fermatean fuzzy transportation problem is of the form: $Minimize \Delta = \sum_{r=1}^m \sum_{s=1}^n \tilde{P}_{rs} y_{rs}$

Subject to the constraint,

$$\sum_{r=1}^n y_{rs} = \kappa_r, r = 1, 2, \dots, m \tag{4}$$

$$\sum_{r=1}^m y_{rs} = \lambda_s, s = 1, 2, \dots, n \tag{5}$$

$$y_{rs} \geq 0, \forall r, s \text{ and integer.} \tag{6}$$

Where \tilde{P}_{rs} is the Fermatean fuzzy cost of transporting one component of commodities from provenance (r) to the terminus (s). y_{rs} is the consignment transportation from provenance (r) to the terminus (s). κ_r is the total accessibility of the commodity at provenance (r). λ_s is the total requirement of the commodity at terminus (s). $\sum_{i=1}^m \sum_{j=1}^n \tilde{P}_{rs} y_{rs}$ is the

total Fermatean fuzzy transportation cost. If $\sum_{r=1}^m \kappa_r = \sum_{s=1}^n \lambda_s$, then Fermatean fuzzy transportation problem is a balanced

Fermatean fuzzy transportation problem. If $\sum_{r=1}^m \kappa_r \neq \sum_{s=1}^n \lambda_s$, then Fermatean fuzzy transportation problem is an unbalanced Fermatean fuzzy transportation problem.

3. MAIN RESULT

Definition 3.1: In this session, a new defuzzification function is introduced to obtain the crisp values for the Fermatean fuzzy number. Use of defuzzification function results in the powerful way to correlate the Fermatean Fuzzy Number.

$(Df)_{FFN}$ is a defuzzification function defined to acquire crisp value for the Fermatean Fuzzy Number(FFN),

$$(Df)_{FFN} : FF(R) \rightarrow R \text{ is defined by } (Df)(\sigma, \delta) = |\sigma^3 - \delta^3|. \tag{7}$$

3.2 Fermatean fuzzy solid transportation problem: A typical Fermatean fuzzy Solid transportation problem is of the form

$$\text{Minimize } \Delta = \sum_{r=1}^m \sum_{s=1}^n \sum_{t=1}^l \tilde{P}_{rst} y_{rst} \tag{8}$$

Subject to the constraint,

$$\sum_{s=1}^n \sum_{t=1}^l y_{rst} = \kappa_r, r = 1, 2, \dots, m \tag{9}$$

$$\sum_{r=1}^m \sum_{t=1}^l y_{rst} = \lambda_s, s = 1, 2, \dots, n \tag{10}$$

$$\sum_{r=1}^m \sum_{s=1}^n y_{rst} = \theta_t, t = 1, 2, \dots, l \tag{11}$$

Where \tilde{P}_{rst} is the Fermatean fuzzy cost of transporting one component of commodities from provenance (r) to the terminus (s) through shipment (t). y_{rst} is the consignment transportation from provenance (r) to the terminus (s) through shipment (t). κ_r is the total accessibility of the commodity at provenance (r). λ_s is the total requirement of the commodity at terminus (s). θ_t is the total shipment of the commodity from the provenance (r) to the terminus (s).

$\sum_{r=1}^m \sum_{s=1}^n \sum_{t=1}^l \tilde{P}_{rst} y_{rst}$ is the total Fermatean fuzzy transportation cost.

$$\begin{aligned} \sum_{s=1}^n \kappa_{st} &= \sum_{r=1}^m \lambda_{tr}, \sum_{t=1}^l \lambda_{tr} = \sum_{s=1}^n \theta_{rs}, \sum_{r=1}^m \theta_{rs} = \sum_{t=1}^l \kappa_{st} \\ \sum_{s=1}^n \sum_{t=1}^l \kappa_{st} &= \sum_{t=1}^l \sum_{r=1}^m \lambda_{tr} = \sum_{r=1}^m \sum_{s=1}^n \theta_{rs} \end{aligned}$$

$$y_{rst} \geq 0, \forall r, s, t; r = 1, 2, \dots, m; s = 1, 2, \dots, n; t = 1, 2, \dots, l. \tag{12}$$

The Fermatean fuzzy solid transportation table for r=s=t=3, i.e., for 3 provenance, 3 terminus through 3 shipments is given by,

FERMATEAN FUZZY SOLID TRANSPORTATION TABLE

TABLE 1

Shipment	S ₁			S ₁			S ₁			θ ₁
		S ₂			S ₂			S ₂		θ ₂
			S ₃			S ₃			S ₃	θ ₃
Provenance	T ₁			T ₂			T ₃			Accessibility
P ₁	\tilde{P}_{111}	\tilde{P}_{112}	\tilde{P}_{113}	\tilde{P}_{121}	\tilde{P}_{122}	\tilde{P}_{123}	\tilde{P}_{131}	\tilde{P}_{132}	\tilde{P}_{133}	κ ₁

P ₂	\tilde{P}_{211}	\tilde{P}_{212}	\tilde{P}_{213}	\tilde{P}_{221}	\tilde{P}_{222}	\tilde{P}_{223}	\tilde{P}_{231}	\tilde{P}_{232}	\tilde{P}_{233}	κ_2
P ₃	\tilde{P}_{311}	\tilde{P}_{312}	\tilde{P}_{313}	\tilde{P}_{321}	\tilde{P}_{322}	\tilde{P}_{323}	\tilde{P}_{331}	\tilde{P}_{332}	\tilde{P}_{333}	κ_3
Requirement	λ_1			λ_2			λ_3			$\sum \kappa_r = \sum \lambda_s = \sum \theta_t$

3.3 A proposed Ultra novel Fourier innovative optimization technique using Fermatean fuzzy operators with solid transportation problem:

Step 1: Construct the Mathematical Fermatean fuzzy Solid transportation table for the given situation or consider the given FFSTP.

Step 2: Render the appropriate significant value (weight) v_1, v_2, \dots, v_n for all the shipment (κ) so that $\sum_{r=1}^n v_r = 1$.

Step 3: For the FFSTP in Step 1, reconstruct the conventional transportation problem by using the novel Fermatean fuzzy operators such as, Fermatean Fuzzy Iterative Weighted Geometric Operator [FFIWGO] or Fermatean Fuzzy Einstein Iterative Weighted Geometric Operator [FFEIWGO]. Thus, obtain the reconstructed Fermatean fuzzy transportation problem in which the costs are Fermatean fuzzy number.

Step 4: For the Fermatean fuzzy conventional transportation acquire the reformulated crisp Fermatean fuzzy transportation problem using the futuristic defuzzification functions specified by, $(Df)(\sigma, \delta) = |\sigma^3 - \delta^3|$

Where σ and δ expresses the acceptance and non-acceptance function of Fermatean fuzzy number.

Step 5: Multiply each cost in the crisp FFTP with the corresponding Neutralization seminal constant to obtain the Conventional TP known as advanced Conventional transportation problem or Ultra Novel conventional transportation problem.

Step 6: Check the Ultra Novel Conventional transportation problem obtain in Step 5 is either balanced or unbalanced. If it is balanced, then follow case (i) otherwise follow case (ii).

Case (i): If the UNCTP is balanced, then construct the pure Integer LPP for the UNCTP obtained in Step 5 given by,

$$\text{Minimize } \Delta = \sum_{r=1}^m \sum_{s=1}^n P_{rs} y_{rs}$$

$$\text{Subject to the constraint, } \sum_{s=1}^n y_{rs} = \kappa_r, r = 1, 2, \dots, m \tag{13}$$

$$\sum_{r=1}^m y_{rs} = \lambda_s, s = 1, 2, \dots, n, y_{rs} \geq 0, \forall r, s \text{ and integer.} \tag{14}$$

Case (ii): If the UNCTP is unbalanced, then construct the pure integer LPP with mixed constraints for the UNCTP obtained in Step 5. Go to next Step.

Step 7: Obtain the correlative maximization problem given by

$$\text{Maximize } \Delta^* = \text{Minimize}(-\Delta) = -\sum_{r=1}^m \sum_{s=1}^n P_{rs} y_{rs} \tag{15}$$

Step 7a: (i) Reformulate the Maximization PILPP for (15) using the subject to the constraint $\sum_{r=1}^m \sum_{s=1}^n P_{rs} y_{rs} + \Delta^* \leq 0$ and

also constraints (9), (10) and (11) in case of Step 6 case (i) or constraints from of the unbalanced TP of Step 6 case (ii) are satisfied.

(ii) Form the Fourier elimination table for the PILPP acquired in step (7a)(i).

Step 7b: Now using Modified Fourier Elimination Method and theorem 3.2 choose and extract variable from the Fourier Elimination table formulated in Step(7a)(ii). Also neglect the inefficient constraint (if any) from the table.

Step 8: Follow the Steps (7a), (7b) upto all variables of y_{rs} are neglected from the table excluding the objective function variable w.

Step 9: Compute the supreme value of all maximum feasible estimate of w represented by Δ^* which is the optimum solution w. Now, compute all y_{rs} using backward substituting method and basic Algebraic method. Say y_{rs}^i ; $r = 1, 2, \dots, m$ and $s = 1, 2, \dots, n$.

Step 10: Thus, the obtained optimal solution for the Fermatean fuzzy solid transportation problem is given by $y_{rs} = y'_{rs}, r = 1, 2, \dots, m, s = 1, 2, \dots, n$ and $Minimize \Delta = -\Delta^* \text{ or } |\Delta^*|$. Stop the process.

4. NUMERICAL EXAMPLE

Consider the following Fermatean fuzzy Solid Transportation Problem

TABLE 5

Shipment	S ₁			S ₁			S ₁			11
		S ₂			S ₂			S ₂		14
			S ₃			S ₃			S ₃	9
Provenance	T ₁			T ₂			T ₃			Accessibility
P ₁	(0.4,0.978)	(0.7,0.869)	(0.8,0.787)	(0.3,0.991)	(0.9,0.647)	(0.7,0.869)	(0.6,0.922)	(0.2,0.997)	(0.7,0.869)	11
P ₂	(0.4,0.978)	(0.2,0.997)	(0.6,0.922)	(0.1,0.997)	(0.3,0.991)	(0.8,0.787)	(0.8,0.787)	(0.4,0.978)	(0.5,0.957)	13
P ₃	(0.8,0.787)	(0.1,0.977)	(0.3,0.991)	(0.4,0.978)	(0.7,0.869)	(0.3,0.991)	(0.5,0.957)	(0.6,0.922)	(0.4,0.978)	10
Requirement	7			15			12			34

Solution:

Method-I: In this method, the optimum solution for the Fermatean Fuzzy Solid Transportation Problem is obtained by using FFIWG operator and the Defuzzification function (*Df*).

Step 1: Consider the given Fermatean Fuzzy Solid Transportation Problem.

Step 2: Allot appropriate weights v_1, v_2 and v_3 for all the shipments as $v_1 = 0.4, v_2 = 0.3, v_3 = 0.3$, so that $v_1 + v_2 + v_3 = 0.4 + 0.3 + 0.3 = 1$.

By using step 3 to step 10, we get, Compute the value of $y_{rs}, r = 1, 2, \dots, m, s = 1, 2, \dots, n$ using backward substitution method and basic algebraic method, we get,

$$y_{11} = 7, y_{12} = 4, y_{13} = 0, y_{21} = 0, y_{22} = 1, y_{23} = 12, y_{31} = 0, y_{32} = 10, y_{33} = 0.$$

Thus, the optimal solution obtained is $\Delta = 231$.

Method II: In this method, the optimum solution for the Fermatean Fuzzy Solid Transportation Problem is obtained by using FFEIWG operator and the defuzzification function (*Df*).

Step 1: Consider the given Fermatean Fuzzy Solid Transportation Problem.

Step 2: Allot appropriate weights v_1, v_2 and v_3 for all the shipments as $v_1 = 0.4, v_2 = 0.3, v_3 = 0.3$, so that $v_1 + v_2 + v_3 = 0.4 + 0.3 + 0.3 = 1$.

By using step 3 to step 10, we get, the optimal allocations $y_{11} = 7, y_{12} = 4, y_{22} = 1, y_{23} = 12, y_{32} = 10$ and the optimum solution is given by, $y_{11} = 7, y_{12} = 4, y_{13} = 0, y_{21} = 0, y_{22} = 1, y_{23} = 12, y_{31} = 0, y_{32} = 10, y_{33} = 0$ and $\Delta = 231$.

5. Computational Result comparison and analysis:

Comparative Analysis of Computational result of METHOD-I and METHOD-II with manifold methodologies:

The MATLAB graphical comparative result analysis of METHOD-I and METHOD-II is given by,

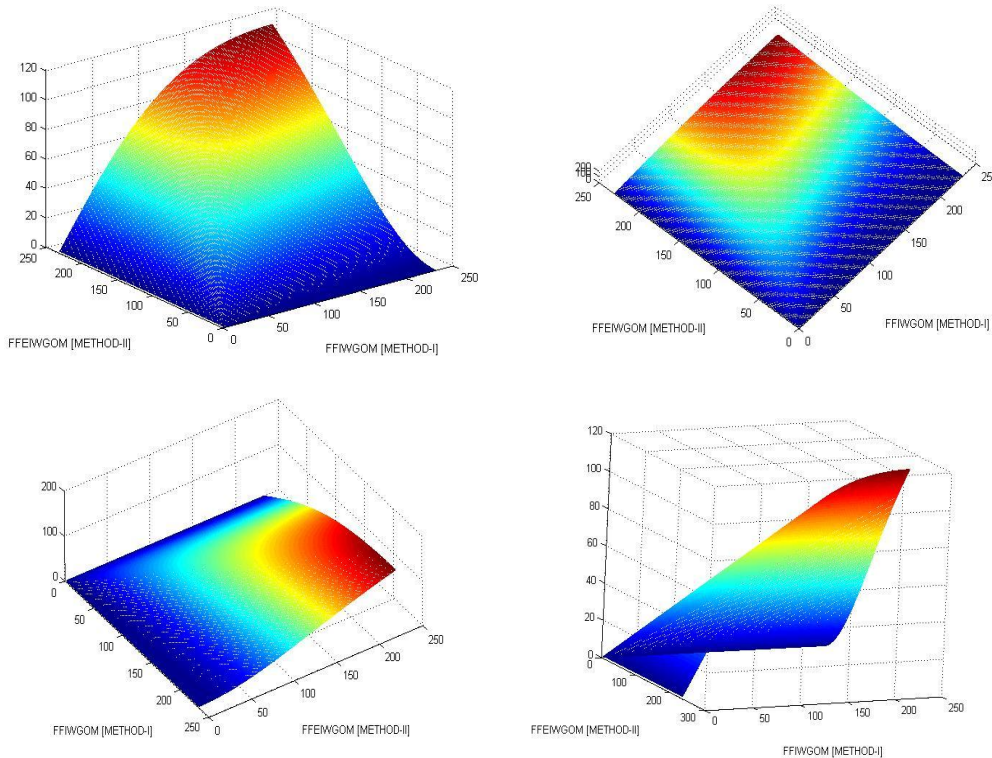


TABLE 3

S. No	METHODS	COMPUTATIONAL RESULTS
1.	Vogel's Approximation Method	231
2.	North West Corner Rule Method	261
3.	Least Cost Method	231
4.	Row Minima Method	231
5.	Column Minima Method	231
6.	Russell's Approximation Method	231
7.	Heuristic Method	231
8.	MODI Method	231
9.	Stepping Stone Method	231
10.	FFIWGO Method (Method-I)	231
11.	FFEIWGO Method (Method-II)	231

CONCLUSION

We have built Fermatean Fuzzy solid transportation on the basis of Fermatean fuzzy number. The two prominent operators in particular Fermatean fuzzy iterative weighted geometric operator has been implemented to acquire the reformulated Fermatean fuzzy conventional transportation problem, A novel defuzzification function for Fermatean fuzzy number is elucidated and utilized to acquire crisp transportation problem. A new Ultra novel Fourier innovative technique involving Fourier elimination table and modified Fourier elimination method harnessing FFIWG and FFEIWG operators have proposed. The proposed technique has been described with numerical values. For prospective research, we augment this methodology to multi objective solid transportation problems and generalized solid transportation problem and furthermore this method can be applied in miscellaneous research fields and correspondingly to scrutinize and unravel the real life tribulations.

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