A Stochastic Approach to Determine Along-Wind Response of Tall Buildings

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Abstract: Wind velocity is turbulent in nature and hence its variation with time is random. The interaction of the turbulent wind with the building generates random vibrations in the building. When the frequency of the turbulent wind force comes in line with the natural frequency of the building, it produces resonant amplification of the building response that is undesirable for structural safety and dwellers comfort. This paper probes the application of theory of random vibration in determining the peak along wind response of a conventional tall building, rectangular in configuration. Velocity power spectral density function is a focal point in mathematical modeling of turbulent wind velocity. Different expressions for velocity power spectral density function were proposed by Simiu, Harris and Kaimal. They are used to derive corresponding displacement power spectral density functions. A peak in the displacement power spectral density function plot is observed at a frequency equal to the natural frequency of the building. The expected value of peak along wind response is calculated using principles of random vibration theory. Numeric computing software MATLAB is used for solving the mathematical equations involved in the analysis. All the three velocity power spectral density functions produced nearly equal peak displacement at the tip of the building wherein, Harris velocity power spectral density function has an edge over the others. A brief comparison of all the velocity power spectral density functions is also presented.

Keywords: Wind analysis, Power Spectral Density function, Random vibration, Stochastic process, Tall buildings, Along wind response

1. Introduction

Tall buildings are vulnerable to heavy sway caused by the wind. Basically, wind flow is designated with its speed or velocity. The flow of wind in a terrain can be described using the boundary layer theory, where the ground acts as a viscous boundary layer. The wind flow is naturally turbulent. But, there is a mean wind speed, be it hourly mean or 3-sec averaged, that carries the turbulent wind. Thus, one can say that, the wind speed is fluctuating randomly about its mean wind speed. Fig. 1 shows a typical wind velocity time history. Thus one can find a scope in applying the random vibration theory to analyze flow properties of wind. As per that theory, any periodic function having a definite periodicity can be written as summation of sines and cosines of different frequencies which is well known as fourier expansion of periodic functions. It even can be extended to non periodic functions. A random process can explicitly be defined as a periodic function with infinite periodicity. A random process is said to be stationary if its statistical properties (mean, standard deviation, etc) doesn't change with time [1]. Wind velocity time history is assumed to be a stationary random process about the mean speed.

Velocity power spectral density function (PSD) has been a pivot point in mathematical modeling of wind velocity. As mentioned earlier, in a typical wind velocity time history, the velocity at any point of time can be decomposed as mean wind velocity (be it hourly mean, 3-sec averaged, etc) summed up with fluctuating component of velocity about the mean value. Now, using fourier expansion, this fluctuating component, at any instant of time can be written as the summation of eddies of different frequencies (circular frequencies). Thus each eddy has a definite wave number. The velocity of eddy per unit wave number is called as velocity power spectral density function. Mathematically it is the spectrum of power of fourier coefficients of velocity fluctuations for different frequencies. Various expressions for velocity power spectral density function were proposed by many researchers, of which expressions proposed by Simiu, Harris and Kaimal are used here. Thus, in an average wind storm of 1hr duration, at a given height, the expected value of peak wind velocity can be obtained using velocity power spectral density function from stochastic approach [2]. Extreme winds resulted from thunder storms are non stationary and are dealt under non stationary random vibration theory [3].

As a consequence of stochastic wind velocity, the response of a structure to wind is also random with time. Hence one can define displacement power spectral density function as the displacement produced by each eddy per unit wave number. As each of these eddies possess a unique frequency, the interaction of each of these eddy frequency with the natural frequency of the building can be visualized with the help of displacement power spectral density function. Thus, expected value of peak displacement response, which is produced as a result of resonance between the wind force and flexibility of the building, is obtained using displacement power spectral density function. Displacement power spectral density functions can be derived from velocity power spectral density functions using the principles of random vibration. Many international codes have implicitly specified formulations, for calculating peak response, in terms of Gust factor or dynamic response factor, which represents the ratio of peak response to mean response of the building. Australian code [4] and IS code of 2015 [5] has dynamic response factor, Whereas, Canadian code has gust effect factor [6].

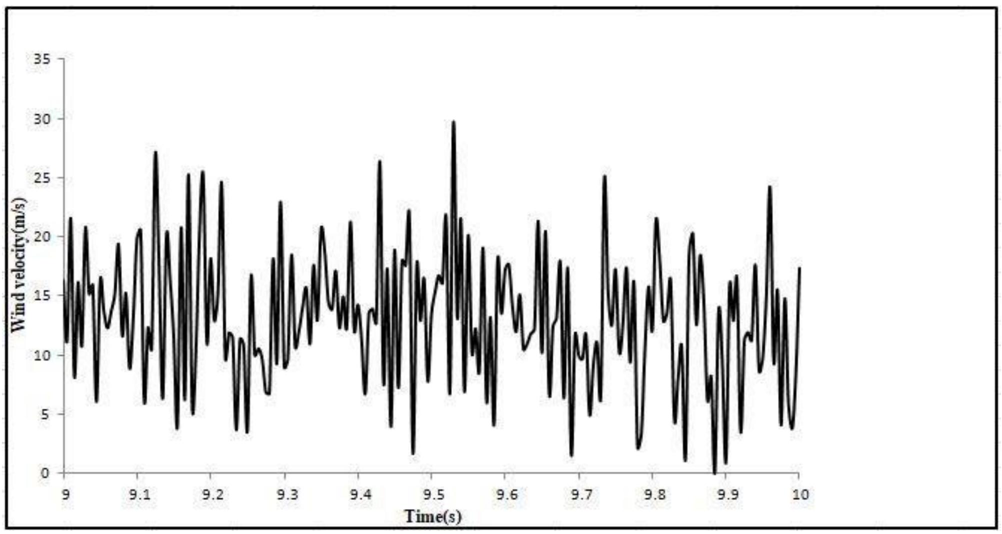

Fig. 1 Typical wind velocity time history [10]

Codes have used those factors in calculating peak wind forces and hence those forces are displacement based and conservative. Stochastic method is one of the several types of methodologies to perform the numerical modeling of structures. Prominent researchers used rigorous methods for application to structural modeling that are based on Probabilistic theories, Artificial Neural Networks [7] or Linear and Non Linear Regression analyses [8]. While stochastic methods are more suitable when smaller data is available, ANN and Regression methods are data intensive. Saiful Islam et al developed simple approach for evaluating the wind response statistics that are important for building serviceability [9]. In the next section, a tall building from Sitaram Vemuri and Srimaruthi Jonnalagadda., 2023 [10] is considered and peak along wind response of that structure is determined using theory of random vibration. Comparison of different velocity power spectral density functions is also presented. Followed by that, there will be discussion on results and then finally conclusions are presented.

2. Along-wind response using random vibration theory

A tall building, rectangular in configuration from Sitaram Vemuri and Srimaruthi Jonnalagadda., 2023 [10] is considered. Along wind response of the structure is worked out in similar lines specified by Simiu, E and Robert, H.S., 1996. Table 1 gives the details of the building.

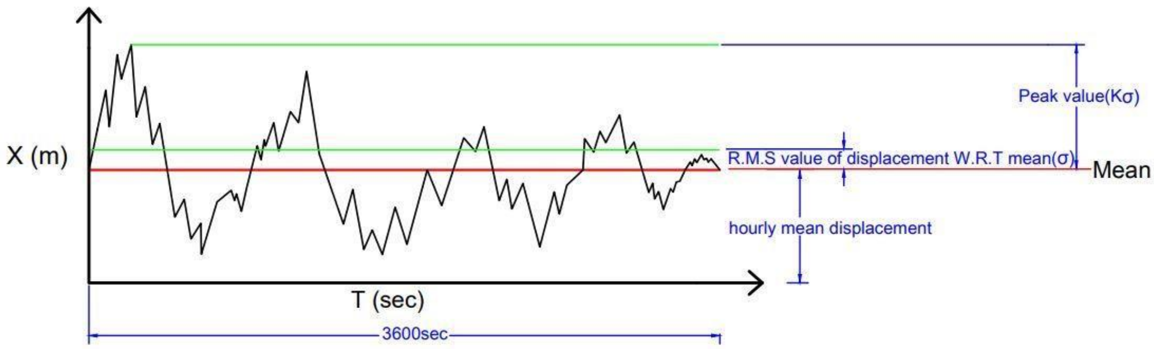

Fig. 2 Typical along wind response time history

Fig. 2 shows a typical along wind response time history. From figure, it is clear that, response variation with time is fluctuating about mean response. Thus, the total peak response is given by summation of mean

Table 1 Details of the building

Location	Mumbai
Height (H)	183m
Width (b)	40m
Depth (d)	55m
Basic Wind Speed (V_b)	44m/s
Terrain category	IV
Roughness length(Z_o)	2m
Friction velocity(u)	3m/s
Fundamental Frequency (f)	0.51Hz
Building Density (ρ_b)	200 (Kg/m^3)
Damping Ratio (ξ)	0.05
C_d (Drag coefficient)	1.837
C_L (Leeward direction)	0.713
C_w (Windward direction)	1.124

response and peak value of fluctuating response about the mean. This response analysis is based on two assumptions. 1. The shape of fundamental mode shape is linear and 2. The response to wind loading is generally dominated by the fundamental mode [1]. The second assumption holds if the ratios of natural frequencies in the second and higher modes to the fundamental frequency are sufficiently large. The mean response is given by,

$$\bar{X}(z) = \frac{1}{2}\rho(C_w + C_l)B\,\Sigma\frac{\int_0^H U^2(z)x_i(z)dz}{4\Pi^2 n_i^2 M_i} \tag{1}$$

$$M_i = \int_0^H x_i^2(z)m(z)dz \tag{2}$$

Where,

'ρ' is the density of air = 1.225kg/m^3.
'C_w' is the windward drag coefficient.
'C_l' is the leeward drag coefficient.
'B' is the width of the building.
'$x_i(z)$' is the mode shape.
'U (z)' is the hourly mean wind speed given by logarithmic law.
'n_i' is the modal frequency of ith mode.
'$m(z)$' is the mass of the building per unit height.

The hourly mean wind speed is given by the expression,

$$\mathrm{U}\ (\mathrm{z}) = \frac{1}{k}\times u_*\times\ln\frac{z}{z_0} \tag{3}$$

Where,

'k' is the Karman constant = 0.4.
'u_*' is the friction velocity of the terrain.
'z' is the height of various levels of the building.
'z_0' is the roughness length for various terrains.

'z_0' for various terrains are given in table Table 2.

Table 2 Terrain Roughness length for various Terrains

Terrain category	I	II	III	IV
z_0	0.03-0.1	0.2-0.4	0.8-1.2	2-3

Since we have assumed the mode shape as linear, the expression for the mode shape is given by,

$$x_i(z) = \frac{z}{H} \tag{4}$$

After obtaining the peak responses along the height of the building, a mode shape correction factor is applied to this linear mode shape mentioned in (4), to obtain the actual mode shape and corresponding values of peak response. The mass of the building per unit height is worked out from the density of the building. The peak expected value of the fluctuating displacement about mean is given by,

$$x'_{peak} = k\sigma_x \quad (5)$$

Where, 'σ_x' is the root mean square displacement with respect to mean and is represented in Fig. 2. It is also called as standard deviation with respect to mean value. 'k' is the expected value of largest peak factor for fluctuating displacement component as shown in Fig. 2. The expected value of largest peak factor in a given 'T' interval is given by,

$$k = \sqrt{2\ln(\nu T)} + \frac{0.577}{\sqrt{2\ln(\nu T)}} \quad (6)$$

Here, 'T' is taken as 3600sec since average wind storms last for 1 hour. ν'' from random vibration theory is given by,

$$\nu = \sqrt{\frac{\int_0^\infty n^2 s_x(n)dn}{\int_0^\infty s_x(n)dn}} \quad (7)$$

$$S_x(z,n) = \frac{\rho^2}{16\Pi^4}\Sigma\frac{x_i^2(z)C_{Df}^2}{n_i^4 M_i^2\left\{\left[1-\left({}^{n}/_{n_i}\right)^2\right]^2 + 4\eta_i^2\left({}^{n}/_{n_i}\right)^2\right\}} \times\left(\int_0^H\int_0^H\int_0^B\int_0^B x_i(z_1)\,x_i(z_2)\bar{U}(z_1)\bar{U}(z_2)S_u^{1/2}(z_1)S_u^{1/2}(z_2)coh(y1,y2,z1,z2,n)dy_2dy_1dz_1dz_2\right) \quad (9)$$

and

the R.M.S value of displacement is given by,

$$\sigma_x = \sqrt{\int_0^\infty S_x(z,n)} \quad (8)$$

Where,

'$S_x(z,n)$' is the displacement power spectral density function.

The displacement power spectral density function is given by,

Where,

'$x_i(z)$' is the mode shape defined in equation (4) 'n' is the fundamental natural frequency of the building 'ŋ' is the damping ratio.

'n_i' is the frequency of i[th] harmonic eddy

'C_{Df}' is given by,

$$C_{Df}^2 = C_w^2 + C_l^2 + 2C_wC_lN(n_1) \quad (10)$$

$$N(n_1) = \frac{1}{\xi} - \frac{1}{2\xi^2}\left(1 - e^{-2\xi}\right) \tag{11}$$

$$\xi = \frac{15.4 n_i \Delta x}{\bar{U}} \tag{12}$$

Where

U'_' is the hourly mean wind velocity at a height of $2/3\,H$.

'H' is the height of the building.

Δx'' = min (B, H, D)

$S'_u(z)$' is the velocity power spectral density function at height 'z'.

co'$h(y1, y2, z1, z2, n)$' is the coherence function.

Coherence function gives the measure of the extent to which two random signals are correlated. Let say, $(y1, z1)$, $(y2, z2)$ be two points on wind ward face of the building, z-axis being oriented along height of the building. When two wind velocity random signals hits the building at those two points, the effect of one on each other is dealt by coherence function and it is given by,

$$coh(y1, y2, z1, z2, n) = e^{-\bar{f}} \tag{13}$$

$$\bar{f} = \frac{n\left[C_z^2(Z_1 - Z_2)^2 + C_y^2(y_1 - y_2)^2\right]^{1/2}}{\frac{1}{2}[\bar{U}(Z_1) + \bar{U}(Z_2)]} \tag{14}$$

Where, C_z, C_y are the exponential decay coefficients and are equal to 10 and 16.

Equation (9) clearly shows that the displacement power spectral density function is dependent on velocity power spectral density function. Different expressions for velocity power spectral density function are given by Simiu, Kaimal and Harris.

Simiu's velocity power spectral density function is given by,

$$\frac{nS_u(z,n)}{u_*^2} = \frac{200f}{(1+50f)^{\frac{5}{3}}} \tag{15}$$

Where,

'n' is the eddy frequencies that make the fluctuating velocity components. Where,

$$f = \frac{nz}{u(z)} \tag{16}$$

'u(z)' being the hourly mean wind speed given in equation (3) and,

Kaimal's velocity power spectral density function is given by,

$$\frac{nS_u(z,n)}{u_*^2} = \frac{105f}{(1+33f)^{\frac{5}{3}}} \tag{17}$$

Where,

$$f = \frac{nz}{u(z)} \tag{18}$$

Whereas, Harris velocity power spectral density function, which is independent of height is given by,

$$\frac{nS_u(z,u)}{u_*^2} = \frac{4x}{(2+x^2)^{\frac{5}{6}}} \quad (19)$$

Where,

$$x = \frac{1200n}{\bar{U}\,(10)} \quad (20)$$

Where,

$\bar{U}(10)$ is hourly mean wind speed at 10m height of the building.

Fig. 3 depicts the variation of all the above velocity PSD functions, with frequency of the eddy, at tip of the building.

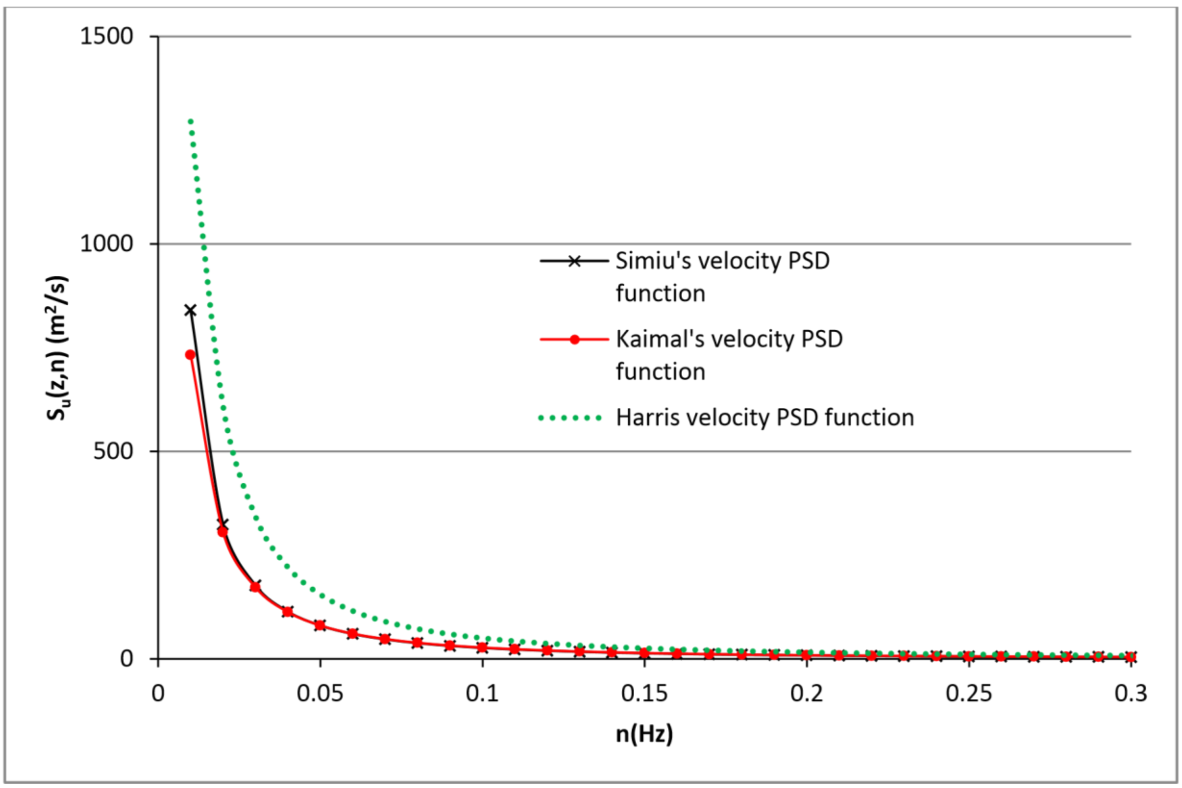

Fig. 3 Different velocity PSD Function at 183m height of the building

The above equation (9) explicitly shows the conversion of velocity power spectral density function into displacement power spectral density function. Simiu's, Kaimal's and Harris velocity power spectral density functions are used to generate corresponding displacement power spectral density functions. Working out all the above formulations and using MATLAB for solving the quadruple integrals, peak displacement at the tip of the building is obtained using equation (5). Peak displacements at different heights of the building can be obtained by multiplying the peak displacement at the tip of the building with the mode shape with correction factor '$\propto$'. Equation (21) gives the peak displacements as a function of height of the building. The mode shape correction factor equals '0.4' for terrain category-IV [1].

$$x_{i,act}(z) = x_{total}\left(\frac{z}{H}\right)^{\propto} \quad (21)$$

3. Results & Discussion

Displacement power spectral density functions corresponding to Simiu's, Kaimal's and Harris velocity power spectral density functions are determined at the tip of the building. The multiple integrals involved in determining the displacement PSD functions are solved with the help of MATLAB computing software. Fig. 4 shows the displacement power spectral density function variation with frequency of the eddy, corresponding to different velocity PSD functions. In the figure, a peak is observed at a frequency of around 0.5Hz. It is due to the fact that the natural frequency of the building considered is 0.51Hz and eddy of frequency 0.51Hz produces resonance. The harris displacement PSD function is dominating all the other PSD functions.

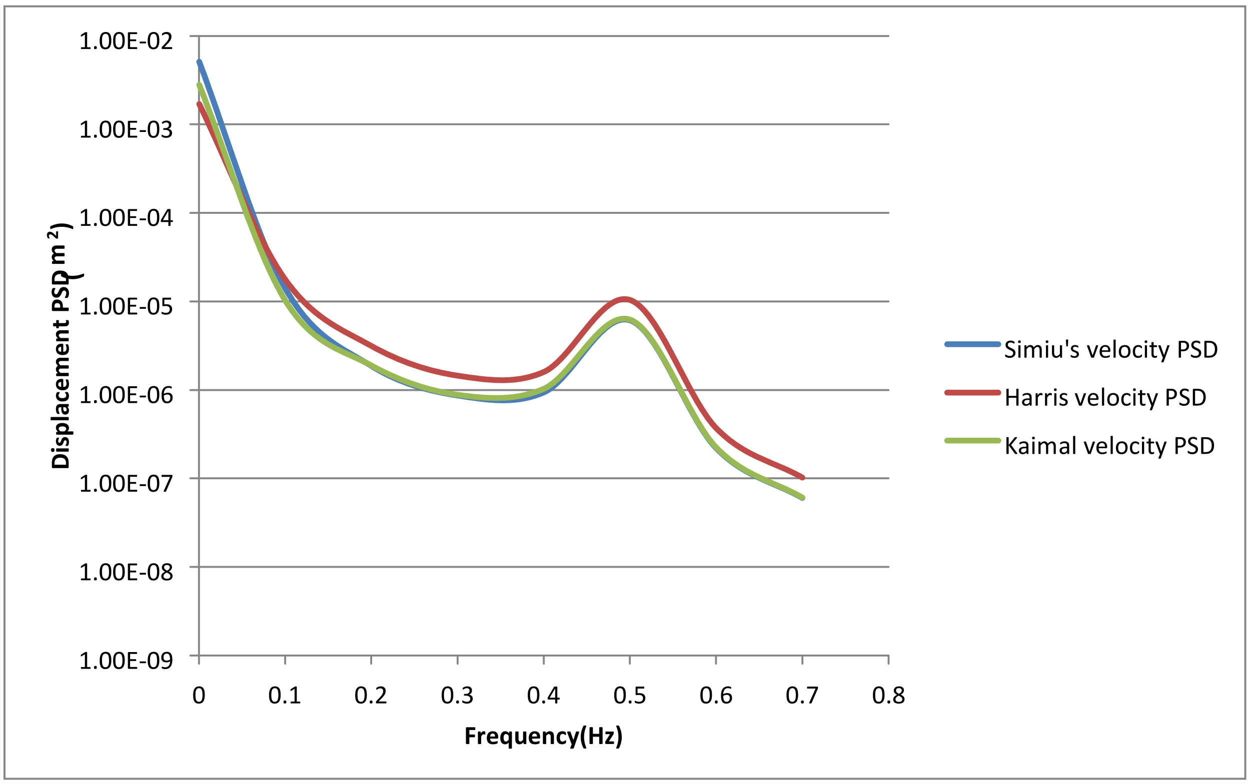

Fig. 4 Displacement power spectral density function

The mean displacement response and peak fluctuating response are obtained from displacement power spectral density functions corresponding to the three velocity PSD functions and results are compared. The results are tabulated in Table 3.

Table 3 Peak tip displacements using different PSD functions

Parameter	Simiu's velocity PSD	Harris Velocity PSD	Kaimal's velocity PSD
$\bar{x}$(m)	0.0137	0.0137	0.0137

K	2.9387	3.046	2.95
σ_x(m)	0.0050	0.0051	0.0049
x'_{peak}(m)	0.01470	0.0155	0.0144
$x_{Total}(m)$	0.02839	0.029	0.0282

It is observed that the peak tip displacement obtained using all the above velocity PSD functions are nearly same. However, peak tip displacement obtained using Harris PSD took an edge over the others.

4. Conclusions

In this paper, a tall building is considered and peak along wind response of the structure is worked out using theory of random vibration. The peak displacement is what the matter of concern for the design engineers to assess structural safety and dwellers comfort. So, there is a need to keep a check on response of tall buildings to wind forces. So, an effort has been made in working out the peak along wind response of a 183m tall building. The envelope of wind response predicted by this model could be useful to tall buildings. For typical cast-in-place concrete housing buildings in India [11], the influence of wind is negligible as most of them are short buildings.

The following observations are made from this study.

1. Velocity power spectral density function plays a key role in determining the peak along wind response of the building
2. Simiu's and Kaimal's velocity PSD function is a function of height of the building where as Harris velocity PSD is independent of height of the building.
3. For any eddy frequency, Harris velocity PSD function has the highest magnitude compared to other PSD functions.
4. Even in case of displacement PSD functions, Harris displacement PSD function holds high over the others.
5. The obtained peak along wind tip displacement of the tall building is almost same in case of all the three PSD functions of which, Harris PSD function took an edge over the others.

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