Analysis of Hysteresis and Eddy Current losses in ferromagnetic plate induced by time-varying electromagnetic field

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Abstract - *Whenever electric devices are exposed to time-varying electromagnetic field, it brings out two impacts on the devices: Firstly, conducting currents appears in the device, which ultimately gives rise to Joule's heat which rises the temperature of the device in terms of Eddy current loss, and, secondly due to the time lag between magnetization and demagnetization of the ferromagnetic plate some amount of energy is lost which is termed Hysteresis loss. In this paper, we have treated this total loss as the heat source for the problem. Based on Maxwell's equations a three-dimensional mathematical model for the magnetic field, temperature field, and elastic field in the plate was established. Then the governing equations for determining magnetic field intensity, temperature, and stresses inside the plate were solved by integral transform technique. The results obtained are displaced graphically to illustrate the influence of wave frequency, skin depth, electrical conductivity, magnetic permeability, hysteresis loss, and Eddy current loss of steel plate on the various fields considered in the problem.*

Key Words: Eddy current, hysteresis loss, time-varying electromagnetic field, magneto-thermoelasticity, integral transform

1.INTRODUCTION

In many electromagnetic equipment such as magnetic circuits of motors, generators, inductors, magnetically levitated high-speed terrestrial vehicles, energy storage devices in electromagnetic fields, fusion reactors, and devices using electro-magnetic propulsion etc, ferromagnetic materials are widely used. Analogous to conventional structures which experience mechanical loads, the ferromagnetic structures inside the strong magnetic fields typically are exposed to the magnetic force arising due to mutual interaction between time-varying magnetic fields and the magnetization of ferromagnetic materials. Due to this strong magnetic force, the ferromagnetic structures undergo deformation which affects their stability drastically [1].

Eddy current induced in conducting mediums (such as metallic plates) by time-varying magnetic field and its consequences is of great practicable significance due to its magnificent role in a vast range of technical and industrial applications. Many mechanical structures get activated when immersed in the electromagnetic field. Such structures or conducting mediums, when moving through the electromagnetic field or when exposed to the time-varying electromagnetic field, Eddy currents are induced. This eddy current creates an internal magnetic field that exactly opposes the external magnetic field. This gives rise to a skin effect in which the current density near the outside of the conducting medium becomes higher than the inside. The eddy current creates a kind of power loss in the medium known as Eddy current loss. The heat produced by Eddy current is used in various applications like heat treatment of metal products, induction furnaces, Induction hardening in steel parts, induction welding/brazing as a way of connecting metal components, or induction annealing which can selectively soften a region of a steel portion, etc. [2-5]. The estimation of Eddy current is also used for one of the inspection methods which is non-destructive testing which serves for a variety of purposes, including flaw detection, measuring material and coating thickness and determining and laying out the heat treatment condition for conducting materials.

Magneto-thermoelasticity is a subject where we study the interactions between magnetic, thermal, and mechanical fields in a thermoelastic solid in the presence of a magnetic field. The theories like heat conduction theory, classical elasticity theory, and electromagnetic theory which are applied to solve the coupling problems of temperature field, electromagnetic field, and elastic field of conductive elastic solids are also included in Magneto-thermoelasticity. The theoretical idea of magneto-thermoelasticity was introduced by [6-7] and later on was developed by [8]. Paria [9] considered a thermo-elastic solid inside a magnetic field and studied the propagation of the plane waves and gave a theoretical framework for the advancement of magneto-thermoelasticity. Wilson [10] studied the propagation of magneto–thermoelastic waves in a non-rotating medium. The above studies were based on the theory of classical coupled thermoelasticity, with interaction among the electromagnetic field, the thermal field, and the elastic field, as well as the dispersion relation, taken into consideration. Nayfeh et al. [11] used the Perturbation technique to study the effect of small couplings related to thermoelasticity and magneto-elasticity of an unbounded isotropic medium. Sherief et al. [12] discussed a one-dimensional thermal shock problem of generalized thermoelastic electrically conducting half-space permeated by a primary uniform magnetic field with thermal relaxation. Biswas et al. [13] exemplify a three-dimensional electro-magneto-thermoelastic coupled problem for homogeneous orthotropic thermally and electrically conducting solid subjected to time-dependent thermal shock. Xu et al. [14] proposed an approximate analytical solution method to investigate the behavior of three-dimensional simply supported rectangular plates with variable thickness subjected to thermo-mechanical loads. Baksi et al. [15] studied the three-dimensional problems of magneto-thermoelasticity in the infinite rotating elastic medium with thermal relaxation and heat source. The study of the interaction between the magnetic field and the strain field in a thermoelastic solid is receiving considerable attention in recent years due to its wide range of applications in various fields. Especially in nuclear fields, the extremely high temperatures and temperature gradients, as well as the magnetic field originating inside nuclear reactors, influences their design and operations. Roychoudhuri [16] studied magnetoelastic plane waves in rotating media with uniform angular velocity. Othman et al. [17] developed a three-dimensional model of the equations of the generalized thermoelasticity to study the effect of magnetic field and thermal relaxation for a homogeneous isotropic elastic half-space solid in the context of Lord-Shulman theory in the absence of body forces or heat source. Ezzat et al. [18] developed a new mathematical model for the equations of the two-temperature magneto-thermoelasticity theory. Das et al. [19] investigated the interaction of a homogeneous and isotopically perfect conducting half-space with rotation, in the context of Lord-Shulman theory. Bawankar et al. [20] studied a two-dimensional problem of a thermosensitive conducting plate with eddy current loss in the context of magneto-thermoelasticity. Higuchi et al. [21] studied the stresses aroused due to transient magnetic fields in an infinite conducting plate using the theory of magneto-thermoelasticity. Mitik et al. [22] have obtained Joule's heat as a thermal loading of a thin elastic, isotropic, ferromagnetic plate subjected transversally to the homogeneous, time-varying magnetic field. Mitik [23] has studied the influence of plate thickness, wave frequency, and hysteresis factor on the temperature field of the thin metallic partially fixed plate which is induced by Harmonic Electromagnetic wave.

The present article is an attempt to study the effect of eddy current loss (aroused due to Joule heat generated by Eddy current) and Hysteresis loss (aroused due to time lag between magnetization and demagnetization) on a three-dimensional ferromagnetic plate placed in time-varying electro-magnetic field as an extension to the research work by Bawankar et al. [20]. The various determined expressions are obtained numerically for steel material and results are introduced graphically. Effects of Eddy current loss along with Hysteresis loss and magnetic field quantities are also analyzed. The integral transform technique is used to find the temperature solution.

2. PROBLEM FORMULATION

We consider an isotropic, homogeneous, thermally, and perfectly conducting elastic thin rectangular ferromagnetic plate with length a width c and thickness b occupying the space $D: 0 \le x \le a, 0 \le y \le b, 0 \le z \le c$ as shown in figure 1. The plate is subjected to time-dependent exponentially varying magnetic field $H_0 e^{\omega t}$ uniformly distributed along X- and Z-direction and acts on the plate in a positive Y-direction as shown in the figure. Here H_0 is the magnetic field strength at the outer surface of the plate and ω is the angular frequency given by $\omega = 2\pi / T = 2\pi f$. Let the magnetic field be given by $\vec{H} = \left(0, H_y, 0\right)$ where $H_y = H_0 e^{\omega t}$ and the induced electric field vector be given by $\vec{E} = \left(E_x, 0, E_z\right)$.

Ampere-Maxwell's equation which states two possible ways of generation of magnetic field: one is due to electric current and the other is due to changing electric field (called the displacement current) is given by:

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{1}$$

The component form (in the absence of displacement current) of the Ampere-Maxwell's equation takes the following form:

$$J_x = -\frac{\partial H_y}{\partial z}, \quad J_z = \frac{\partial H_y}{\partial x} \tag{2}$$

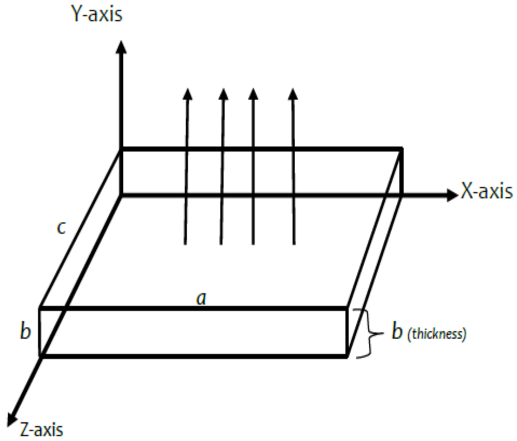

Fig -1: Geometry of the problem.

The displacement vector $\vec{u}$ has the components:

$$u = u_x, v = u_y, w = u_z \tag{3}$$

Following [24], we considered the modified Ohm's law which outlines the impact of temperature gradient and charge density ignoring the seemingly small consequence of temperature gradient on the conduction current $\vec{J}$ as:

$$\vec{J} = \sigma\left(\vec{E} + \dot{\vec{u}} \times \vec{B}\right) \tag{4}$$

Using (2) the component form of the above equation can be written as:

$$J_x = \sigma\left(E_x - B_y \dot{w}\right), \quad J_z = \sigma\left(E_z - B_y \dot{u}\right) \tag{5}$$

Similarly, the component of magnetic flux density is given by:

$$B_y = \mu H_y \tag{6}$$

Solving (1) and (4) we obtain the components of electric field intensity as:

$$E_x = -\frac{1}{\sigma}\frac{\partial H_y}{\partial z}, \quad E_z = \frac{1}{\sigma}\frac{\partial H_y}{\partial x} \tag{7}$$

Faraday's law of electromagnetism which describes how time-varying magnetic field induces an electric field gives:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{8}$$

Using (6) and (7), the above equation reduces to:

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial z^2} = \sigma\mu\frac{\partial H_y}{\partial t} \tag{9}$$

Equation (9) represents the uncoupled equation of the magnetic field. Using $H_y(x,z,t) = H_0 e^{\omega t}, \quad \mu = \mu_0\mu_r$ in (9), we obtain:

$$\delta_z^2 \frac{\partial^2 H}{\partial x^2} + \delta_x^2 \frac{\partial^2 H}{\partial z^2} = 2H \tag{10}$$

where $\delta_z = \sqrt{2/\omega\sigma_z\mu_0\mu_r}$, $\delta_x = \sqrt{2/\omega\sigma_x\mu_0\mu_r}$, are the skin depth in the z-axis and x-axis directions respectively. In particular, for isotropic material $\sigma_x = \sigma_z$, hence $\delta_x = \delta_z = \delta = \sqrt{2/\omega\sigma\mu_0\mu_r}$ with this, Eq. (10) modifies to:

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2} = \frac{2}{\delta^2}H \tag{11}$$

The pre-requisite boundary conditions are considered as:

$$H(0,z,t) = H(a,z,t) = H_0 \tag{12}$$

$$H(x,0,t) = H(x,\mathrm{c},t) = H_0 \tag{13}$$

As a result of a time-varying electromagnetic field conducting currents appear in the plate material, referred to as Eddy current. Eddy current produces the resistive deprivation that converts some form of energy, for example, kinetic energy into heat energy which is known as Joule heat. Joule heat diminishes the efficiency of the conducting material. Joule heat gives rise to the Eddy current loss W_{Eddy}. For the considered problem the distribution of Joule's heat in terms of Eddy current loss given by [23]

$$W_{Eddy} = \frac{1}{2\sigma}\|J\|^2 = \frac{1}{2\sigma}\left[J_x^2 + J_z^2\right] \tag{14}$$

When a magnetization force is applied to a magnetic material, the molecules of the magnetic material are aligned in one particular direction. Switching of magnetic force in the opposite direction causes the internal friction of the molecular magnets. This friction, in fact, resists the change in the direction of magnetism which finally gives rise to the Magnetic Hysteresis. To defeat this resistance caused due to internal friction, a part of the magnetizing force is utilized which results in the work done by the force. Heat is generated when this kind of work done is performed by magnetizing force. This extra heat generated results in the wastage of energy in the form of heat called as Hysteresis loss. It is known that Hysteresis loss W_{Hyst} is proportional to the square of magnetic field amplitude and frequency [23], therefore it is given by

$$W_{Hyst}(z,t) = k_H \mu\, f\, H_y^2 \tag{15}$$

Neglecting the coupling term η between the temperature and the deformation fields [22], the governing equation of the temperature field along with the boundary and initial conditions is given by:

$$\nabla^2 T + \frac{W_{Total}}{\lambda_0} = \frac{C\rho}{\alpha}\frac{\partial T}{\partial t} \tag{16}$$

$$T(0,y,z,t) = T(a,y,z,t) = 0 \tag{17}$$

$$T(x,0,z,t) = T(x,b,z,t) = 0 \tag{18}$$

$$\left(\frac{\partial T}{\partial z}\right)_{z=0,c} = 0 \tag{19}$$

$$T(x,y,z,0) = 0 \tag{20}$$

The term $W_{Total} \equiv W_{Total}(x,y,z,t)$ is treated as the heat source of the problem. Also, it consists of two parts the hysteresis loss and Joule's Heat (responsible for Eddy Current loss). Both are taken within the plate for $t > 0$ subject to the initial and boundary conditions prescribed for the problem considered. Thus

$$W_{Total}(x,y,z,t) = W_{Eddy} + W_{Hyst} \tag{21}$$

Apart from Eddy current loss and Hysteresis loss the metallic plate placed in a time-varying magnetic field also suffers the Lorentz force. The components of Lorentz force are given by the expressions:

$$f_x = -\frac{\mu}{2}\frac{\partial}{\partial x}\left(H_y\right)^2 \tag{22}$$

$$f_z = -\frac{\mu}{2}\frac{\partial}{\partial z}\left(H_y\right)^2 \tag{23}$$

The constitutive equation for stress-displacement-temperature relation and strain-displacement relation is given by [9]:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda\,\mathrm{e} - \beta\mathrm{T})\delta_{ij} \tag{24}$$

$$e = \vec{\nabla}\cdot\vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \tag{25}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{26}$$

Using (24), the components of the stress field are written as:

$$\sigma_{xx} = 2\mu\frac{\partial u}{\partial x} + \lambda\,\mathrm{e} - \beta\mathrm{T} \tag{27}$$

$$\sigma_{yy} = 2\mu\frac{\partial v}{\partial y} + \lambda e - \beta T, \tag{28}$$

$$\sigma_{zz} = 2\mu\frac{\partial w}{\partial z} + \lambda e - \beta T \tag{29}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{30}$$

$$\sigma_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{31}$$

$$\sigma_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \tag{32}$$

The displacement equation of the theory of elasticity, considering the Lorentz force takes the following form:

$$\rho\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k} + (J \times B)_i \tag{33}$$

For considered three-dimensional problem, the above equations give:

$$\rho\ddot{u} = \left[(2\mu+\lambda)\frac{\partial^2 u}{\partial x^2} + \lambda\left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right) - \beta\frac{\partial T}{\partial x}\right] - \frac{\mu}{2}\frac{\partial}{\partial x}\left(H_y\right)^2 \tag{34}$$

$$\rho\ddot{v} = \left[(2\mu+\lambda)\frac{\partial^2 v}{\partial y^2} + \lambda\left(\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 w}{\partial y \partial z}\right) - \beta\frac{\partial T}{\partial y}\right] \tag{35}$$

$$\rho\ddot{w} = \left[(2\mu+\lambda)\frac{\partial^2 w}{\partial z^2} + \lambda\left(\frac{\partial^2 u}{\partial z \partial x} + \frac{\partial^2 v}{\partial z \partial y}\right) - \beta\frac{\partial T}{\partial z}\right] - \frac{\mu}{2}\frac{\partial}{\partial z}\left(H_y\right)^2 \tag{36}$$

We consider the plate is at rest before $t = 0$, and we suppose that the surfaces are traction free i. e.

$$\sigma_{xx} = \sigma_{zz} = 0 \tag{37}$$

And the mechanical boundary conditions and the initial conditions are:

$$\left(\frac{\partial u}{\partial x}\right)_{x=0,a} = \left(\frac{\partial v}{\partial y}\right)_{y=0,b} = \left(\frac{\partial w}{\partial z}\right)_{z=0,c} = \left(\frac{\beta}{2\mu+\lambda}\right)T \tag{38}$$

$$(u)_{t=0} = (v)_{t=0} = (w)_{t=0} = 0 \tag{39}$$

3. SOLUTIONS

3.1 Determination of Magnetic field

To find the solution to the magnetic field described in (11), (12) and (13), we need to transform the inhomogeneous boundary conditions into homogeneous ones. For this, we assume the solution of (11) as

$$H(x,z,t) = h(x,z) + H_0 \tag{40}$$

Using (40) in (11) to (13) we obtain:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = \frac{2}{\delta^2}\left(h + H_0\right) \tag{41}$$

$$h(0,z,t) = h(a,z,t) = 0 \tag{42}$$

$$h(x,0,t) = h(x,c,t) = 0 \tag{43}$$

Applying double finite Fourier sine transform [21] to (41) we obtain:

$$\hat{h}(m,n) = -\frac{2}{\delta^2}H_0\,\chi_{mn} \tag{44}$$

We now apply the inverse of double finite Fourier sine transform to (44) to obtain:

$$h(x,z) = -c_1\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\chi_{mn}\sin(\zeta_m x)\sin(\zeta_m z) \tag{45}$$

Using (45) in (40), we obtain the expression for the magnetic field as:

$$H(x,z) = H_0 - c_1\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\chi_{mn}\sin(\zeta_m x)\sin(\zeta_m z) \tag{46}$$

Consequently, the magnetic field intensity inside the rectangular plate at any time t is given by:

$$H(x,z) = \left[H_0 - c_1\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\chi_{mn}\sin(\zeta_m x)\sin(\zeta_m z)\right]e^{\omega t} \tag{47}$$

Current density expressions are modified to:

$$J_x(x,z) = ac_2\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}n\chi_{mn}\sin(\zeta_m x)\cos(\zeta_m z)\right]e^{\omega t} \tag{48}$$

$$J_z(x,z) = -c\,c_2\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}m\chi_{mn}\cos(\zeta_m x)\sin(\zeta_m z)\right]e^{\omega t} \tag{49}$$

Using (14) and (15), the expressions for Eddy current and Hysteresis losses are given by:

$$W_{Eddy} = c_3\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\chi_{mn}^2\left[\left(\frac{n}{c}\sin(\zeta_m x)\cos(\zeta_n z)\right)^2 + \left(\frac{m}{a}\cos(\zeta_m x)\sin(\zeta_n z)\right)^2\right]e^{2\omega t} \tag{50}$$

$$W_{Hyst} = k_H\mu f\left[H_0 - c_1\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\chi_{mn}\sin(\zeta_m x)\sin(\zeta_m z)\right]^2 e^{2\omega t} \tag{51}$$

where $\zeta_m = \frac{m\pi}{a}$, $\zeta_n = \frac{n\pi}{c}$, $c_1 = \frac{8H_0}{ac\delta^2}$, $c_2 = \frac{4\mu\sigma\omega\pi H_0}{a^2c^2}$,

$c_3 = 8\sigma\left(\frac{\mu\omega\pi H_0}{ac}\right)^2$, $\Delta_{mn} = \left[\frac{\left((-1)^m - 1\right)\left((-1)^n - 1\right)}{mn\pi^2}\right]$,

$\chi_{mn} = \Delta_{mn} / \left[\pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{c^2}\right) + \frac{2}{\delta^2}\right]$

3.2 Dimensionless Quantities

It is more convenient to introduce the dimensionless variables as below:

$$\bar{x}=\frac{x}{b},\bar{y}=\frac{y}{b},\bar{z}=\frac{z}{b},\bar{c}=\frac{c}{b},\bar{H}_y=\frac{H_y}{H_0},\tau=\frac{t}{\sigma\mu b^2},$$

$$\bar{J}_x=-\frac{bJ_x}{H_0},\bar{J}_z=\frac{bJ_z}{H_0},\bar{W}_E=\frac{\sigma b^2 W_E}{H_0^2},\bar{W}_H=\frac{W_H}{k_H\mu fH_0^2}$$

$$\bar{T}=\frac{C\rho T}{\mu H_0^2},\left(\bar{f}_x,\bar{f}_z\right)=\frac{b\left(f_x,f_z\right)}{\mu H_0^2},\bar{\sigma}_{ij}=\frac{\bar{\sigma}_{ij}}{\mu H_0^2/2} \tag{52}$$

3.3 Determination of Temperature Field

The temperature field along with the initial and boundary conditions in dimensionless form is expressed as (from here onwards we drop the bar notation for the sake of convenience):

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+c_4 W(z,\tau)=c_5\frac{\partial T}{\partial\tau} \tag{53}$$

$$T(0,y,z,\tau)=T(1,y,z,\tau)=0 \tag{54}$$

$$T(x,0,z,\tau)=T(x,1,z,\tau)=0 \tag{55}$$

$$\left(\frac{\partial T}{\partial z}\right)_{z=0,1}=0 \tag{56}$$

$$T(x,y,z,0)=0 \tag{57}$$

where $c_4=\frac{a^2 c\rho}{\mu H_0^2\lambda_0},c_5=\frac{C\rho}{\sigma\alpha\mu}$

Applying double finite Fourier sine [25] transform to (53), we obtain:

$$-\pi^2\left(m^2+n^2\right)\hat{T}+\frac{\partial^2\hat{T}}{\partial z^2}+c_4\hat{W}(\bar{z},\tau)=c_5\frac{\partial\hat{T}}{\partial\tau} \tag{58}$$

We now apply the Finite Fourier cosine transform with respect to z,

$$-\pi^2\left(m^2+n^2+p^2\right)\hat{\hat{T}}+\frac{\partial^2\hat{\hat{T}}}{\partial z^2}+c_4\hat{\hat{W}}=c_5\frac{\partial\hat{\hat{T}}}{\partial\tau} \tag{59}$$

Applying Laplace Transform to the above equation with respect to time co-ordinate, we obtain:

$$\hat{\hat{T}}^*(m,n,p,s)=c_6\hat{\hat{W}}^*\left[\frac{1}{\left(s+c_{mnp}\right)\left(s-2\omega\right)}\right] \tag{60}$$

Applying the inverse Laplace transform, inverse finite Fourier cosine and inverse double finite Fourier cosine transform to the above equation we obtain:

$$T(x,y,z,\tau)=c_6\bar{W}_{Total}\times\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[I_{mn}+2\sum_{p=1}^{\infty}I_{mnp}\cos\left(p\pi z\right)\right]\times\sin\left(m\pi x\right)\sin\left(n\pi y\right) \tag{61}$$

Where

$$c_6=\frac{\alpha\sigma a^2}{\pi^2 H_0^2\lambda_0},c_{mnp}=\frac{\alpha\sigma\mu}{C\rho}\left(m^2+n^2+p^2\right),$$

$$I_{mnp}=\int_0^\tau e^{-[(2\omega+c_{mnp})u-2\omega\tau]}du,\ I_{mn}=\int_0^\tau e^{-[(2\omega+c_{mn,p=0})u-2\omega\tau]}du$$

3.4 Determination Displacement and Stresses

Simplifying (34)-(36) further and dropping the inertia term, we obtain:

$$\frac{\partial^2 u}{\partial x^2}=\frac{\beta}{(2\mu+\lambda)}\frac{\partial T}{\partial x}+\frac{\mu}{2(2\mu+\lambda)}\frac{\partial}{\partial x}\left(H_y\right)^2 \tag{62}$$

$$\frac{\partial^2 v}{\partial y^2}=\frac{\beta}{2\mu+\lambda}\frac{\partial T}{\partial y} \tag{63}$$

$$\frac{\partial^2 w}{\partial z^2}=\frac{\beta}{2\mu+\lambda}\frac{\partial T}{\partial z}+\frac{\mu}{2(2\mu+\lambda)}\frac{\partial}{\partial z}\left(H_y\right)^2 \tag{64}$$

The expressions for displacement components are given by:

$$u\equiv u_x=\frac{\beta}{2\mu+\lambda}\int T(x,y,z,\tau)dx+\frac{\mu}{2(2\mu+\lambda)}\int H(x,z,\tau)^2 dx \tag{65}$$

$$v\equiv u_y=\frac{\beta}{2\mu+\lambda}\int T(x,y,z,\tau)dy \tag{66}$$

$$w\equiv u_z=\frac{\beta}{2\mu+\lambda}\int T(x,y,z,\tau)dz+\frac{\mu}{2(2\mu+\lambda)}\int H(x,z,\tau)^2 dz \tag{67}$$

Following [21], the quasi-static solutions of displacement due to temperature change and Lorentz force in terms of its thermal and magnetic components are given by:

$$u^T=\frac{\beta}{2\mu+\lambda}\int T(x,y,z,\tau)dx, \tag{68}$$

$$u^M=\frac{\mu}{2(2\mu+\lambda)}\int H(x,z,\tau)^2 dx \tag{69}$$

$$v^M=\frac{\beta}{2\mu+\lambda}\int T(x,y,z,\tau)dy \tag{70}$$

$$w^T=\frac{\beta}{2\mu+\lambda}\int T(x,y,z,\tau)dz, \tag{71}$$

$$w^M=\frac{\mu}{2(2\mu+\lambda)}\int H(x,z,\tau)^2 dz \tag{72}$$

Using the above equations along with (25) in (27)-(32), we obtain:

$$\sigma_{xx}^T=\sigma_{zz}^T=\frac{2\beta\lambda}{2\mu+\lambda}T(x,y,z,\tau) \tag{73}$$

$$\sigma_{xx}^M=\sigma_{zz}^M=\frac{\mu(\mu+\lambda)}{2\mu+\lambda}H(x,z,\tau)^2 \tag{74}$$

$$\sigma_{yy}^T=\frac{2\beta\lambda}{2\mu+\lambda}T(x,y,z,\tau) \tag{75}$$

$$\sigma_{yy}^M=\frac{\lambda\mu}{2\mu+\lambda}H(x,z,t)^2 \tag{76}$$

$$\sigma_{xx} = \sigma_{zz} = \frac{1}{2\mu+\lambda}\left[\mu(\mu+\lambda)H(x,z,\tau)^2 + 2\beta\lambda T(x,y,z,\tau)\right] \tag{77}$$

$$\sigma_{yy} = \frac{\lambda}{2\mu+\lambda}\left[\mu H(x,z,\tau)^2 + 2\beta T(x,y,z,\tau)\right] \tag{78}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\ \sigma_{xz} = \mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right),$$

$$\sigma_{yz} = \mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) \tag{79}$$

4. NUMERICAL RESULTS AND DISCUSSION

To illustrate and compare the theoretical results obtained, we now present some numerical results which depict the variations of displacement, temperature, and stress components. The material chosen for the purpose of numerical evaluations is steel, for which we take the following values of the different physical constants, using the physical data given in [22]. Figure 2, shows the magnetic field inside the rectangular plate. It is found that the magnetic field intensity is symmetric in both directions. Figure 3, shows the variations of specific heat losses with frequency. Frequency has a huge impact on both Eddy current loss and Hysteresis loss as well as on the general properties of ferromagnetic materials. At high frequency, eddy current losses are dominating whereas at low frequency, hysteresis losses are determining. At low frequency, the Hysteresis loss is a bit larger than the eddy current loss (see Figure 3), while the magnetic flux density remains the same. Meanwhile, when the frequency reaches above 110 *Hz*, eddy current loss is predominant as compared to hysteresis loss. Further, as frequency increases slowly, both losses also increase.

Table -1: Material Constants

Physical Constants	Value
Permeability of vacuum (μ_0)	1.26×10^{-4} H/m
Lame's constant (μ)	79.3 GPa
Electric conductivity (σ)	7.7×10^{6} S/m
Specific Heat (C)	502.416 J/kg K
Density (ρ)	7663 kg/m^3
Coefficient of Thermal Intensity (κ)	1.4×10^{-3}m^2/sec
Poisson's Ratio (ν)	0.28
Thermal Diffusivity (α) Heat conduction coefficient (λ_0) Electric field intensity (E)	12×10^{-6} K^{-1} 50 W/mK 205 GPa

Eddy current losses have a powerful impact on total heat loss. However, Hysteresis losses are important parts of total heat losses at low frequencies. The graph of hysteresis loss is linear as it is proportional to the frequency while the eddy current losses are proportional to the square of the frequency. Figure 4, shows that the total heat losses increase when magnetic flux density increases, whereas frequency is kept as constant. It also shows that the total heat loss gets larger and larger as the frequency of input supply increases, meanwhile, the magnetic flux density remains constant. Thus, steady growth is observed in total heat loss with an increase in magnetic flux density and the frequency increases from $50\,Hz$ to $250\,Hz$. This is because as per equation (17) the total heat loss term consists of two parts, Heat loss due to Eddy current and due to Hysteresis phenomenon. And Eddy current loss is directly proportional to the frequency of the supply whereas Hysteresis loss is proportional to the square of the frequency (see equation (14)). Therefore, this will lead to an increase in total heat loss with an increase in frequency. Figure 5 shows the effect of the resistivity of the material on the variation of Eddy current loss. Resistivity is the main factor that affects the Eddy current loss of the electrical machines which work under the time-varying electromagnetic field. The Eddy current loss mainly depends on the resistivity of the thin lamination plates of insulation material which are used in the machines like transformers where the thermal effect produced by the Eddy current loss affects the working environment of the machine drastically.

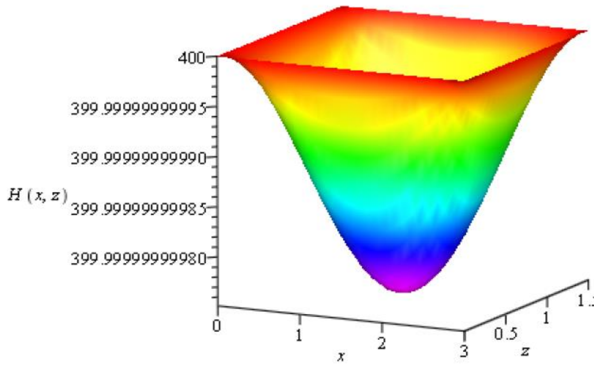

Fig -2: Distribution of magnetic field intensity along x and z direction

It is also observed that the higher the resistivity of the insulation/lamination material the lesser the Eddy current loss produced. Most of the ceramics which are non-metals like porcelain have a high tendency to resist the flow of electric current. With the use of such high resistivity materials for insulation one can reduce the Eddy current loss completely. Eddy current loss is greatest on the surface and decreases when we go deep inside the plate material. This is because of the phenomenon so-called as "Skin Effect". The non-uniform distribution of the Electric current over the surface or skin of the material plate carrying the current is called the Skin effect.

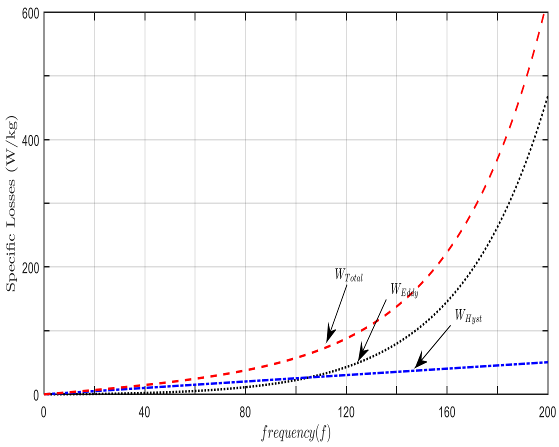

Fig-3: Variation of Specific heat losses versus frequency

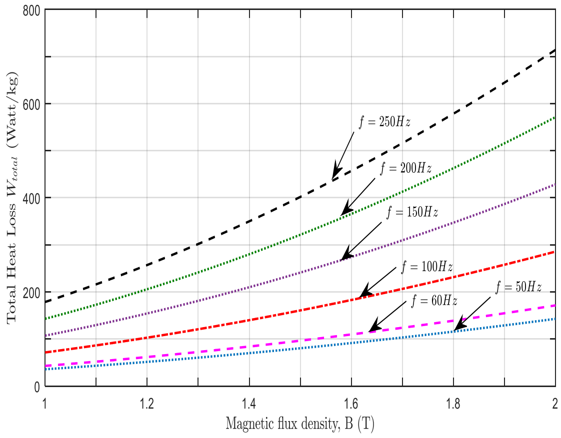

Fig-4: Distribution of Total Heat Loss versus flux density

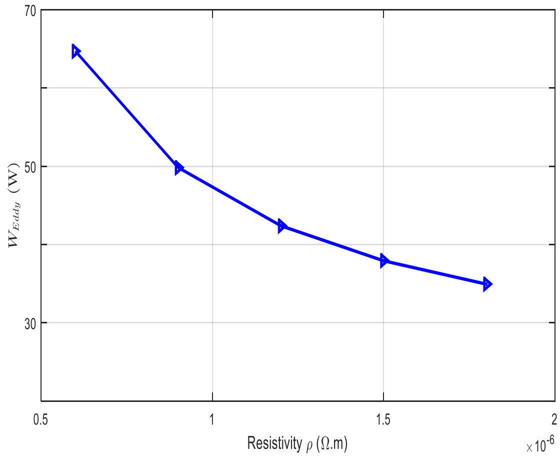

Fig-5: Effect of Resistivity on Eddy Current loss

The skin depth (δ) of the magnetic material defines a certain distance to which the strength of the electromagnetic field suffers the resistance in the amplitude and it reduces to $1/e$ of its original amplitude.

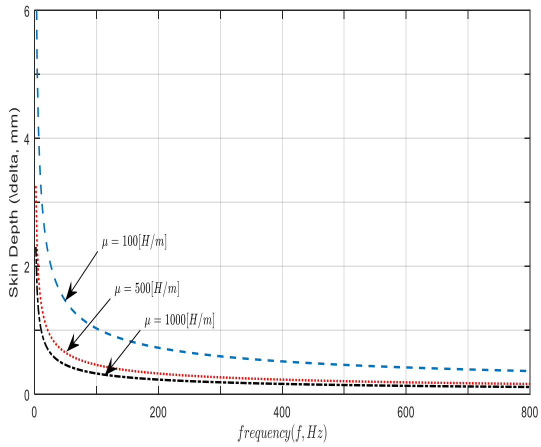

Fig-6: Variation of Skin depth with frequency

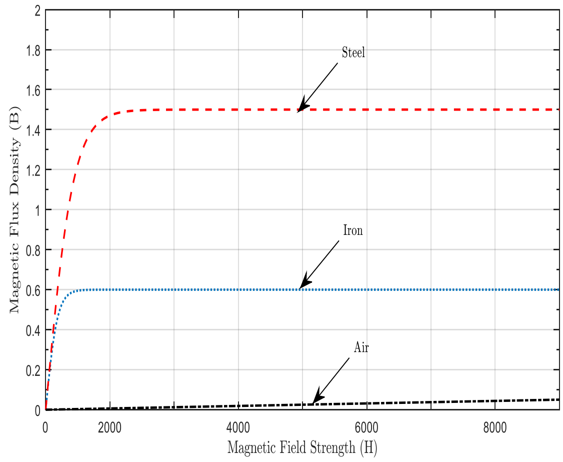

Fig-7: B-H Curve for Steel, Iron and Air

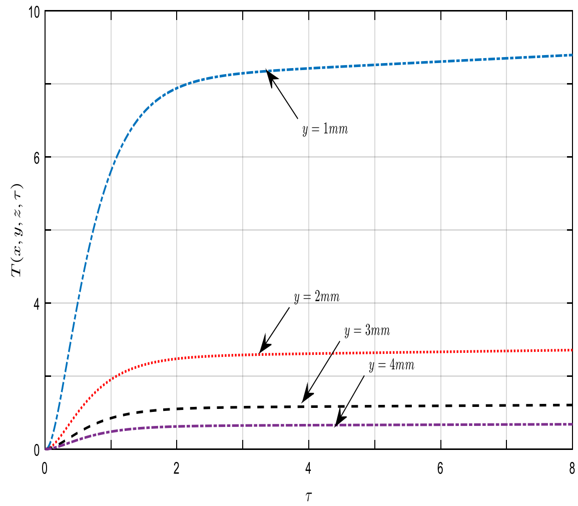

Fig-8: Time variation of non-dimensional temperature along the thickness of the plate

Or in other words, skin depth defines the distance below the surface of the current-carrying conductor where the amplitude of the electromagnetic field strength decreases to 1/e of its value at the surface.

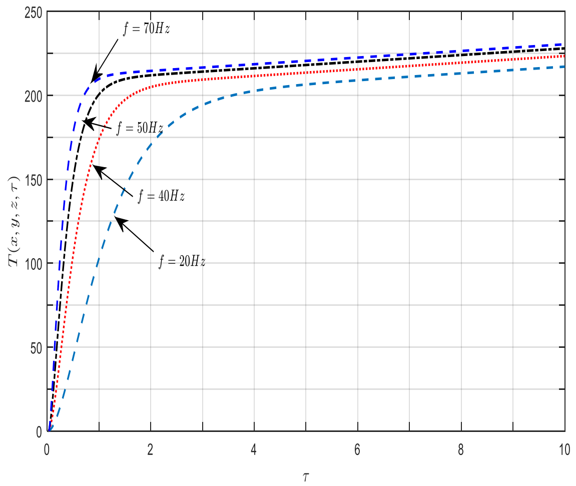

Fig-9: Time variation of non-dimensional temperature for different values of frequency

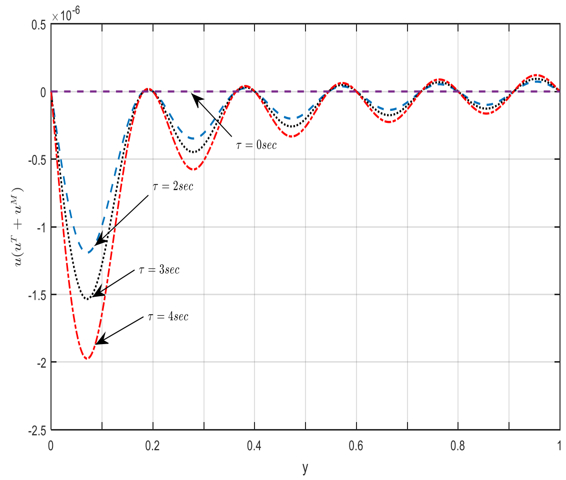

Fig-10: Variation of displacement component u along the thickness of plate for different values of time

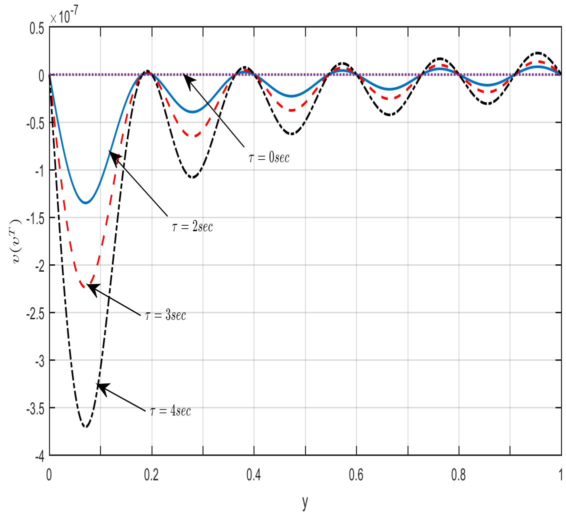

Fig-11: Variation of displacement component v along the thickness of plate for different values of time

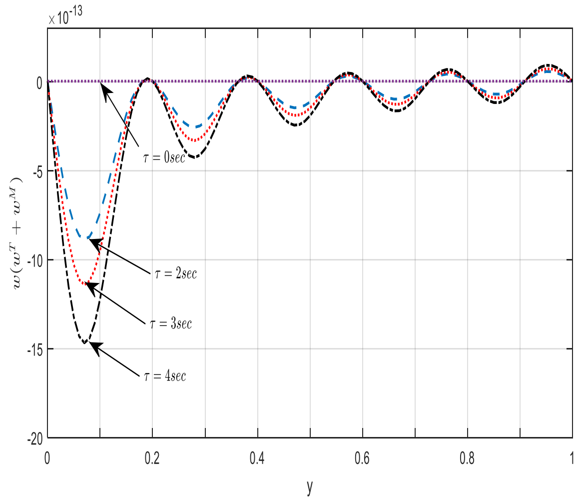

Fig-12: Variation of displacement component w along the thickness of plate for different values of time

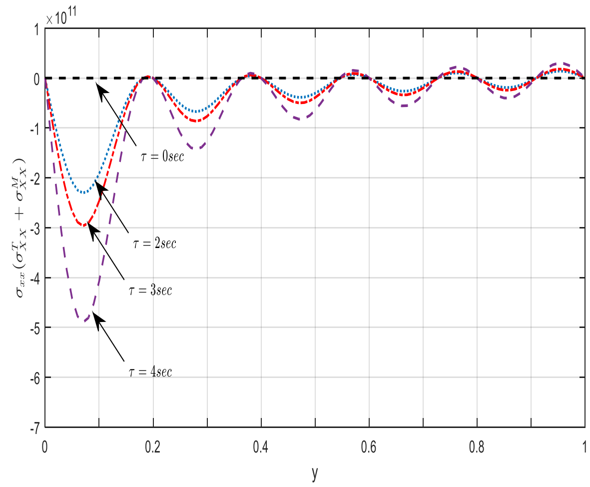

Fig-13: Variation of stress component along the thickness of plate for different values of time

This decrease is referred to as the decay of one Naper (0.368). Theoretically, it is given by [23]:

$$\delta = \frac{1}{\sqrt{\sigma\mu\pi f}} \tag{80}$$

At higher frequencies, the electromagnetic waves can penetrate only near the surface of the plate material and the strength of electromagnetic waves decreases exponentially with an increase in the thickness of the plate material. Figure 6 shows the variation of skin depth with the frequency of electromagnetic waves for various values of magnetic permeability. It shows that at higher frequencies the skin depth becomes much smaller. Therefore, the Eddy current loss or heating aroused due to it can be controlled to any required depth of the material simply by changing the supply of input frequency. Figure 7 represents the variation of Magnetic flux density (B) with the Magnetic field strength (H) for Steel, Iron, and Air. From the figure, we can notice that variation in magnetic field density is proportional to the Magnetic field strength until it reaches a certain value from which it does not increase anymore and becomes almost constant and steady while the Magnetic field strength continues to increase. From the figure, it can be seen that for Steel the magnetic flux density increases up to 1.5T and this point in the graph is referred to as the **Magnetic Saturation.** Thus, Magnetic Saturation is the point of maximum flux density attained by the ferromagnetic material in the B-H Curve. Figure 8 shows the time variation of temperature for the different values of thickness. It can be observed from the figure that temperature first increases and then it becomes steady with time. Also, since wave frequency is inversely proportional to the skin depth of the plate material, therefore as soon as the plate thickness increases the temperature of the plate decreases. From the figure, it can be observed that for higher plate thickness the temperature is lower as compared to that for small plate thicknesses. Figure 9 shows the time variation of temperature at the middle of the plate with different values of frequency. Due to the effect of wave frequency the plate heats up gradually to attain the maximum temperature then after a few seconds, the temperature becomes steady. Figure 10 displays the distribution of the displacement component u versus distance y. The displacement component always begins from zero for the four values of time coordinate and satisfies the boundary condition at $y=0$ as well as the initial condition at $\tau=0$. For $\tau=2,3,4$sec the displacement component u decreases for the range $0 \le x \le 0.06$ and $0.2 \le x \le 0.26$ while it increases in the range $0.06 \le x \le 0.2$ and $0.26 \le x \le 0.38$. It can be observed that the displacement component u decreases as time increases. And it is showing oscillatory behavior and finally converging towards zero with the increase of y. In Figure 11 and 12, variation of v and w with respect to y is presented for various values of time coordinate. It is noticed that v and w decrease with time. Displacement components exhibit oscillatory behavior and tend towards zero with the increment in y. Figure 13 shows the variation of Stress components along the thickness of the plate for different values of time.

5. CONCLUSIONS

In this paper, a three-dimensional model of Magneto-thermo-elasticity under the influence of time-varying magnetic field is established. Taking Steel plate as an example of ferromagnetic material, a series of analyses were carried out. According to the results obtained in this paper, the following conclusions can be obtained:

1. The subject of this paper is obtaining Eddy current loss and Hysteresis loss as the total heat loss of the ferromagnetic plate.
2. The distribution function of Magnetic field intensity in a rectangular plate of steel material is expressed in context of Maxwell's equations using Double finite Fourier sine transform.
3. It is observed that the time-varying electromagnetic field is responsible for the generation of Joule heat which ultimately gives rise to Eddy current loss.
4. The influence of the frequency, magnetic field intensity, and the plate thickness on the total heat loss is discussed.
5. One of the very suitable methods for solving the problem as discussed in this paper is the integral transform technique (Finite Fourier cosine transform, Double finite Fourier Sine transform and Laplace transform).
6. It is found that the temperature increases with an increase in wave frequency and decreases with an increase in plate thickness.
7. The conductivity of the material affects the depth of penetration significantly.

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