NURBS BASED ISOGEOMETRIC ANALYSIS TO PERFORM TOPOLOGY OPTIMIZATION OF CONTINUUM STRUCTURES USING EVOLUTIONARY ALGORITHMS

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Abstract - *This research paper provides a comprehensive literature review on the application of "Non-Uniform Rational B-Splines (NURBS)-based Isogeometric Analysis (IGA)" for topology optimization of continuum structures utilizing evolutionary algorithms. Topology optimization aims to determine the optimal distribution of materials within a given design domain to achieve desired performance criteria. Traditional finite element methods (FEM) have been widely used for structural analysis and optimization; however, they often suffer from geometrical inaccuracies owing to the need for mesh generation. Isogeometric Analysis, which employs NURBS to represent geometry and analysis fields, offers seamless integration of design and analysis. When coupled with evolutionary algorithms such as genetic algorithms (GA) and particle swarm optimization (PSO), it facilitates efficient and effective topology optimization. This paper reviews the theoretical background, computational techniques, recent advancements, and challenges associated with employing NURBS-based IGA and evolutionary algorithms for topology optimization of continuum structures.*

Keywords: **Non-Uniform Rational B-Splines (NURBS), Isogeometric Analysis (IGA), Topology Optimization, Evolutionary Algorithms, Genetic Algorithms (GA), Continuum Structures, Finite Element Methods (FEM).**

1. INTRODUCTION

Topology optimization is a pivotal methodology in engineering design that, aims to determine the optimal distribution of materials within a designated design domain to fulfill prescribed performance criteria. While traditional finite element methods (FEM) have been the cornerstone of structural analysis and optimization, their reliance on mesh generation often introduces geometric inaccuracies. In response, Isogeometric Analysis (IGA) emerged as a transformative paradigm, leveraging Non-Uniform Rational B-Splines (NURBS) to seamlessly integrate geometry and analysis. The amalgamation of NURBS-based IGA with evolutionary algorithms presents a potent framework for efficient and effective topology optimization, enabling exploration of intricate design spaces.

2. OBJECTIVE OF STUDY

I. Improving the productivity of the model with less time, effort and consumption using the proposed concept helps to improve the optimal solution.

II. To improve the ability of the optimization process by using a metaheuristic algorithm called firefly algorithm to the proposed concept which improves input to the optimization process.

III. To determine the nodal displacements of two-dimensional plate carrying in-plane loading.

IV. To Optimize the cross-section dimensions of the beam carrying transverse loading.

3. LITERATURE REVIEW

[1] Michael Kocvara approached "The structural optimization problem as a non-convex and nonlinear optimization challenge. Nonlinearity arises from equilibrium conditions contingent on displacement and thickness [52]". The primary objective of this study is to reframe non-convex mixed-integer nonlinear problems into integer linear conic problems, employing semi-definite, binary/integer linear, or second-order cone formulations. Demonstrating the tractability of truss topology optimization problems using YALMIP software, renowned for its adeptness in modeling both convex and non-convex problems, this paper presents numerical findings across various example scenarios including cantilever beams, truss bridges, single mass, and multiple mass problems. Additionally, it showcases initial design

configurations alongside binary and relaxed solution visualizations. An example is shown in Fig.1, where a cantilever beam is given below and the binary shown in Fig.2.

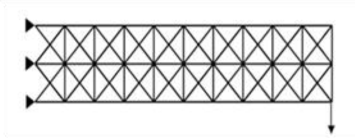

Fig.1 Cantilever with initial design

Fig.2 Cantilever with relaxed and binary solution

[2] W. Prager and J.E. Taylor present a method for addressing typical issues concerning sandwich structures [88]. They considered displacement or velocity and variable specific stiffness for conditions of minimum weight. When the specific structural weight is linearly related to specific stiffness, stiffness is not a factor. Consequently, the optimal condition for minimum weight is determined first by determining the displacement, followed by the variable specific stiffness derived from the differential equations of the structure. This paper outlines the conditions for "optimal elastic design for maximum stiffness, maximum fundamental frequency, maximum buckling load, and optimal plastic design for maximum safety, accompanied by relevant examples."

Prager outlined the optimal condition for maximum stiffness, derived from the principle of minimum potential energy, as $e[q(x;0)] = C^2/2$, where $e[q(x;0)]$ represents the scalar measure of strain energy at point x. For maximum fundamental frequency, the optimal condition, guided by the Rayleigh principle, is $e[q(x;0)] - b^2/2\ w^2[u(x;0)]2 = C^2/2$, indicating a constant difference between specific strain and specific kinetic energy per unit stiffness throughout the structure. The optimal conditions for maximum buckling load and plastic design for maximum safety are framed in a similar manner to the optimal condition for maximum stiffness, with a redefined meaning of $e[q(x;0)]$, which ensures maximum safety.

[3] Topology optimization originated in 1904 with Michell [51], who initially employed a lower volume fraction and illustrated its principles using a truss-like structure. In 1974, Rozvany expanded this concept by elucidating the general theory of topology optimization. He extended the optimal layout theory from truss-like structures to beam systems using a grillage. Rozvany collaborated with Prager in 1977 and Birker in 1994 to publish research on the theory. Initially applied to analytical solutions in grid-type structures, the theory later evolved to encompass higher volume fractions, particularly continuum structures. This evolution led to the application of numerical methods, as seen in Rozvany's generalized shape optimization proposal in 1992 and Haber's variable topology optimization concept in 1996.

[4] "Evolutionary Structural Optimization (ESO)" was introduced by Xie and Steven in 1992. This methodology constantly eliminates elements with low stress in the design domain. Building upon this approach, bidirectional evolutionary structural optimization (BESO) was developed by Querin et al. in 1998. BESO differs by simultaneously adding and removing elements. Both methods fall under gradient-based algorithms, converging to local optima, which represent the best solutions within a neighboring set of candidate solutions.

In contrast, genetic algorithms, proposed by Sandgren et al. in 1990, operate by generating a population of potential solutions rather than focusing on a single solution. Genetic algorithms explore a larger number of design variables, making them suitable for comparatively smaller problems owing to their ability to approach a broad range of potential solutions.

[5] As computational methodologies advanced, Hughes introduced Isogeometric Analysis (IGA), a novel approach incorporating NURBS basis functions, also known as Non-Uniform Rational B-Splines. Isogeometric Analysis bridges the gap between computer-aided design (CAD) and finite element analysis. Unlike the standard finite element method, which relies on Lagrange polynomial basis functions, Isogeometric Analysis (IGA) leverages NURBS functions commonly used in CAD. These functions accurately represent geometry, enabling direct utilization of CAD data to provide approximate solutions.

[6] Aleksey V. Pichugin, Andrew Tyas and Matthew Gilbert have presented a paper on "Michell structure which is carrying a uniformly distributed load which has an equal spaced pinned supports [3]." A.G.M Michell in his paper, formulated a criterion having equal tensile and compressive trusses and satisfied by a least volume truss. T: $f' < 0$, $f'' > 0$, $\varepsilon' = -\epsilon$, $\varepsilon'' = \epsilon$; RC: $f' = 0$, $f'' < 0$, $|\varepsilon'| \le \epsilon$, $\varepsilon'' = -\epsilon$; RT: $f' > 0$, $f'' = 0$, $\varepsilon' = \epsilon$, $|\varepsilon''| \le \epsilon$; These three regions are Michell's criteria which are used to get an optimal structure where f' and f" are force components, ε' and ε" are principal strains and ϵ is positive infinitesimal. The term 'Michell structure' means that it satisfies the Michell criteria. To calculate a

uniformly distributed load that carries an infinite set of equally distributed pin supports, a half-plane that transmits a uniformly distributed load to two pinned supports is considered.

[7] A.Tyas, A.V. Pichugin and M. Gilbert presented another paper to prove the existence of a least volume structure which is carrying a uniformly distributed load by satisfying optimality criteria [9]. Many studies have been conducted to find the least volume structure but in most of the cases it is assumed that load is directly applied to the suspended cable in tension or to an arch which is in compression. According to Rozvany and Prager in their research papers it is proved that the least volume solution is determined when the angle is 30 degrees between support and parabolic funicular.

[8] A.S.L.Chan presented a paper on design of Michell optimum structure. For any structural design, the main aim is to develop an optimum design by taking minimum weight criteria with equilibrium conditions when there is known external forces [8]. The application of Michell theory is done previously to elastic problems by Drucker and Shield and Prager has applied to limit design. Now, in this paper the analysis of geometrical layout of two-dimensional structures with the theory of plane plastic flow is done. Basically, two assumptions are done in Michell theorem. The first one is the stresses in all the members are equal to ± allowable stresses for tension and compression members. The second one is existence of virtual deformation of the region with vanishing displacement and strains along the members are equal to ± e, where e is the linear strain.

[9] Evolutionary algorithm is a population-based metaheuristic algorithm inspired by natural evolution. These are suitable for large complex problems such as multimodal and discontinuous etc., In this algorithm, the population of individuals are randomly generated and then in the iteration process, the first step is to evaluate the fitness of each individual and in the second step, new population is generated according to their fitness by selecting pairs of individuals from the current population and they are mated by recombination and mutation operator to generate new individuals and these individuals placed in place of least fit population [70]. This iteration process is continued until it reaches the termination criteria and the maximum number of generations is reached.

[10] "The inception of Isogeometric analysis was originally proposed by Hughes TJR [31]". This pioneering method employs fundamental functions derived from non-uniform Rational B-Splines to craft precise geometric models, transforming the analysis of solid, fluid, and structural problems governed by partial differential equations. Notably, this approach not only facilitates the creation of accurate geometric model but also simplifies mesh refinement by eliminating the need for continual interaction with the CAD geometry post-initial mesh construction.

This study explores several significant characteristics of NURBS, including:

1. NURBS basis functions satisfy the property of forming a partition of unity, ensuring an accurate representation of geometry.

2. Applying affine transformations to the control points enables corresponding transformations in physical space, illustrating the affine covariance property inherent in NURBS.

3. The continuity and support characteristics of NURBS basis functions align with those of B-splines, maintaining smoothness and compact support.

4. When all weights within a NURBS representation are equal, it simplifies to a B-spline form, characterized by piecewise polynomials.

5. NURBS surfaces and solids can be interpreted as the result of projective transformations applied to tensor product representations of piecewise polynomial entities, facilitating the modeling of intricate geometries and smooth surfaces.

[11] Nguyen and colleagues provided a comprehensive explanation of Isogeometric Analysis (IGA), detailing its foundational concepts and implementation, as well as its distinctions from conventional Finite Element Analysis (FEA) using MATLAB. They introduced Non-Uniform Rational B-Splines (NURBS) functions and highlighted their relevance, particularly in addressing plate and shell problems due to their capability for straightforward construction and rotation-free formulation, which is beneficial for thin shells. Additionally, they underscored the advantages of NURBS in structural vibration problems and the utility of IGA in solving partial differential equations with fourth-order derivatives. However, the paper also acknowledges drawbacks associated with NURBS, such as the challenge of producing water-tight geometries, which can complicate mesh generation. To facilitate understanding, the authors provided MATLAB implementation examples for one, two, and three-

dimensional Isogeometric Finite Element Analysis applicable to structural and solid mechanics.

4. CONCLUSION

In summary, the fusion of NURBS-based IGA with evolutionary algorithms has transformed topology optimization, providing a robust platform for crafting streamlined and lightweight structures. Despite notable advancements, hurdles surrounding computational demands, multi-objective optimization, and manufacturability limitations persist. Tackling these obstacles requires collaborative research and interdisciplinary co-operation. Embracing evolving methodologies and harnessing hybrid optimization approaches will enable researchers to fully exploit the capabilities of NURBS-based IGA in topology optimization, fostering innovation and progress in engineering design.

This study demonstrates the effective and efficient application of Isogeometric Analysis (IGA) for solving structural mechanics problems. The fundamental benefit of employing IGA lies in deriving basic functions directly from the geometry, which are then utilized to derive displacements. By accurately representing geometry using Non-Uniform Rational B-Splines (NURBS), IGA yields result that are more precise compared to Finite Element Analysis (FEA).

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