RADIUS OF CURVATURE IN AUTOMORPHISM STRUCTURE

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Abstract - *In mathematics, curvature is any concept that is closely related to geometry. Intuitively, a bend is the number of curves that deviate from a straight line, or an area that deviates from a plane. In curves, a canonical pattern is a circle, with a curve equal to the frequency of its surface. The smaller circles bend sharply, which is why they are so bent. Bending in place of a split curve is the bending of its osculating circle, which is the circle that best balances the curve closest to this point. Bending a straight line is zero. In contrast to the tangent, which is the vector quantity, the point curvature is usually the scalar size, i.e., expressed by one real number. As for the areas (and, especially the high-altitude masses), embedded in the Euclidean space, the concept of bending is more complex, as it depends on the choice of the surface surface or the masses. This leads to the concepts of high bending, low bending, and meaning bending. In most parts of the Riemannian (of at least two sizes) that are not included in the Euclidean space, one can define the inner bending, which does not refer to the outer space. See the Curvature of Riemannian manifolds for this definition, which is made according to the length of the curves followed by the repetition, and expressed, using straight algebra, by the Riemann curvature tensor.*

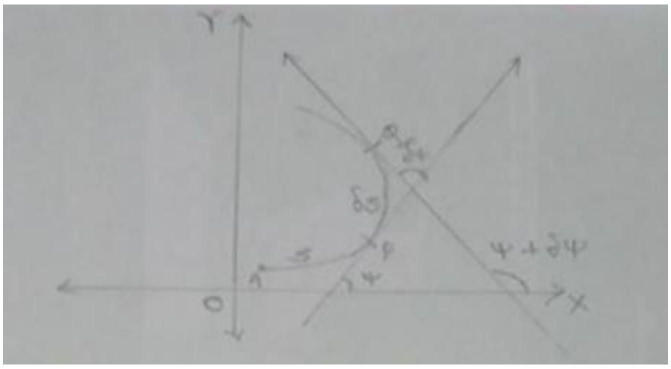
Key Words: *Total curvature, Average curvature, Circle of curvature, Centre of curvature.*

1.INTRODUCTION

Understandably, a curved curve describes any part of a curve as the curve of a curve shifts in the minimum distance traveled (e.g. angle to rad / m), so it is the fastest rate of change in the direction of a forward point curve: the larger the curve, the greater the degree of change. In other words, the curvature measures the speed of the unit tangent veggies [4] (faster depending on the curve). It can be proven that this rate of rapid change is precarious. Specifically, suppose a point moves in a curve at the constant speed of a single unit, i.e., the position of the point P (s) is a parameter function, which can be considered as time or as the length of an arc from a given origin. Let the Ts be the unit tangent vector of the P (s) curve, which is also found in the Ps relative to s. After that, the output of Ts (s) concerning s is a normal vector in a curve and its length is curved. To have a definition, the definition of a curve and its separation by various factors requires that the curve be separated continuously close to P, having a continuous tangent; it also requires the curve to be split twice in P, by ensuring the availability of the affected limits, as well as the availability of T (s). The curvature of the coefficient according to the discovery of the unit tangent vector may not be more accurate than the definition of a circular vein, but curvature application formulas are easier to obtain. Therefore, and also because of its use in kinematics, this feature is often given as a definition of bending.

1.2 Some Basic Definitions:

- **CURVATURE:**



Let P be a point on the curve and let Q be its neighboring point.

Let A be a reference point on the curve.

Further,

Let Arc AP = s and Arc PQ = δs

Let Tangent at P and Q makes angle Ψ and $\Psi + \delta\Psi$ with the x- axis.

Hence angle between the two tangents is,

$$(\Psi + \delta\Psi) - \Psi = \delta\Psi$$

This means that, the point on the curve moves from P to Q the tangent at P turns through an angle $\delta\Psi$

∴ If the curve is sharper then angle will be longer.

∴ $\delta\Psi$ = Total curvature

∴ $\frac{\delta\Psi}{\delta s}$ = Average Curvature

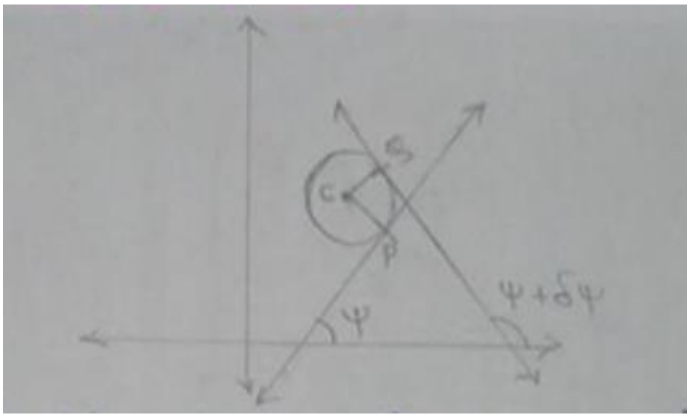
∴ Its limit as Q→P as $\delta s \to 0$ is called the curvature of curve at P.

$$\therefore \textit{Curvature at } P = \lim_{\delta s \to \infty} \frac{\delta\Psi}{\delta s}$$

$$\textit{Curvature} = \frac{\delta\Psi}{\delta s}$$

Where $\frac{\delta\Psi}{\delta s}$ *is actual curvature.*

- **CURVATURE OF CIRCLE :**



Consider a circle of center C and radius r. Let P and Q be two adjoint points on the circle.

Let Arc PQ = δs.

Let Tangent at P and Q make angle Ψ and $\Psi + \delta\Psi$

The angle between the tangent is $\delta\Psi$,

Since, radii CP and CQ are perpendicular to the Tangent.

∴< PCQ = $\delta\Psi$

We know that, Arc PQ = radius (< PCQ)

$$\Rightarrow Arc\ PQ = r\ (\delta\Psi)$$

$$\Rightarrow Arc\ PQ = r\ (\delta\Psi)$$

$$\Rightarrow Arc\ PQ = \delta s = r.\delta\Psi$$

$$\Rightarrow \quad \delta s = r.\delta\Psi$$

$\Rightarrow \frac{1}{r} = \frac{\delta\Psi}{\delta s}$ taking limit as $\delta s \to 0$

$$\Rightarrow \lim_{\delta s \to \infty} \frac{\delta\Psi}{\delta s} = \frac{1}{r}$$

$$\Rightarrow \frac{d\Psi}{ds} = \frac{1}{r}$$

∴ Curvature of the circle is constant at every point and its equal to the reciprocal of its radius.

The reciprocal of curvature is called the radius of curvature it is denoted by,

Radius of curvature $\rho = \frac{\delta s}{\delta\Psi}$

- **Radius of curvature for Cartésien Equation :**

A) If the tangent at point is parallel to x- axis,

∴ $\frac{dx}{dy} = \infty$ i.e. $\frac{dy}{dx} = 0$ in that case,

By defination of derivative,

$$\Rightarrow \quad y = f(x)$$

$$\Rightarrow \quad \frac{dy}{dx} = tan\Psi$$

Diff. w.r.t. x we get,

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \sec^2\Psi \frac{d\Psi}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = (1 + \tan^2\Psi)\frac{d\Psi}{ds}.\frac{ds}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \left(1 + \left(\frac{dy}{dx}\right)^2\right)\frac{1}{\rho}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow \left(1+\left(\frac{dy}{dx}\right)^2\right)\frac{1}{\rho}\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \left(1+\left(\frac{dy}{dx}\right)^2\right)\frac{1}{\rho}\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{1+\frac{1}{2}}}{\frac{d^2y}{dx^2}} = \rho$$

$$\Rightarrow \boxed{\rho = \frac{\left(1+\left(\frac{dx}{dy}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2x}{dy^2}}}$$

B) If the tangent at point is parallel to the y-axis, $\frac{dy}{dx} = \infty$ i.e. $\frac{dx}{dy} = 0$ In that case,

By defination of derivative,

$$\Rightarrow x = f(y)$$

$$\Rightarrow \frac{dx}{dy} = cot\Psi$$

$$\Rightarrow \boxed{\rho = \frac{\left(1+\left(\frac{dx}{dy}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2x}{dy^2}}}$$

- **Applicational exemples of radius of curvature :**

Ex. For the curve $y = c\cosh\frac{x}{c}$ find the radius of curvature.

Solution: Let the given equation of the curve is,

$$y = c\cosh\frac{x}{c} \qquad ....(1)$$

We know the formula for radius of curvature,

$$\boxed{\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}}$$

Therefore, diff. equation (1) w.r.t.x

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(c\cosh\frac{x}{c}\right)$$

$$\Rightarrow \frac{dy}{dx} = c.\frac{d}{dx}\left(\cosh\frac{x}{c}\right)$$

$$\Rightarrow \frac{dy}{dx} = c.\sinh\frac{x}{c}.\frac{1}{c}\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx} = \sinh\frac{x}{c} \qquad \text{then,}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{c}\cosh\frac{x}{c}$$

$$\therefore \boxed{\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}}$$

$$\Rightarrow \rho = \frac{\left(1+\sin^2 h\frac{x}{c}\right)^{\frac{3}{2}}}{\frac{1}{c}\cosh\frac{x}{c}}$$

$$\Rightarrow \rho = \frac{c\left(\cos^2 h\frac{x}{c}\right)^{\frac{3}{2}}}{\cosh\frac{x}{c}}$$

$$\Rightarrow \rho = c\frac{\left(\cos^3 h\frac{x}{c}\right)}{\cosh\frac{x}{c}}$$

$$\Rightarrow \rho = c\left(\cos^2 h\frac{x}{c}\right)$$

This is required solution.

Ex. Find the radius of curvature of the curve at any point on the curve, $y = a\log\sec\frac{x}{a}$

Solution: Let the given equation of the curve is,

$y = a \log \sec \frac{x}{a}$ ....(1)

We know the formula for radius of curvature,

$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

Therefore, diff. equation (1) w.r.t.x

$$\Rightarrow \frac{dy}{dx} = a \cdot \frac{1}{\sec \frac{x}{a}} \cdot \sec \frac{x}{a} \cdot \tan \frac{x}{a} \cdot \frac{1}{a}$$

$$\Rightarrow \frac{dy}{dx} = \tan \frac{x}{a}$$ then,

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^2\left(\frac{x}{a}\right) \cdot \left(\frac{1}{a}\right)$$

$$\therefore \rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

$$\Rightarrow \rho = \frac{\left(1 + \tan^2\left(\frac{x}{a}\right)\right)^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right) \cdot \left(\frac{1}{a}\right)}$$

$$\Rightarrow \rho = \frac{\left(\sec^2\left(\frac{x}{a}\right) \cdot\right)^{\frac{3}{2}}}{\sec^2\left(\frac{x}{a}\right) \cdot \left(\frac{1}{a}\right)}$$

$$\Rightarrow \rho = \frac{\text{a} \sec^3\left(\frac{x}{a}\right) \cdot}{\sec^2\left(\frac{x}{a}\right) \cdot}$$

$$\Rightarrow \rho = a \sec\left(\frac{x}{a}\right)$$

This is a required solution.

Ex. Find the radius of curvature of the curve $y = x^2 - 3x + 1$ at (1, -1)

Solution: Let the given equation of the curve,

$y = x^2 - 3x + 1$ ....(1)

We know the formula for the radius of curvature,

$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

Therefore, diff. equation (1) w.r.t.x

$$\Rightarrow \frac{dy}{dx} = 2x - 3$$ then,

$$\Rightarrow \frac{d^2y}{dx^2} = 2$$

Now,

$$\left(\frac{dy}{dx}\right)_{at(1,-1)} = 2(1) - 3$$

$$\left(\frac{dy}{dx}\right)_{at(1,-1)} = -1$$

$$\left(\frac{d^2y}{dx^2}\right)_{at(1,-1)} = 2$$

$$\therefore \rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2_{at(1,-1)}\right)^{\frac{3}{2}}}{\left(\frac{d^2y}{dx^2}\right)_{at(1,-1)}}$$

$$\Rightarrow \rho = \frac{(1 + (-1)^2)^{\frac{3}{2}}}{2}$$

$$\Rightarrow \rho = \frac{(1 + 1)^{\frac{3}{2}}}{2}$$

$$\Rightarrow \rho = \frac{2^{\frac{2}{3}}}{2}$$

$$\Rightarrow \rho = \frac{\sqrt{2}\,\sqrt{2}\,\sqrt{2}}{2}$$

$$\Rightarrow \rho = \frac{\sqrt{4}\,\sqrt{2}}{2}$$

$$\Rightarrow \rho = \frac{2\,\sqrt{2}}{2}$$

$\Rightarrow \rho = 2$

This is required solution.

Ex. If the curve is $y = \frac{ax}{a+x}$ then show that $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$

Solution : Let given equation of curve is,

$$y = \frac{ax}{a+x} \qquad ......(1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{(a+x) - ax(1)}{(a+x)^2} = \frac{a^2}{(a+x)^2}$$

$$so, \quad \frac{d^2y}{dx^2} = \frac{(a+x)^2(0) - a^2(0+2a+2x)}{(a+x)^4} = \frac{-a^2(2(a+x))}{(a+x)^4}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2a}{(a+x)^3}$$

But from equation (1)

$$(a+x) = \frac{ax}{y}$$

$$\therefore \frac{dy}{dx} = \frac{a^2}{\left(\frac{ax}{y}\right)^2} = \frac{a^2}{\frac{a^2x^2}{y^2}} = \frac{a^2y^2}{a^2x^2} = \frac{y^2}{x^2}$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{2a^2}{\left(\frac{ax}{y}\right)^3} = -\frac{2a^2}{\frac{a^3x^3}{y^3}} = -\frac{2a^2y^3}{a^3x^3} = -\frac{2y^3}{ax^3}$$

$$\therefore \boxed{\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}}$$

$$\Rightarrow \rho = \frac{\left(1+\frac{y^4}{x^4}\right)^{\frac{3}{2}}}{\frac{-2y^3}{ax^3}}$$

$$\Rightarrow -\frac{2\rho}{a} = \frac{x^3}{y^3}\left(1+\frac{y^4}{x^4}\right)^{\frac{3}{2}}$$

$$\Rightarrow \left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x^3}{y^3}\right)^{\frac{2}{3}}\left(1+\frac{y^4}{x^4}\right)$$

$$\Rightarrow \left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x^2}{y^2}\right)\left(1+\frac{y^4}{x^4}\right)$$

$$\Rightarrow \left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$$

This is required solution.

Ex. Find radius of curvature at (a,0) on the curve $y = x^3(x-a)$

Solution: Let given curve equation is,

$$y = x^3(x-a) \qquad ......(1)$$

$$\Rightarrow \quad y = x^4 - ax^3$$

$$\Rightarrow \quad \frac{dy}{dx} = 4x^3 - 3ax^2$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 12x^2 - 6ax$$

Now,

$$\left(\frac{dy}{dx}\right)_{at(a,0)} = 4a^3 - 3a^3 = a^3$$

$$\left(\frac{d^2y}{dx^2}\right)_{at(a,0)} = 12a^2 - 6a^2 = 6a^2$$

$$\boxed{\rho \; at \; (a,0) = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}}$$

$$\Rightarrow \rho \; at \; (a,0) = \frac{\left(1+(a^3)^2\right)^{\frac{3}{2}}}{6a^2}$$

$$\Rightarrow \rho \; at \; (a,0) = \frac{(1+a^6)^{\frac{3}{2}}}{6a^2}$$

This is required solution

3. CONCLUSIONS

In a different geometry, the radius of curvature, R, is a recurring method of bending. In a curve, it is equal to the width of a circular arc that better balances the curve at the same time. In extreme cases, a curvature is a circle that fits snugly in a common category or in a combination.

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