KINEMATICS OF ELECTRONIC DIFFERENTIAL SYSTEM THE RELATIONSHIP BETWEEN TRACK LENGTH AND THE ROTATIONAL DIFFERENCE OF THE REAR WHEELS

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Abstract - - *"A vehicle whose propulsion system is fully or partially powered by electricity is termed as an electrical vehicle" [1]. They were introduced around a century ago but due to the depletion of the fossil fuel resources and their adverse impact on the environment the electric vehicles have suddenly taken a boom. The conventional electrical vehicles use a set of electric motors with a combination of a rechargeable electric battery pack to propel the vehicle. As the industry is trying to improve the range and performance of the electrical vehicle there are some mechanical components that can be replaced with more compact electric components to serve the same purpose but with less frictional losses.*

In traditional drive-train a differential is used. It is a set of gears arranged in a specific manner with three shafts it felicitates different rotational speeds to each rear wheel while cornering. It uses a gearing ratio which determines the difference in the speeds of each wheel. The electronic differential system is a mechanism which uses electric control unit and set of sensors to determine the optimal torque distribution between the wheels.

By using clothoid[2] also known as Euler's curve[3] we can understand the transition curves[5] which are actively used in designing the highways and tracks. Using vector calculus, we can establish a relationship between the number of rotations of the rear wheels.

***Key Words*: Electric vehicles, Clothoid, Euler's curve, Transition curves, curved paths, Track length, Electronic Differential**.

1.INTRODUCTION

The weight of Electric Vehicles, including their structure and materials, has always been an area of interest for designers. They focus on reducing the body weight by optimizing the structure and form, often using aluminium as a preferred material. There have been advancements in motor design and control technology in vehicles. One notable improvement is the use of Electronic Differential (ED) instead of the traditional gearbox, which allows for a set of traction motor to drive both wheels. The speed of these individual-wheel can be managed by the ED, aligning the torque and rotation speed of the electric motor with the resistance and speed of the vehicle.

1.1 Why Electronic Differential System?

An Electronic Differential System (EDS) offers several advantages over traditional mechanical differentials such as enhanced traction control, improved stability and handling, increased safety, the main purpose of the electronic differential (ED) is to replace the mechanical differential with a system with individual-wheel drives.

1.2 Kinematics and its application in Electric Differential system.

Kinematics is a study of the system without considering forces that causes the motion. In electronic differential system the rotational motion of wheels, turning radius of the path and the curvature of the path are all deduce into a relation by using simple vector calculus[4].

The "clothoid[2] is a curve whose curvature changes linearly with its curve length". It has various applications in differential computation. They are also actively used in designing roads and railway tracks since they require transitional curves to connect linear paths to curved paths.

By understanding the transitional curves[5] and their effects on the kinematics of the vehicle we can understand the working of a differential system. Whenever a vehicle is on a curved path the front wheels turned in the direction of the curve and the angle of deviation of the front wheel from the mean position is estimated using Ackermann steering geometry[6] which provides us the angle of turn for the front wheels from the mean position based upon the curvature of the path.

The Transitional curves[5] are curves whose curvature is continuously changing from the start of the curve at the tangent having infinite curvature to a smooth curve having a nominal curvature. They are always having a gradual transition without any sudden or abrupt change in relation to the automobiles it can result in the change of centripetal force on the vehicle. At any instant when the vehicle is on the curved path, we can establish a relationship for the rotation of the two rear wheels and the curvature of the path.

2. METHODOLOGY

Majority of electric four wheelers have a set of electric motors at the rear wheels that is one for each wheel. This facilitates individual traction control to each wheel which is controlled by the motor-controller. The working concept behind the electronic differential can be easily understood by understanding the interaction between the vehicle and the surface, in this case the surface is the highway. The highways and railways are designed using clothoid[2] also known as Euler's curve[3] which is nothing but a curve whose curvature changes linearly with its curve length. It can be also stated as the curve whose curvature is the reciprocal of the radius of the curve.

By taking an analytical approach and performing some computations we can relate the difference between the rate of rotation of the track wheels to the Euler's curve. The curvature of a straight path is infinite and the radius of curvature of the path is zero.

In electronic differential system the overall power to both the electric motors at the rear wheels is same, the torque and angular velocity may vary considerably. In electric vehicles the electric current represented by (I) is responsible for tractional force and the potential difference represented by (V) is responsible for angular velocity of the motor.

When the vehicle is running on a circular path the difference between the angular velocities of the rear wheels can be computed by considering a simple vehicle on a circular path. Consider a vehicle let a vehicle is moving with a uniform velocity on a circular path with no slip condition and deviation from the path, the specification of the vehicle is taken from a real vehicle Tata Tiago EV[7] represented in *Table -1: Technical Specification of the reference vehicle.* The path of the vehicle is shown in. **Figure 1:** Curve traced by the reference vehicle on a circular path.

Parameters	Technical Information
Torque	114 Nm
Tyres [1175/65 R14] diameter	0.5842 m
Wheel base	2.4 m
Track length	1.502 m
Minimum turning circle diameter	5.1 m

Table 1: Technical Specification of the reference vehicle.

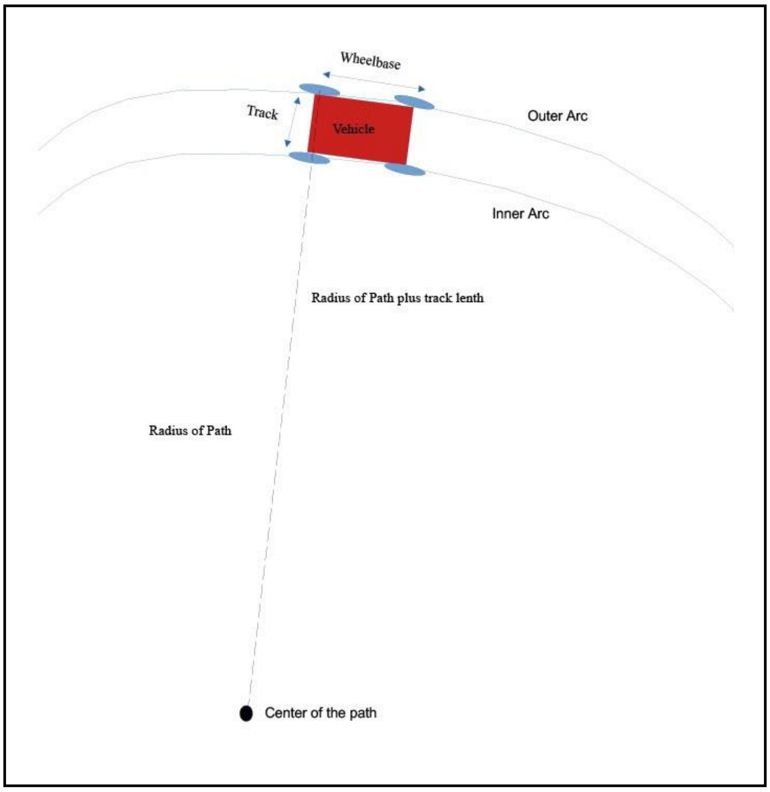

Figure 1: Curve traced by the reference vehicle on a circular path.

3. CALCULATIONS

The calculations are based on clothoid[2] and Vector Calculus[4] we consider a simple vehicle moving on a circular path having a uniform velocity, the distance covered by the vehicle in one minute is considered to eliminate the time derivative (velocity). The length of arcs is denoted by (S) using subscript 1, 2 for inner and outer arc. Let the outer wheel trace an arc be $\mathbf{S_1}$ and the inner wheel trace an arc be $\mathbf{S_2}$ the radius of curvature of the path be (r) and the angle subtended by the arc at the centre be (ϴ) in radians, consider the track length of the vehicle be (dr) since the value of track length as compare to the radius is very small (dr <10% of (r)).

3.1 Equation (1): -

The first equation is for the outer arc, it gives the displacement of the outer wheel covered in one minute. The outer arc equation is derived from clothoid[2] relation which is as follows.

The length of the circular arc (S), is the product of the radius of the path (r) and the angle subtended by the arc at the centre (ϴ).

The first equation is for the outer arc denoted by S_1

$$\mathbf{S_1 = [\,\Theta\,(r + dr)]}$$

3.2 Equation (2): -

The second equation is for the inner arc, it gives the displacement of the inner wheel covered in one minute. The inner arc equation is also derived from clothoid[2] relation. The second equation is denoted by S_2.

$$\mathbf{S_2 = [\,\Theta\,(r)]}$$

3.3 Equation (3): -

The difference between the number of rotations of the outer wheel and the inner wheel is the required relation we can deduce this by taking the difference of the arcs traced by each wheel denoted by (ϕ)and dividing it by the circumference (C) of the wheel which gives us the difference between the number of rotations of the rear wheels of the vehicle.

$$\mathbf{[(\phi_1) - (\phi_2)] = \left\{\frac{S1-S2}{C}\right\}}$$

By using equation 1, equation 2, equation 3 and
Table *1*: Technical Specification of the reference vehicle. following table is calculated.

Number of observations	Track Length (dr) m.	Radius of Path (r) m.	Angle subtended at the centre (Θ) Radian.	Outer Arc length (S_1) m.	Inner Arc length (S_2) m.	Circumference of the wheels (C) m.	Difference Of number of rotations of wheels $[(\phi_1) - (\phi_2)] = \frac{[S1 - S2]}{(C)}$
I	1.502	5.1	1.57079	10.37035	8.011029	1.835318	1.28551348
II	1.502	150	1.57079	237.9778	235.6185	1.835318	1.28551344
III	1.502	300	1.57079	473.5963	471.2370	1.835318	1.28551348
IV	1.502	600	1.57079	944.8333	942.4740	1.835318	1.28551347

Table 2: Calculations for difference of number of rotations for rear wheels.

4. RESULTS

In each subsequent set of computation the radius is increasing, starting with the minimum turning radius of the reference vehicle that is 5.1 meters reaching up to 600 meters which are big turns a vehicle must take on highways and ring roads. And for each of these circular turns the vehicle traces an arc one from the inner wheel and one from the outer wheel for each turn with increasing radius of the path the arc length increases since radius and arc is directly proportional. But the difference between the number of turns of the outer wheel and the inner wheel is same up to six decimal places in all the four computations i.e. (1.285513 turns)

This is because in each computation the radius of the path changes but the track length denoted by (dr) remains same the track length is the factor which determines the difference between the number of rotations of each rear wheel.

$[(\phi_1) - (\phi_2)]$ is proportional to the **track length**

As the curvature of the path increases the difference between the number of rotations of the wheel's remains constant because the track length remains constant**.**

3. CONCLUSION

By understanding the observations from the calculation, we can establish a relationship between the number of rotations of the wheel and the track length. We can justify from our observations that: -

The difference between the number of rotation of wheels is directly proportional to the track length of vehicle.

$[(\phi_1) - (\phi_2)]$ is proportional to **Track length**

The results contribute towards better understanding of the kinematics of electric differential system, the design, and the physics behind it. Hence track length is the factor which will help in developing the algorithm for the electric control unit.

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