

# Comparative Analysis of Waffle Slab Behavior Using Analytical Theories and Structural Design Approaches

Alifa Mansoori <sup>1</sup>, Smita B. Patil <sup>2</sup>

<sup>1</sup>ME, Structural Engineering, Dept. of Civil Engineering, Datta Meghe College of Engineering, Airoli, Navi Mumbai, India

<sup>2</sup>Associate professor, Dept. of Civil Engineering, Datta Meghe College of Engineering, Airoli, Navi Mumbai, India

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**Abstract** - This study explores the structural behavior and analysis of waffle slabs by various approaches. Waffle slabs are well known for their high strength-to-weight ratio and material efficiency. This study investigated by analytical theories and numerical simulations using FEM. The research performs comparative analysis of deflection predictions using Kirchhoff's Plate Theory, Timoshenko's Plate Theory, and Finite Element Analysis (FEA) and concludes that FEA provides better accuracy. Timoshenko's theory with shear deformation shows close results with FEA than Kirchhoff's theory, especially for thinner slabs.

The study also optimized waffle slab design parameters—rib spacing, depth, and slab thickness—according to ACI 318-14 provisions. FEA in Abaqus analyzes failure modes under ultimate loads, verifying results against experimental results and also carried out analysis in ETABS to determine the impact of design variables on structural performance. Major conclusions are the need for FEA methods for accurate deflection prediction, the efficiency of automated optimization in minimizing material costs. The results offer real-world advice for engineers and highlighting the incorporation of computational techniques to optimize structural efficiency in waffle slab design. The research fills gaps in conventional analytical solutions and presents a complete framework for waffle slab design and analysis.

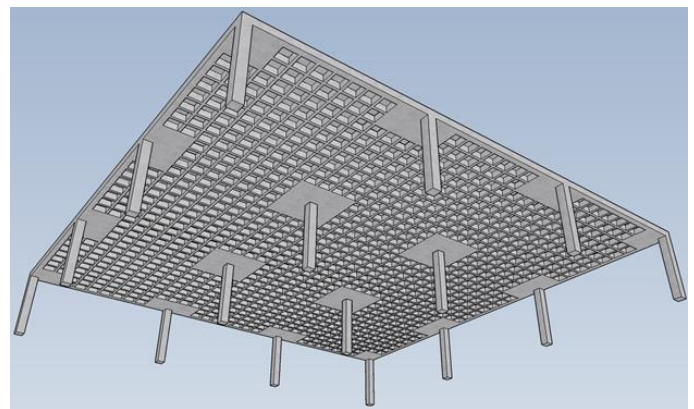
**Key Words:** Waffle slabs, Structural behaviour, High strength-to-weight ratio, Material efficiency, Analytical theories, Numerical simulations, Finite Element Method (FEM), Comparative analysis

## 1. INTRODUCTION

Waffle slabs have proved to be an effective structural solution for buildings with large column with longer span.[1]. They are also known for their improved load-carrying capacity and minimized material use. These slabs have a monolithic form of a thin top slab and crossing ribs in two perpendicular directions [2]. These are extensively used in commercial as well as industrial buildings. Waffle slabs' shape reduces dead weight without compromising high structural efficiency that is why they are an ideal choice over conventional flat slabs [3]. Recent advances in structural analysis and design software, particularly finite element modelling (FEM) and design software have enabled engineers to perform thorough

analyses of waffle slabs [4]. The application of nonlinear analysis and parametric studies has helped understand behaviour of these slabs and their failure modes [5]. These programs have also helped optimization methods for them further. Despite these advancements there is still a limitation on accurately predicting waffle slab responses. Also, there is a knowledge gap in design variable optimization, and cost saving improvement.

Waffle slabs also known as a grid slabs are reinforced concrete slab system made by a network of ribs running in perpendicular directions [6]. These ribs create a creating a grid-like pattern, resembling like a Waffle, hence they are called as Waffle slabs. A typical rib and deck arrangement of these slabs is as shown in figure below.



**Fig-1:** A two-way waffle slab with ribs running in perpendicular direction (Olawale et al, 2020)

## 2. INTRODUCTION

The article [6] "Two-Way Joist Concrete Slab Floor (Waffle Slab) System Analysis and Design" by StructurePoint is a complete method of designing waffle slabs based on ACI 318-14. It is centered on the Equivalent Frame Method (EFM) for two-way joist system analysis, explaining the structural response under different loading conditions. The article describes the step-by-step procedure for calculating flexural strength, shear strength, and serviceability criteria like deflection limits. By combining theoretical computations with software analysis, the research guarantees precision and adherence to design specifications. A major contribution

of this paper is its application in practice of ACI 318-14 provisions, and as such, it is a worthy reference for engineers dealing with waffle slabs. The authors present design examples of manual calculation and compare it with software analysis. It shows the critical role of exact rib spacing, depth, and proper reinforcement placing. This process maximizes structural effectiveness. This paper includes an in-depth guideline for designers and researchers on how to get an understanding of and implement ACI 318-14 criteria in waffle slab design.

This research by Olawale et al [7] the optimization of reinforced concrete waffle slab design by using genetic algorithms. It is done to obtain cost-effective and efficient designs. The authors check different design parameters. These parameters were rib spacing, slab thickness and reinforcement details etc. By the combination of this optimization methods with conventional design methods the research showing how genetic algorithms make waffle slab design more efficient than traditional methods. The authors also find the problems of traditional design methodologies and suggest a new approach with reduced material use. This research is significant to understand the role of artificial intelligence in structural engineering. This method especially useful in optimizing slab structures under ACI 318-14 standards. The study is an addition to the expanding body of research on automated design processes and is hence a valuable reading for practicing engineers and researchers interested in novel methods for waffle slab optimization.

This research by Salomão et al [8] study into slab system selection utilizing a multi-criteria decision-making method. The authors shows that even if structural engineers usually make their decisions based on material consumption and design compatibility, other important elements like productivity, aesthetics, sustainability, and user comfort are mostly neglected. There is also a research gap in their opinion. To close this gap the research uses the Analytic Hierarchy Process (AHP). They also verified their results with using the Delphi Method and machine learning applied to a database of more than 2000 already built slabs.

The research has proven that AHP works well to incorporate subjective and objective elements, giving a systematic approach for comparing alternative slab systems. The Delphi Method has also been extensively applied to improve decision models making sure the credibility of chosen criteria. Including of machine learning in this work improves forecasting accuracy by the identification of patterns in vast datasets of construction activities. The current research indicates that solid slabs are often most commonly used based on ease of construction and overall usage, with waffle slabs being attractive in terms of lower weight and higher span effectiveness. As per these authors most studies typically fail to be based on a holistic model encompassing large-scale construction and operation factors. In integrating AHP, expert examination, and machine learning techniques,

this paper progresses earlier investigations through the implementation of a holistic and applicable construction industry decision model.

This research by Pechorskaya and Sophia [9] discusses the structural analysis of high-rise buildings using ETABS and RSA software. The aim of the paper is to compare their performance in design use. The authors say that despite many software applications being available for structural analysis it is still challenging for engineers to select appropriate application. Due to the simplicity and graphical interface of 3D analysis programs, structural engineers increasingly apply them. This research is comparing the performance of a thirty-story reinforced concrete building under gravitational and lateral loads, including wind loads, using both ETABS and RSA.

These researchers are based on existing studies that have investigated the contribution of software. Most studies are based on increasing modeling efficiency, load analysis, and design accuracy. Research has indicated that ETABS and RSA are commonly applied in high-rise building analysis, with enhanced simulation capabilities. Existing literature indicates limitation in the results produced by various software as a result of differences in their algorithms and modeling strategies. This research makes an addition to this area by contrasting axial forces and moments derived from both programs and thereby indicating that RSA tends to yield greater force and moment values compared to ETABS. This presentation of the differences, the study offers structural engineers' useful information for choosing the best-suited software depending on the needs of their projects.

This research by Anjali Mishra and Anurag Bajpai [10] examine the investigation of waffle slabs from various computational and theoretical approaches. This study is based on assessment of structural performance. A two-way joist slab structure made up of intersecting ribs at regular grid points, made to create column-free spaces. The study aims to compare analysis methods such as Rankine Grashoff's method (as an approximate), Timoshenko's plate theory (as an exact), and the stiffness approach with ETABS 2018 software. From a comparison of reinforced concrete slabs of differing sizes—12×18m, 18×24m, and 24×30m—the work studies significant flexural parameters, namely bending moments ( $M_x$  &  $M_y$ ), shear forces ( $Q_x$  &  $Q_y$ ), and mid-span deflections. These researchers expand on previous research that identifies the benefits of waffle slabs in minimized material cost. They also showed how waffle slab enhanced structural efficiency, and better load transfer. Past studies have proven that waffle slabs provide superior structural behavior in large-span structures through minimized deflection at the same strength. The present investigation further enhances the understanding of the behavior of waffle slabs in the light of IS 456 2000.

### 2.1 Gap in the literature

While waffle have many applications because of their efficiency in structural application, especially under long-span buildings. So, they have been analyzed for years but some gaps in knowledge exist within current literature. Few studies concentrate either on simplified analyses or numerical analysis without taking a complete comparative approach towards both. Conventional analytical models like Kirchhoff's and Timoshenko's plate theories are usually used separately and not often compared to highly accurate finite element simulations. This reduces the knowledge of their respective accuracy and applicability.

In addition, even though finite element analysis is now widely used in structural engineering, few research studies have comprehensively compared its outcome with analytical predictions using design codes like ACI 318-14. The significance of geometric parameters—rib spacing, rib depth, and slab thickness—in designing waffle slab behaviour is also not yet fully explored. Also, even with the existence of software such as ETABS and Abaqus, there isn't much recorded workflow with optimization.

### 3. METHODS AND MATERIALS

The analysis of these slabs involves understanding parameters such as deflection bending moments shear forces and stress distribution. There are multiple methods to analyze waffle slabs viz: Grashoff method, Timoshenko's Plate Theory, Yield Line Theory and Finite Element Analysis (FEA). According to [10] the Rankine-Grashoff method is a simplified method for analyzing slabs. It treats the waffle slab as a series of connected beams. This assumption simplifies the calculation of moment and deflection. To understand the behaviour of these slabs, lets assume a1 and b1 are the spacing along X and Y directions respectively.

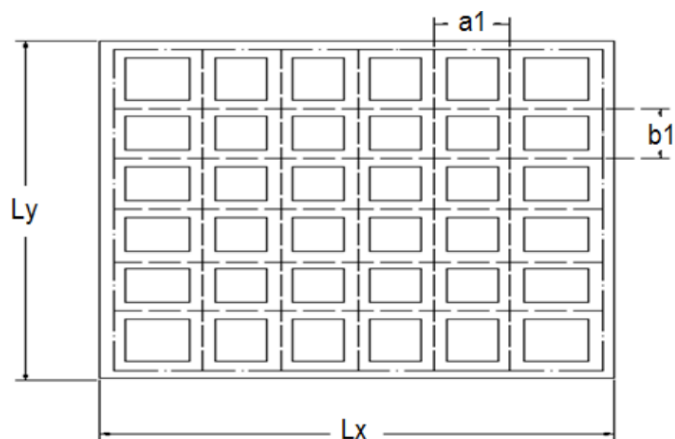


Fig -2: Top view of waffle slabs from bottom direction showing longitudinal and transverse ribs [23].

Kirchhoff's plate theory [15]

According to Rankine-Grashoff theory deflections of ribs at junctions are made equal and is given by

$$\Delta = \frac{5q_1a^4}{384} = \frac{5q_2b^4}{384} \quad \text{----- (1)}$$

Where,, q1 and q2 are the loads shared in X and Y directions, respectively. These values of the loads are given by,

$$q_1 = q \left( \frac{b^4}{a^4 + b^4} \right) \quad \text{----- (2) and}$$

$$q_2 = q \left( \frac{a^4}{a^4 + b^4} \right) \quad \text{----- (3)}$$

The bending moments for the central ribs are given by,

$$M_{AB} = \left( \frac{q_1 b_1 a^2}{8} \right) \quad \text{----- (4) and}$$

$$M_{BD} = \left( \frac{q_2 a_1 b^2}{8} \right) \quad \text{----- (5)}$$

These are equivalent formulae for design of two way slabs because Rankine-Grashoff method is an approximate analytical technique used for the analysis of two-way slabs. Because of this it has several limitations when applied to waffle slabs, which have a grid-like ribbed structure rather than a uniform plate.

Timoshenko's Plate theory [11]

Timoshenko's Plate Theory is an advanced structural analysis method used to study the behavior of plates. It extends the classical Kirchhoff Plate Theory. It does it by incorporating the effects of shear deformation and rotary inertia. This makes it more accurate for analyzing complex structural systems like waffle slabs.

$$q(x, y) = D \nabla^4 w - \beta \nabla^2 w + ph \frac{\partial^2 w}{\partial t^2} \quad \text{----- (6)}$$

Where,

D - Flexural Rigidity of the Plate given by

$$D = \frac{Eh^3}{12(1 - \nu^2)} \quad \text{----- (7)}$$

$\nabla^4 w$  – Biharmonic Operator (Fourth-Order Derivative) given by,

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \quad \text{----- (8)}$$

This formulation represents the bending deformation in the plate. Also  $\beta$  – Shear Deformation Coefficient is given by,

$$\beta = \frac{Gk_s h}{D} \quad \text{----- (9)}$$

Kirchhoff's theory assumes plane sections remain normal to the mid-surface, but Timoshenko's theory considers transverse shear deformations. Due to this there is a development of bending and shear. The rotations due to bending and shear are given by,

$$\theta_x = \frac{\partial w}{\partial x} + \frac{Q_x}{k_s G h} \quad \text{----- (10)}$$

$$\theta_y = \frac{\partial w}{\partial y} + \frac{Q_y}{k_s G h} \quad \text{----- (11)}$$

where  $Q_x$  and  $Q_y$  are shear forces per unit length. Also the bending moments per unit width in the x and y directions are given by:

$$M_x = -D \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad \text{----- (12)}$$

$$M_y = -D \left( \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \quad \text{----- (13)}$$

$$M_{xy} = -D(1-\nu) \frac{\partial^2 w}{\partial xy} \quad \text{----- (14)}$$

$M_x, M_y$  = Bending moments per unit width in the x and y directions (N·m/m).

To understand the difference between Kirchhoff's theory and Timoshenko's theory, we simulated waffle slab was simulated with equal loading conditions. The deflection was calculated incrementally for varying slab thicknesses from 100 mm to 300 mm. Kirchhoff Plate Theory with transverse shear deformation assumed to be negligible, was contrasted with Timoshenko's Plate Theory which considers transverse shear effects. Calculations were done based on established governing equations for both theories. From the graph, Kirchhoff's theory is seen to predict values of deflection that

are slightly lower compared to those predicted by Timoshenko's theory.

### Finite Element Analysis

Finite Element Analysis (FEA) differs highly from both Kirchhoff's Plate Theory and Timoshenko's Plate Theory in its approach to analyzing waffle slabs. Finite Element Analysis (FEA) is quite different from Kirchhoff's Plate Theory and Timoshenko's Plate Theory in how it goes about analyzing waffle slabs. Whereas Kirchhoff's and Timoshenko's theories are based on analytical solutions with pre-established assumptions. FEA gives a numerical solution by breaking down the slab into smaller finite elements and solving for their interactions [16]

FEA tends to yield data closer to experimental findings as it does not make limiting assumptions about the slab's performance [17]. FEA can model different boundary conditions, uneven loads and complex structural geometries with ease. However, FEA accuracy depends on the mesh, the type of elements, and the resources available, and these need to be well assessed while performing such analyses [18].

The first step in FEA is to divide the waffle slab into finite elements (typically shell, solid, or beam elements). The nodes of these elements represent the degrees of freedom (DOFs) of the structure. Each node has displacement components  $u, v, w$  and, in some cases, rotational DOFs  $\theta_x, \theta_y$ . The element deflection then can be calculated using,

$$w(x, y) = \sum_{i=1}^n N_i(x, y) w_i \quad \text{----- (15)}$$

Where,  $w(x, y)$  = deflection at a given point,  $N_i(x, y)$  = shape functions,  $w_i$  = nodal displacement.

The general equilibrium equation for plate bending in FEA is derived from the weak form of the governing differential equation. Using the principle of virtual work, the equilibrium equation can be written as [19].

$$[K]\{d\} = \{f\} \quad \text{----- (16)}$$

= Global stiffness matrix of the waffle slab,  $\{d\}$  = displacement vector (unknowns to be solved),  $\{F\}$  = applied force vector.

For a thin plate, the stiffness matrix is derived from the bending strain energy:

$$[K] = \int_A [B]^T [D] [B] dA \quad \text{----- (17)}$$

Where,  $[B]$  = strain-displacement matrix,  $[D]$  = material stiffness matrix,  $dA$  = differential element area.

For a plate bending element, the stiffness matrix includes bending terms:

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad \text{----- (18)}$$

Where, E =Young’s modulus, h= slab thickness = Poisson’s ratio. Once the global stiffness matrix is assembled, boundary conditions and loads are applied. The system of equations is solved to obtain the displacements at each node. Bending moments in the slab are obtained from.

$$M_x = -D \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad \text{----- (19)}$$

$$M_y = -D \left( \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \quad \text{----- (20)}$$

$$M_{xy} = -D(1-\nu) \frac{\partial^2 w}{\partial xy} \quad \text{----- (21)}$$

The graph shows a comparative analysis of deflection values in a waffle slab. It is obtained using three different methods: Kirchhoff’s Plate Theory, Timoshenko’s Plate Theory, and Finite Element Analysis (FEA). As slab thickness increases the difference between Kirchhoff and Timoshenko’s models reduces. This is indicating that shear deformations become less significant. This confirms work done by [20]. The deflection values are slightly different from both analytical theories. This happens due to the consideration of additional factors such as material non-linearity, boundary conditions, and load distribution [21]. For thicker slabs, all three methods show relatively similar results, as the influence of shear deformation reduces as thickness increases [22].

#### 4. RESULTS AND DISCUSSION

The analysis of the waffle slab was carried out using ETABS version 20.0.0. The structure was modeled considering a Cartesian grid system with grid lines spaced at 10 meters in the X-direction and 8 meters in the Y-direction. The slab was divided into multiple panels based on point coordinates defined precisely to match the bay layout typical for a waffle slab system. The resultant slab system can be seen in the figure below.

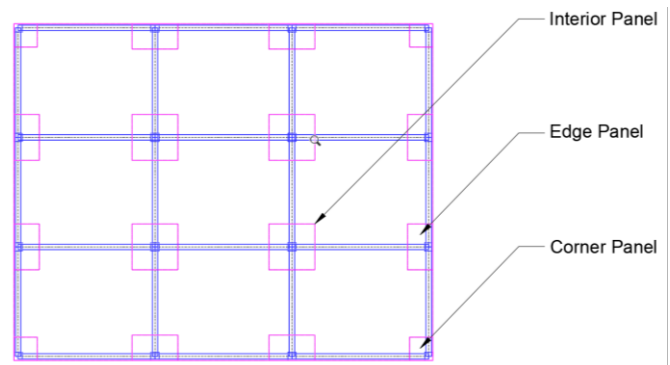


Fig -3: layout of the waffle slab used for analysis.

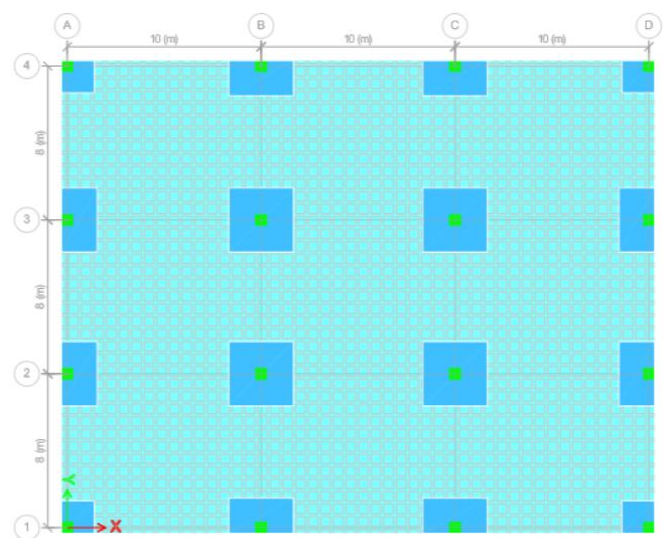


Fig -4: Fig Top view of the waffle slab system.

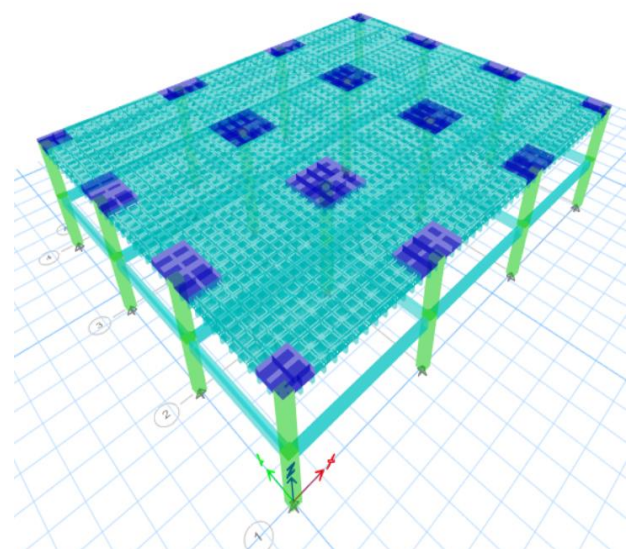


Fig -5: 3D view of the waffle slab system.

**Table-1:** Summary of sections used for analysis

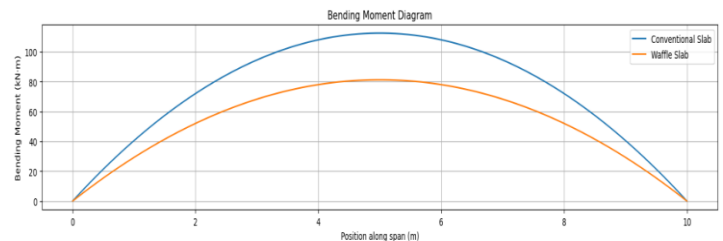
Element	Section Name	Material	Shape	Dimensions/ Properties
Column	COL 600	M25 Concrete	Rectangular	600 mm × 600 mm
Beam	Floor Beam	M25 Concrete	Rectangular	450 mm × 350 mm
Beam (Plinth)	Plinth Beam	M25 Concrete	Rectangular	600 mm × 400 mm

The material properties used included M25 grade concrete for the slab, beams, and columns, along with Fe500 grade reinforcement. The slab was modeled as a shell element with the material property set to M25 concrete, and appropriate thickness was assigned to represent the waffle slab's behavior accurately. Dead loads and live loads were assigned through area load patterns defined in the model. Load combinations were generated as per standard design practices to ensure the structure was checked for all possible critical load conditions.

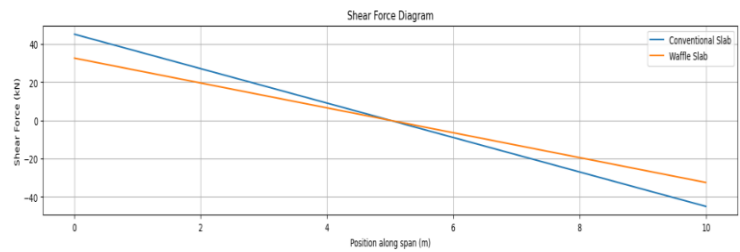
The mass source for dynamic analysis was defined by considering both self-weight and applied loads, without lumping mass at nodes, ensuring a realistic distribution of seismic forces. Modal analysis was conducted to determine natural frequencies, modal mass participation ratios, and mode shapes. After defining the model, structural analysis was performed to obtain results for base reactions, story drifts, story forces, and point displacements. The slab behavior under different load cases was evaluated by studying the deflection patterns, bending moments, and shear

forces. All relevant outputs were extracted and interpreted to assess the performance of the waffle slab system under the applied loads. The analysis results were then used to validate the adequacy of the design and to check serviceability criteria such as deflection limits.

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**Fig -6:** Variation of bending moment across the span of waffle slabs.



**Fig -7:** Variation of shear force across the span of waffle slabs.

**Table-2:** Summary of ETABS analysis results

Position (m)	Shear Conv (kN)	Shear Waffle (kN)	Moment Conv (kN.m)	Moment Waffle (kN.m)	Deflection Conv (mm)	Deflection Waffle (mm)
0	45	32.5	0	0	0	0
0.90909	36.81818	26.59091	37.19008	26.8595	16.77652	9.693487
1.91919	27.72727	20.02525	69.7888	50.40302	33.58837	19.40739
2.929293	18.63636	13.4596	93.20478	67.31456	46.87896	27.08671
3.939394	9.545455	6.893939	107.438	77.59412	55.45374	32.04122
4.9495	0.454545	0.328283	112.4885	81.24171	58.58658	33.85138
5.9596	-8.63636	-6.23737	108.3563	78.25732	56.01983	32.36831
6.9697	-17.7273	-12.803	95.04132	68.64096	47.9643	27.71382
7.9798	-26.8182	-19.3687	72.54362	52.39261	35.09926	20.28039
8.9899	-35.9091	-25.9343	40.86318	29.5123	18.57243	10.73117
10	-45	-32.5	0	0	0	0

## 5. CONCLUSIONS

This study completely investigated the structural behavior of waffle slabs by using analytical methods and numerical methods with ETABS analysis too. A comparison by using Kirchhoff's Plate Theory, Timoshenko's Plate Theory, and Finite Element Analysis (FEA) showed that FEA is the best for making deflection predictions but ETABS is best for practical design and analysis. Among analytical techniques, Timoshenko's theory, considering shear deformation, exhibited better conformity with FEA outcomes than Kirchhoff's theory, especially when dealing with thin slabs. The study also emphasized important design parameters like rib spacing, rib depth, and slab thickness according to ACI 318-14 recommendations, resulting in better utilization of materials. ETABS analysis simulations confirmed the failure mechanisms and structural behavior under various loads. Overall, the results highlight issues with the classical methods of analysis. In understanding behavior of waffle slabs and recommend the use of sophisticated computational tools in the design process. The paper offers useful practical guidelines for engineers who seek to attain material efficiency and structural security in waffle slab systems.

## REFERENCES

- [1] Design of solid ribbed and waffle slabs for residential buildings: A review. *Indian Journal of Engineering*, 2023, 20, e14ije1014 doi: <https://doi.org/10.54905/disssi/v20i53/e14ije1014>
- [2] Curth, Alexander & Hartwell, Ashley & Brodesser, Tim & Mueller, Caitlin. (2022). Parametric waffle slabs: Optimal geometry materialized with additive construction.
- [3] Portland Cement Association, Flat Slabs and Waffle Slabs: Analysis and Design: Reinforced Concrete Design Programs for Desk Calculators. Portland Cement Association, 1972.
- [4] Plos, Mario & Johansson, Morgan & Zandi, Kamyab & Shu, Jiangpeng. (2024). Recommendations for structural assessment of RC slabs using the finite element method. 10.1201/9781003483755-453.
- [5] Al-Dala'ien, Rayeh Nasr, Agusril Syamsir, Mohd Supian Abu Bakar, Fathoni Usman, and Mohammed Jalal Abdullah. 2023. "Failure Modes Behavior of Different Strengthening Types of RC Slabs Subjected to Low-Velocity Impact Loading: A Review" *Journal of Composites Science* 7, no. 6: 246. <https://doi.org/10.3390/jcs7060246>.
- [6] Singh, S. (1994). Flat Slab and Waffle Slab Systems. In: Cost Estimation of Structures in Commercial Buildings. Macmillan Building and Surveying Series. Palgrave, London. [https://doi.org/10.1007/978-1-349-13030-6\\_4](https://doi.org/10.1007/978-1-349-13030-6_4)
- [7] Olawale, Simon & Akintunde, Olutosin & Afolabi, Mutiu & Tijani, Murtadha. (2020). Design Optimization of Reinforced Concrete Waffle Slab Using Genetic Algorithm. 4. 46-62. 10.22115/SCCE.2020.224460.1195.
- [8] Nithyambigai G., Rameshwaran P.M., Stella Mary F., Behaviour of waffle slab, *Materials Today: Proceedings*, Volume 46, Part 9, 2021, Pages 3765-3768, ISSN 2214-7853, <https://doi.org/10.1016/j.matpr.2021.02.016>.
- [9] J. Prasad, S. Chander, and A. K. Ahuja, "Optimum dimensions of waffle slab for medium size floors," *Asian Journal of Civil Engineering (Building and Housing)*, vol. 6, no. 3, pp. 183-197, 2005.
- [10] Ali Alraie, Manoranjan Barik, Effect of continuity on reduction factors of bending moments and shear forces in grid slabs, *Journal of Building Engineering*, Volume 13, 2017, Pages 291-297, ISSN 2352-7102, <https://doi.org/10.1016/j.job.2017.08.010>.
- [11] S. P. Timoshenko and S. Woinowsky-Krieger, *Theory of Plates and Shells*, 2nd ed. New York, NY, USA: McGraw-Hill, 1959.
- [12] S. P. Timoshenko and D. H. Young, *Theory of Structures*, 2nd ed. New York, NY, USA: McGraw-Hill, 1965.
- [13] W. J. M. Rankine, *A Manual of Applied Mechanics*, 6th ed. London, UK: Charles Griffin & Company, 1872.
- [14] M. Yoosaf K.T., R. S., and J. Ramanujan, "Finite element analysis and parametric study of grid floor slab," *American Journal of Engineering Research (AJER)*, vol. 3, pp. 20-27, 2014.
- [15] J. N. Reddy, *Shear Deformable Beams and Plates: Relationships with Classical Solutions*, 1st ed. Oxford, UK: Elsevier, 2000.
- [16] Y. M. Desai, T. I. Eldho, and A. H. Shah, *Finite Element Method with Applications in Engineering*. New Delhi, India: Pearson Education India, 2011.
- [17] O.O.R. Famiyesin, K.M.A. Hossain, Y.H. Chia, P.A. Slade, Numerical and analytical predictions of the limit load of rectangular two way slabs, *Computers & Structures*, Volume 79, Issue 1, 2001, Pages 43-52, ISSN 0045-7949, [https://doi.org/10.1016/S0045-7949\(00\)00113-9](https://doi.org/10.1016/S0045-7949(00)00113-9).
- [18] Liu, Yucheng. (2013). Effects of Mesh Density on Finite Element Analysis. *SAE Technical Papers*. 2. 10.4271/2013-01-1375.

- [19] C. S. Jog, Introduction to the Finite Element Method. Bangalore, India: Indian Institute of Science 2012.
- [20] Ramsay, Angus & Maunder, E.A.W.. (2016). An Error in Timoshenko's 'Theory of Plates and Shells'. The Structural Engineer. 94. 36-39. 10.56330/UYQU3957.
- [21] Kk, Riyas & Dewangan, Uk. (2016). Effect of Material Nonlinearity on Deflection of Beams and Frames. Indian Journal of Science and Technology. 9. 10.17485/ijst/2016/v9i28/92375.
- [22] Birkle, Gerd & Dilger, Walter. (2008). Influence of slab thickness on punching shear strength. ACI Structural Journal. 105. 180-188.
- [23] S. C. Woodson, Tests and Evaluation of Upgraded Flat-Plate and Waffle-Slab Floor Systems. PN, 1983.