Formulation of Approximate, Generalized Field Data Based Mathematical Model for PVC Pipe Manufacturing Process

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Abstract - Polyvinyl-Chloride (PVC) is a plastic product which has matchless versatility. It effectively replaces wood, paper and metal in several applications. As such plastic pipes have been progressively replacing conventional pipes like G.I., Cast Iron, Asbestos Cement or Stone-ware for a number of important and uses. Among the various types of plastic pipes which are commonly used for such applications PVC pipes are the most widely used all over the world on account of their most favorable balance of properties. PVC pipes are light in weight, rates for use under pressure, easy to install, low frictional loss, low on maintenance cost, and have low frictional loss. PVC pipes have wide variety of uses in fields like city/town/rural water supply scheme, spray irrigation, deep tube well schemes and land drainage schemes. PVC pipes are used for a variety of purposes e.g. water supply schemes, spray irrigation, deep tube well schemes and land drainage schemes. A of comprehensive mathematical need model development is always felt important in this system operation and its performance analysis. The theory of experimentation is a good approach of representing the responses in terms of proper interaction of various inputs involved in any phenomenon. by using this approach, the present investigation is to develop approximate, generalized field databased mathematical model using dimensional analysis for the pvc pipe extruding operation. Simulation model does not take into account the electrical motor related factors, machine specifications and raw material related information's. To investigate the effect of these variables on productivity the dimensional analysis is used. The independent variables are identified and are grouped together. The groups formed are electric motor related specifications, extruder machine specifications, specifications of the raw materials, dependent process parameters in pvc pipe plant. The parameters which were constant during the process

were recorded first. The process was planned to record the processing time, pip weight, pipe dimensions, measurement of process parameters and productivity. The model is developed to express the responses as a function of identified inputs. This paper presents a work executed for establishing mathematical model for pvc pipe extrusion process for responses of the system such as dependent process parameters (IID1), pipe dimensions (IID2), pipe weight (IID3) and productivity (IID4). The model is a strong estimator to simulate the process.

Key Words: Field Data Based Mathematical Modeling, PVC

Extruder, Dimensional analysis...

1. INTRODUCTION

In the extrusion of plastics (figure 1), raw thermoplastic material in the form of small beads is gravity fed from a top mounted hopper into the barrel of the extruder. Additives such as colorants and UV inhibitors are often used and can be mixed into the main material prior to arriving at the hopper. The material enters through the feed throat (a funnel shaped opening near the rear of the barrel) and comes into contact with the screw. The rotating screw (at around 120 rpm) forces the plastic beads forward into the barrel which is heated to the desired melt temperature of the molten plastic (usually around 200°C). In most processes, a heating profile is set for the barrel in which three or more independently controlled heaters gradually increase the temperature of the barrel from the rear (where the plastic enters) to the front. In most extruders, cooling fans are present to keep the temperature below a set value if too much heat is generated. This extrusion process continues till the die. The die is an upright cylinder with a circular opening similar to a pipe die. The diameter can be a few centimeters to more than three meters across.



2. IDENTIFICATION OF VARIABLES

Data sets contain information, often much more than can be learned from just looking at plots of those data. Models based on observed input and output data (from real life situation) help us abstract and gain new information and understanding from these data sets. When one is studying any completely physical phenomenon but the phenomenon is very complex to the extent that it is not possible to formulate a logic based model correlating causes and effects of such a phenomenon, then one is required to go in for the field data based models. In view of the dynamic nature of the context under investigation (which reveals complex phenomenon), it was decided that to formulate a field data based model in the present investigation rather than using a theoretical approach.

Generalized field databased models for dependent process parameters (IID1), pipe dimensions (IID2), pipe weight (ПD3) and productivity (ПD4) are established adopting methodology of experimentation [5]. It is planned to collect the data by taking extensive observations in the process of PVC pipe extrusion process. The planning is carried out by using the classical plan of experimentation [6]. The response data is collected based on the entire generalized models. The methodology is briefly stated as follows:



Identification of physical quantities includes the process variables which are independent and dependent. Independent variables are the variables which can be changed without changing other variables of the process. Dependent variables are the variables which can only be

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changed by varying one or more independent variables. Dimensional analysis is carried out in two steps (i) Identification of Physical quantities affecting the process, (ii) dimensional consideration & (iii) dimensionless similarity parameters. The process variables for PVC pipe extruder machines were identified and are tabulated in Table1.

Sr. No	Code	Name of the independe nt variables	MLT indices	Type of var.	Rem
1	C1	Motor Power (HP) Motor Power (KW)	M ¹ L ² T ⁻³	Indepe ndent Indepe ndent	
		Motor Power (watts)	M ¹ L ² T ⁻³	Indepe ndent	
2	C2	Mass or weight of the electric motor	M1 L0 T0	Indepe ndent	
3	C3	Distance of electric motor from the extruder machine	Mº L1 Tº	Indepe ndent	Electric motor
4	C4	Motor speed (RPM)	M ¹ L ¹ T ⁻¹	Indepe ndent	variabl es or
_	C5	Torque (Nm)	M ¹ L ² T ⁻²	Indepe ndent	uata
5		Torque (Nmm)	M ¹ L ² T ⁻²	Indepe ndent	
		Acceleration due to gravity m/s ²	M ⁰ L ¹ T ⁻²	Indepe ndent	
6	C6	Acceleration due to gravity mm/s ²	Mº L1 T-2	Indepe ndent	
7	A1	Extruder machine length (mm)	M ⁰ L ¹ T ⁰	Indepe ndent	
8	A2	Extruder machine width (mm)	Mº L1 Tº	Indepe ndent	
9	A3	Extruder machine height (mm)	Mº L1 Tº	Indepe ndent	Extrud m/c specific
10	A4	Barrel centerline from floor (mm)	Mº L1 T0	Indepe ndent	ations

Table 1: List of independent and dependent variables



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11	A5	Hopper capacity (kg)	M ¹ L ⁰ T ⁰	Indepe ndent	
12	A6	Hopper height (mm)	Mº L1 T0	Indepe ndent	
13	A7	Screw outside diameter (mm)	Mº L1 Tº	Indepe ndent	
14	A8	Screw inner diameter (mm)	Mº L1 Tº	Indepe ndent	
15	A9	Screw pitch (mm)	M ⁰ L ¹ T ⁰	Indepe ndent	
16	A10	Barrel length (mm)	M ⁰ L ¹ T ⁰	Indepe ndent	
17	A11	Barrel diameter (mm)	Mº L1 Tº	Indepe ndent	
18	A12	Weight of the extruder machine (kg)	Mº L1 Tº	Indepe ndent	
19	A13	Die diameter or size (mm)	M ⁰ L ¹ T ⁰	Indepe ndent	
20	A14	Die length (mm)	Mº L1 T0	Indepe ndent	
21	W1	Resin wastage (kg)	M ⁰ L ¹ T ⁰	Indepe ndent	
22	W2	Dust (kg)	Mº L1 T0	Indepe ndent	
23	W3	Filter (kg)	M ¹ L ⁰ T ⁰	Indepe ndent	
20		Filter (gm)	M ¹ L ⁰ T ⁰	Indepe ndent	
24	W4	Chemical wax (kg)	M ¹ L ⁰ T ⁰	Indepe ndent	
25	W5	TBLS powder (gm)	M1 L0 T0	Indepe ndent	Raw mat.
26	W6	Steric acid (gm)	M ¹ L ⁰ T ⁰	Indepe ndent	vari. or data
27	W7	Wastage raw material size (mm)	Mº L1 Tº	Indepe ndent	or data
29	W8	Powder size (mm)	Mº L1 T0	Indepe ndent	
30	W9	Filter material size (mm)	Mº L1 Tº	Indepe ndent	
31	D1	Screw speed of extruder machine	M ⁰ L ¹ T ⁻¹	Depend ent	Depend ent process

		(RPM)			variabl
		Screw speed (m/s)	M ⁰ L ¹ T ⁻¹	Depend ent	es or
32	D2	Melt viscosity (N- s/m2) or kg/m-s	M ¹ L ⁻¹ T ⁻¹	Depend ent	parame ters
33	D3	Melt density (kg/m3)	M ¹ L ⁻³ T ⁰	Depend ent	
34	D4	Extruder pressure (MPA)	M ¹ L ⁻¹ T ⁻²	Depend ent	
		Extruder pressure (kg/ms2)	M ¹ L ⁻¹ T ⁻²	Depend ent	
35	D5	Die pressure (MPA)	M ¹ L ⁻¹ T ⁻²	Depend ent	
		Die pressure (kg/ms2)	M ¹ L ⁻¹ T ⁻²	Depend ent	
36	D6	Extruder temp (0C)	Μο Γο Το	Depend ent	
37	D7	Die temp(0C)	Μο Γο Δο	Depend ent	
38	P1	Pipe dia or Pipe size (mm)	Mº L1 Tº	Depend ent	
39	P2	Pipe wall thickness (mm)	Mº L ¹ Tº	Depend ent	
40	Y1	Pipe weight (kg)	M ¹ L ⁰ T ⁰	Depend ent	
41	Y1	Processing time (sec)	Mº Lº T1	Depend ent	
42	T2	Productivity	Mº Lº Tº	Depend ent	

3. DIMENSIONAL ANALYSIS

Dimensional analysis was carried out to established dimensional equations, exhibiting relationships between dependent Π terms and independent Π terms using Buckingham Π theorem. Dimensional analysis can be used primarily as an experimental tool to combine many experimental variables into one [1].

The various independent and dependent variables of the system with their symbols and dimensional formulae are given in nomenclature. There are several quite simple ways in which a given test can be made compact in operating plan without loss in generality or control. The best known and the most powerful of these is dimensional analysis. In the past dimensional analysis was primarily used as an experimental tool whereby several experimental variables could be combined to form one. The field of fluid mechanics and heat transfer were greatly benefited from the application of this tool [3]. Almost every major experiment in this area was planned with its



help. Using this principle modern experiments can substantially improve their working techniques and be made shorter requiring less time without loss of control. Deducing the dimensional equation for a phenomenon reduces the number of independent variables in the experiments [4]. The exact mathematical form of this dimensional equation is the targeted model. This is achieved by applying Buckingham's Π theorem [5] [6].When we apply this theorem to a system involving n independent variables, (n minus number of primary dimensions viz. L, M, T) i.e. (n-4) numbers of Π terms are formed. When n is large, even by applying this theorem number of Π terms will not be reduced significantly than number of all independent variables. Thus much reduction in number of variables is not achieved. It is evident that, if we take the product of the terms it will also be dimensionless number and hence a Π term. This property is used to achieve further reduction of the number of variables. Dimensional analysis is used to reduce the variables and following Pi terms were evolved out of it. Π1 = Electric motor related variables

 $= \frac{(acceleration due to gravity mm/sl) x (Dist of electmotor frextruder machine) x (mass of elect. motor kg)}{Torque N-mm}$ $\Pi 2 = \text{Specifications of extruder machine}$ $= \frac{(wt.of extruder m/c kg)}{(Hopper capacity kg)} x \frac{(Extruder m/c length mm)}{(Extruder m/c width mm)} x \frac{(Machine height mm)}{(Barrel centreline from floor mm)} x \frac{(Moltine height mm)}{(Barrel dia.mm)} x \frac{(Die length mm)}{(Die diameter mm)}$ $\Pi 3 = \text{Raw material related data}$ $= \frac{(Filter material size mm)}{(wastage raw material size mm)} x \frac{(Wastage raw material size mm)}{(Powder size mm)} x \frac{(TBLS powder gm)}{(Barrel centreline from floormm)} x$ $\frac{(Filter gm)}{(Chemical wax gm)} x \frac{(Screw speed \frac{m}{s})}{(Die pressure \frac{kg}{m2})} x \frac{(Screw speed \frac{m}{s})}{(Bie metsion m)} x \frac{(Melt vis.N = \frac{kg}{ms})}{(Nelt viscosity N - s/m2 \text{ or } kg/m - s)}$ $\Pi D2 = \text{Pipe dimensions}$ $= \frac{(Pipe diameter mm)}{(Pipe wall thickness mm)}$ $\Pi D3 = \text{Pipe weight}$

ΠD4 = Productivity = (Processing time sec.)

Table 2: Formulation of dimensionless pi terms

Pi	Code	Description of Pi	Pi term equation
term		terms	
П1	С6	Acceleration due to gravity mm/s2	
	C3	Diastance of electric motor from the extruder machine	$\Pi 1 = \frac{C6 \times C3 \times C2}{C5}$
	C2	Mass or weight of the electric motor (kg)	
	C5	Torque on electric motor (N-mm)	
	A12	Weight of extruder machine (kg)	
	A5	Hopper capacity (kg)	
	A1	Extruder machine length (mm)	
	A2	Extruder machine width (mm)	$\Pi 2 = \frac{A12}{2}$
	A3	Extruder machine height (mm)	$\frac{A5}{41} \times \frac{A3}{43}$
80	A4	Barrel centerline from floor (mm)	A2 A4
112	A7	Screw outside diameter (mm)	$\frac{A7}{A8} \times \frac{A6}{A9}$
	A8	Screw inside diameter (mm)	A10 A14
	A6	Hopper height (mm)	$\frac{11}{411} \times \frac{11}{412}$
	A9	Screw nitch (mm)	X
	Δ10	Barrel length (mm)	
	<u>Λ11</u>	Barrel diameter (mm)	
	Δ1/	Die length (mm)	
		Die diameter or size	
	A13	(mm)	
	W1	Resin wastage (kg)	T2 - W9
	W2	Dust (kg)	$\frac{113}{W7} = \frac{1}{W7}$
	W3	Filter (gm))	$\frac{W7}{\times} \times \frac{W5}{\times}$
	W4	Chemical wax (gm)	W8 W6
	W5	TBLS powder (gm)	W3 W1
П3	W6	Steric acid (gm)	$\times \frac{W_4}{W_4} \times \frac{W_2}{W_2}$
	W7	Wastage raw material size (mm)	~
	W8	Powder size (mm)	
	W9	Filter material size (mm)	
	D1	Screw speed (m/s)	
	D2	Melt viscosity (N- s/m ²) or (kg/m-s)	$\frac{D4}{\Pi D 1 = \frac{D1}{D1} \times \frac{D1}{D5} \times \frac{D2}{D3}}$
	D3	Melt density (ka/m3)	//
IID1	D4	Extruder pressure (kg/ms2)	$\frac{D3}{X} \times \frac{D7}{D6}$
	D5	Die pressure (kg/ms ²)	

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	D6	Extruder temperature (OC)		
D7		Die temperature (OC)		
D8		Pipe diameter or size (mm)		
	D9	Pipe wall thickness (mm)		
D10 Pi		Pipe weight (kg)		
	D11	Processing time (sec)		
	D12	Productivity		
	P1	Pipe diameter (mm)	P1	
ПD2	P2	Pipe wall thickness (mm)	ΠD2 = ^{P2}	
пр2	Y1	Pipe weight (kg)	ПD2 – V1	
1103	Y2	Processing time (sec)	$\Pi D S = I I$	
ПD4	T2	Productivity	$\Pi D4 = \frac{Y1}{Y2} \times \frac{Y2}{Y1}$	

The main purpose of this technique is making experimentation shorter without loss of control. As per dimensional analysis [1], response variables dependent process parameters (IID1), pipe dimensions (IID2), pipe weight ($\Pi D3$) and productivity ($\Pi D4$) are written in the function form as:

ΠD1 = f (C1, C2, C3, C4, C5, C6, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14, W1, W2, W3, W4, W5, W6, W7, W8, W9) ----- (1)

ΠD2 =f (C1, C2, C3, C4, C5, C6, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14, W1, W2, W3, W4, W5, W6, W7, W8, W9) ----- (2)

ΠD3 =f (C1, C2, C3, C4, C5, C6, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14, W1, W2, W3, W4, W5, W6, W7, W8, W9) ----- (3)

ΠD4 =f (C1, C2, C3, C4, C5, C6, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14, W1, W2, W3, W4, W5, W6, W7, W8, W9) ----- (4)

4. FORMULATION OF FIELD DATA BASED

MATHEMATICAL MODELS TERMS OF PI TERM

Three independent pi terms (Π 1, Π 2, Π 3,) and four dependent pi terms (IID1, IID2, IID3, IID4,) have been in the design of experimentation and are available for the model formulation. Independent π terms = (Π 1, Π 2, Π 3), Dependent π terms = ($\Pi D1$, $\Pi D2$, $\Pi D3$, $\Pi D4$,). Each dependent π is assumed to be function of the available independent π terms, For Process Parameters (ΠD1)

 $\Pi D1 = k1 \times (\Pi 1)a1 \times (\Pi 2)b1 \times (\Pi 3)c1 \dots (5)$ For Pipe Dimensions ($\Pi D2$) $\Pi D2 = k2 x (\Pi 2)a2 x (\Pi 2)b2 x (\Pi 3)c2 -----(6)$ For Pipe Weight (ΠD3) ПD3 = k3 x (П3)a3 x (П3)b3 x (П3)c3 -----(7) For Productivity (ΠD4) $\Pi D4 = k4 x (\Pi 4)a4 x (\Pi 4)b4 x (\Pi 4)c4 \dots (8)$

To compute the models for ΠD1 following approach is adopted. $\Pi D1 = k1 \times (\Pi 1)a1 \times (\Pi 2)b1 \times (\Pi 3)c1$

ПD1 = k1 x (П1)a1 x (П2)b1 x (П3)c1

Taking log on the both sides of equation for $\pi 01$, getting eight unknown terms in the equations,

 $Log \Pi D1 = log k1 + a1log \Pi 1 + b1log \Pi 2 + c1log \Pi 3$ ------(9)

Let, $Z1 = \log \Pi Z1$, $K1 = \log k1$, $A = \log \Pi 1$ $B = \log \Pi 2$ $C = \log \Pi 3$

Putting the values in equations 9, the same can be written as

Z1 = K1+ a1 A + b1 B + c1 C ----- (10)

Equation 9 is a regression equation of Z on A, B, and C in an n dimensional co-ordinate system. This represents a regression hyper plane [7]. To determine the regression hyper plane, determines a1, b1, c1, d1, e1 and f1 in equation 9 so that

 $\Sigma Z1 = nK1 + a1^{*}\Sigma A + b1^{*}\Sigma B + c1^{*}\Sigma C$

 $\Sigma Z1^*A = K1^*\Sigma A + a1^*\Sigma A^*A + b1^*\Sigma B^*A + c1^*\Sigma C^*A$

 $\Sigma Z1^*B = K1^*\Sigma B + a1^*\Sigma A^*B + b1^*\Sigma B^*B + c1^*\Sigma C^*B$

 $\Sigma Z1^*C = K1^*\Sigma C + a1^*\Sigma A^*C + b1^*\Sigma B^*C + c1^*\Sigma C^*C$

In the above set of equations the values of the multipliers K1, a1, b1, c1 are substituted to compute the values of the unknowns (viz. K1, a1, b1, and c1). The values of the terms on L H S and the multipliers of K1, a1, b1, and c1 in the set of equations are calculated. After substituting these values in the above equations one will get a set of 4 equations, which are to be solved simultaneously to get the values of K1, a1, b1, c1,. The above equations can be verified in the matrix form and further values of K1, a1, b1, c1, can be obtained by using matrix analysis.

 $X1 = inv (W) \times P1 -----(11)$

The matrix method of solving these equations using 'MATLAB' is given below.

W = 4 x4 matrix of the multipliers of K1, a1, b1, c1, d1, e1, f1 and g1

P1 = 4 x 1 matrix of the terms on L H S and

X1 =4 x 1 matrix of solutions of values of K1, a1, b1, and c1 Then, the matrix obtained is given by,

Matrix

$$Z_{1} \quad x \begin{bmatrix} 1\\ A\\ B\\ C \end{bmatrix} = \begin{bmatrix} n & A & B & C\\ A & A^{2} & BA & CA\\ B & AB & B^{2} & CB\\ C & AC & BC & C^{2} \end{bmatrix} x \begin{bmatrix} K_{1}\\ a_{1}\\ b_{1}\\ c_{1} \end{bmatrix}$$

The same procedure is adopted for other response variables and after solving this matrix in MATLAB the various models are

$$\begin{split} &(\pi_{D1}) = 23.77935114 \left\{ (\pi_1)^{0.4389} \cdot (\pi_2)^{-0.7474} \cdot (\pi_3)^{0.4155} \right\} \\ &(\pi_{D2}) = 48.7416 \left\{ (\pi_1)^{0.4416} \cdot (\pi_2)^{-0.4445} \cdot (\pi_3)^{0.1504} \right\} \\ &(\pi_{D3}) = 3.149 \left\{ (\pi_1)^{0.6958} \cdot (\pi_2)^{-0.8655} \cdot (\pi_3)^{0.5223} \right\} \\ &(\pi_{D4}) = 0.4325 \left\{ (\pi_1)^{-0.0572} \cdot (\pi_2)^{0.0948} \cdot (\pi_3)^{-0.3847} \right\} \end{split}$$

The graphical representation between the actual values of dependent terms and values obtained by model with coefficient of determination are shown in comparative form as below.





Figure 1: Comparison between Model and Experimental Data of ΠD1 (R2= 0.997372)







Figure 3: Comparison between Model and Experimental Data ΠD3 (R2= 0.9917)



Figure 4: Comparison between Model and Experimental Data ΠD4 (R2= 0.9861)

5. CONCLUSION

Thus the mathematical equations are established in reduced or compact mode in order to make the complete experimentation process less time taking having generation of optimum data. Empirical models to predict the performance of pvc pipe extruder machine were established and optimum values of various parameters were arrived at on the basis of experiments and a new theory is proposed. The outcome of these models can significantly contribute to improve all response variables. The model for the phenomenon represents the degree of interaction of various independent variables.

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