An alternative proof for Beal's conjecture

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Abstract - *In the present investigation on* ***alternative proof for beal's conjecture is discussed*** *with numerical examples $A^x+B^y=C^z$ where A ,B ,C are co-primes and x ,y ,z are greater than 2.*

Key Words: Beal's conjecture, Co-prime, odd number

1. Introduction

In Past few decades, Andrew Beal formulated the Beal Conjecture is a proposition within the number theory. according to this theory , if $A^X + B^Y = C^Z$ where A, B, C are co-primes and x, y, z >2 then A,B,C must have a common prime factor[3]. Darmonand Granville solved the super elliptic equation $Z^m = F(x, y)$ where F is a homogeneous polynomial with integer coefficients of the generalized Fermat equation $Ax^p+By^q = Cz^r$ [4]. The aim of this work is to provide a proof of the Beal Conjecture. Fermat's Last Theorem states that if n > 2, $\delta^n + \gamma^n = \alpha^n$ has no solutions in nonzero integers. As Fermat used not to annotate the proofs of his theorems, this and other statements inspired many generations of mathematicians, who went on to develop important math advances while seeking solutions. All statements of Fermat were eventually proved except one that was refuted, but in this case, Fermat did not actually say that he knew a proof [6].Recently Leandro Torres Di Gregorio discussed Proof for the Beal conjecture and a new proof for Fermat's last theorem[2]. In the present investigation an alternative proof for beal's conjecture is discussed.

Problem Formulation

$$A^X + B^Y = C^Z$$

Where A, B, C are co-primes and x, y, z >2

If possible let A, B, C are co-primes. Then there must be two odd co-primes and the other must be even number. Since A, B, C are co-primes, 1 is not the one of the numbers A, B and C. Any odd number greater than 1 can be written as sum of two consecutive numbers such as

[2p + (2p-1)] or [2p + (2p+1)]

i.e., either in the form of

(4p-1) or (4p+1) (where p € N)

Ex: 3 = 4(1)-1, 5 = 4(1)+1; 7 = 4(2)-1, 9 = 4(2) +1.......etc.....

Since n^{th} power of odd number is also an odd number, n^{th} power of any odd number can also be in the form of either (4q-1) or (4q+1) (where q€ N)

Consider an odd number which is in form of (4p -1), if n is any odd positive number then $(4p-1)^n$ can be written in the form of (4q-1) , (q€N)

Ex: 3=4(1)-1 if p=1, 3 is in the form of 4p-1 if n is any odd i.e., n=3 then

$$(4p-1)^3=3^3 =27.$$

27 can be written in the form of 4(7)-1 i.e., is in the form of 4q-1, (where q =7)

If n is any even positive number then $(4p-1)^n$ can be in the form of (4q+1) where (q€N)

Ex: 5= 4(1)+1 if p=1 then 5is in the form of 4p+1. If n is any even number i.e., if n=2 then 5^2= 25 =4(6)+1 i.e., if n is even number $(4p-1)^n$ can be in the form of 4q+1, q€N

But if we consider an odd number which is in the form of (4p + 1), then $(4p + 1)^n$ can always be written in the form of (4s + 1) even for 'n' is either odd or even number. (s € N)

Ex: 9= 4(2)+1 which is in the form of 4p+1 where p=2. If n is either even or odd positive number, $(4p+1)^n$ can always be in the form of 4s+1, (s€N)

i.e., 9^2=81 =4(20)+1 and 9^3=4(182)+1 means $(4p+1)^n$ can always exist in the form of (4s+1), (s€N) even n is either even or odd positive number

Now if possible let A, B and C are co-primes. If possible let $A^x + B^y = C^z$ and let, A is even number and B, C are odd co-primes let A = 2a, B = (4b – 1) and C = (4c + 1)

Case (i):Then from $A^x + B^y = C^z$ here y is odd natural number and x, y and z > 2 then

$$(2a)^x + (4b - 1)^y = (4c + 1)^z$$

$$(2a)^x = (4c + 1)^z - (4b - 1)^y$$

Let $(4c + 1)^z = (4m + 1)$ and let $(4b - 1)^2 = (4k - 1)$ (since powers of odd numbers are also odd numbers)

$(2a)^x = (4m + 1) - (4k - 1)$ (where m,k€N)

$$(2a)^x = 4m - 4k + 2$$

$$2^x.a^x = 2(2m - 2k + 1)$$

$$\frac{2^x a^x}{2} = 2m - 2k + 1$$

$$2^{x-1}.a^x = 2m - 2k + 1$$

If we observe L.H.S is an even number and R.H.S is an odd number.

So it contradicts our supposition.

So our supposition is wrong

So A, B and C are not co-primes and they must have a common prime factor.

Case (ii) Consider $(4b - 1)^y$ here if y is even number, then we can prove the conjecture in the following method.

In this case let A and B are odd co-primes and let be c an even number.

So if possible let A = (4a + 1), B = (4b -1) and C = 2c

Then $A^x + B^y = C^z$ can be written as

$$(4a + 1)^x + (4b - 1)^y = (2c)^z$$

Here if y is an even number, then, let

$(4a + 1)^x = (4m + 1)$, and $(4b - 1)^y = (4k + 1)$

$(4m + 1) + (4k + 1) = (2c)^z$ (m, k€N)

$4m + 4k + 2 = (2c)^z$

$2(2m + 2k + 1) = 2^z.c^z$

$2m + 2k + 1 = \frac{2^z c^z}{2}$

$2m + 2k + 1 = 2^{z-1}.c^z$

Here L.H.S is odd number and R.H.S is even number.

So here also it is a contradiction.

So our supposition is wrong
So A, B, C are not Co-primes and must have a common prime factor.

CONCLUSIONS

In this present investigation an alternative proof for beal's conjecture is discussed.

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BIOGRAPHIES

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