

A locally optimum detector for detection of random signals under a weakly correlated noise model over fading channels

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Abstract - In modern cognitive radio networks, to avoid interference from secondary users to primary holders with license in spectrum, it is need to have an appropriate spectrum sensing. In methods like spectrum sensing, where samples of noise are correlated, the impairments from independent noise samples do not provide optimum performances. So, in case of random signals over a weakly correlated noise model in fading channels requires a locally optimum detection method has been proposed in this paper. A low signal to noise ratio regime has derived based on the probabilities of false alarm and detection of proposed detector. The average probabilities of false alarm and detection of proposed detector are derivated for different channel gains. The simulation and numerical results helps to compare and define that the proposed method is more appropriate than the conventional energy detection method. Finally we take a scenario in which the estimated and real correlations are different. The effect of correlation mismatch on the probabilities of false alarm and detection in proposed method are estimated.

Key Words: Cognitive radio networks, spectrum sensing, fading channels, locally optimum detection, false alarm, correlation mismatch, energy detection.

I. INTRODUCTION

The most expensive and limited resource for wireless communications are Radio Frequency (RF) spectrums. The increase in demands for additional bandwidth has led to these studies which will indicate the spectrum assigned to primary license holders are not utilized properly. RF's spectrums are efficiently utilized by Cognitive radio technology and the spectrums secondary usages to primary users have lower priority. Based on the interaction with the environment, a cognitive radio signals will change the transmitter parameters. In data transmission, secondary users can sense the spectrum and use spectrum holes dynamically from primary users with the help of cognitive radio signals. Any interference of

secondary users are not allowed in primary users frequencies. Therefore secondary users must be aware of the primary user's presence. Primary license holders are detected by their presence and absence with methods like spectrum sensing. The method of spectrum sensing is tedious task as primary users suffer from fading, shadowing, etc. There are other types of spectrum sensing techniques like matched filtering, energy detection, cyclostationarity-based detection and Eigen value-based detection. Among these energy detection is simple technique and optimized with impairment of additive white Gaussian noise (AWGN). The additive noise samples are statistically independent. In this paper we consider AWGN exhibits correlation significantly. In applications like smart grid monitoring, noise are experimentally measured has several characteristics one of them was correlations of the time domain. These noise models are complex and need Markov transition models. Any real type cognitive radio environment signals will have some level of noise correlation. The correlation noise models are considered for ϵ -mixing noise model, for m -independent noise model and for average moving non-Gaussian noise models. First order moving average (MA) of an i.i.d is considered as weakly correlated noise when the dependence is weak. In all these above studies the implementation of detection schemes are designed for known signals. A detection mechanism like locally optimum (LO) of random signals over a weakly correlated noise model over fading channels is implemented. In case of correlated noise environments, rather than simple detection techniques we use to implement LO detection techniques. False alarm and detection probabilities are used to define LO detection. The above said probabilities depend on channel gain hand we need to take average of h in order to get final average false alarm and detection probabilities. In this paper, we derive some averages of these two probabilities in a large no. of channel gains and averages are taken. From the probabilities we can determine that simulation results obtained are matching well with theoretical results. When compared simple energy detection with locally optimum detection it is clear that later has better performance. Section II in this paper includes two hypotheses which details about the presence and absence of primary user and also about the model for correlated noise samples. Section III includes the derivation of locally optimum test statistic in the presence of correlated noise. In section IV, we compare locally

optimum detector and simple energy detector and the performance of energy detector are analyzed and parameters required are derivated. All the results above these sections are not real scenarios in which there may be some errors in the estimation of correlation coefficient. In section VI we give false alarm and detection probabilities in Numerical and simulation results. In section VII conclusions are finally presented.

II. SYSTEM MODEL

The two hypotheses assumed are, H_0 in which the primary user is said to be absent and H_1 once the first user is said to be present, the received signal samples ($n = 1, 2, \dots, N$) at the secondary user for these two types of hypotheses could also be shapely in equivalent complex baseband illustration as:

$$H_0 : x_n = w_n$$

$$H_1 : x_n = h s_n + w_n$$

where, x_n , h , and w_n denote the received signal, the third Baron Rayleigh attenuation channel gain, and also the noise samples at the secondary user and S_n is that the primary user signal. The channel gain h is assumed to be constant throughout the detection method with zero mean and also the variance of $E[|h|^2] = \sigma_h^2$. The primary user signal used to has zero mean, variance σ_s^2 , and it's real and unreal elements square measure statistically freelance and has each variance $\sigma_s^2 / 2$. The zero mean noise samples square measure assumed to own identical variance σ_n^2 . The element samples square measure assumed to be temporally freelance, identically distributed (i.i.d.) .Moreover, we tend to assume that the noise samples, the fading gains, and also the primary user signals square measure reciprocally freelance. In this paper we tend to assume that the noise samples square measure temporally dependent. In straightforward first-order bilateral associate degreed unilateral moving averages (MAs) of an i.i.d. random method square measure accustomed model the feeble correlate noise. They're straightforward and sensible approximations to a feeble correlate noise. We tend to contemplate feeble dependent situation victimization the unilateral MA of i.i.d. random variables. Presumptuous that e_i , $i = 1, 2, \dots, N$, are the i.i.d. random variables with common Probability density perform (pdf) $f_e(\cdot)$, the noise samples w_1, w_2, \dots, w_n can be defined as:

$$w_1 = e_1$$

$$w_n = e_n + \rho e_{n-1}, n = 2, \dots, N,$$

Where $|\rho| < 1$ is the parameter that defines the noise Correlation.

III. TEST STATISTIC

In order to derive a test statistic to recognize between two hypothesis H_0 and H_1 , we start with the globally optimal (GO) decision statistic expressed as

$$\Lambda = \frac{p(X|H_1)}{p(X|H_0)} = E_{h,S} \left[\frac{f_W(X - hS)}{f_W(X)} \right]$$

Where f_W is the multivariate pdf of the noise samples and $X = x_1, \dots, x_N$, $S = s_1, \dots, s_N$. For the hypothesis H_1 , we have

$$f_W(X - hS) = f_W(x_1 - h s_1, x_2 - h s_2, \dots, x_N - h s_N).$$

Since the noise samples are dependent, it is not possible to write the above multivariate pdf as a multiplication of pdfs of its elements.

IV. ENERGY DETECTOR

We have also analyzed an energy detector (ED) used for detection in the presence of correlated noise samples, in order to compare its performance with the proposed LO detector based on We plan to demonstrate the superiority of the proposed locally optimum detector in terms of performance compared to the conventional energy detection, based on analytical expressions. The test statistic for an energy detector can be expressed as follows:

$$\Lambda = \sum_{i=1}^N |x_i|^2 .$$

$$\Lambda_{|H_0} = \sum_{i=1}^N |w_i|^2 ,$$

$$\Lambda_{|H_1} = \sum_{i=1}^N |h s_i + w_i|^2 .$$

We explicitly derive expressions for the false alarm probability and detection probability when this energy detector statistic is used under noise conditions that match our correlated noise model.

V. CORRELATION MISMATCH

So far, all the presented results are based on the assumption that we have the perfect knowledge about the correlation coefficient ρ between noise samples at different times. This may not be valid in real scenarios where it is possible to have error in estimating the correlation coefficient. In order to investigate the effect of correlation mismatch in our proposed detector, we denote the real correlation coefficient with ρ and the estimated one with $\hat{\rho}$. In order to include these two quantities in our analysis, we can start with the test statistics in Equation

$$\Lambda = \sum_{i=1}^N \frac{1 - \hat{\rho}^{2i}}{1 - \hat{\rho}^2} |\hat{y}_i|^2$$

For H_0 hypothesis,

$$\begin{aligned} \hat{y}_i &= \sum_{k=0}^{i-1} (-\hat{\rho})^k w_{i-k} = \sum_{k=0}^{i-1} (-\hat{\rho})^k (e_{i-k} + \rho e_{i-k-1}) \\ &= e_i + \left(1 - \frac{\rho}{\hat{\rho}}\right) \sum_{k=1}^{i-1} (-\hat{\rho})^k e_{i-k}, \end{aligned}$$

and therefore

$$\Lambda_0 = \sum_{i=1}^N \frac{1 - \hat{\rho}^{2i}}{1 - \hat{\rho}^2} \left| e_i + \left(1 - \frac{\rho}{\hat{\rho}}\right) \sum_{k=1}^{i-1} (-\hat{\rho})^k e_{i-k} \right|^2$$

As we can see Λ_0 is a summation of dependent random variables. Similar arguments can be made to prove that Λ_0 can be considered a asymptotically Gaussian random variable when the number of samples N is large enough. It can be proved that the random variables in this summation are ρ -mixing. By defining

$$d_i = \left| e_i + \left(1 - \frac{\rho}{\hat{\rho}}\right) \sum_{k=1}^{i-1} (-\hat{\rho})^k e_{i-k} \right|^2$$

Calculate the mean μ_{0-m} and variance σ_{0-m}^2 of the Gaussian random variable Λ_0 . A similar approach can be followed for Λ_1 , and the parameters of these distributions can be used to compute false alarm and detection probabilities under mismatch conditions

VI. NUMERICAL RESULTS AND DISCUSSIONS

At secondary user a fading channel with weakly correlated noise, $N=500$ samples are taken. In case of slow fading channel, the fading coefficient h is constant for a given sampling period. Let the detection probability is taken as 0.95 and different signal to noise ratios (SNRs, defined as $SNR = \sigma_n^2 \sigma_s^2 / \sigma_n^2$) are calculated to find average false alarm probabilities for both proposed locally optimum detector model and conventional energy detector model. Theoretical analysis are found and compared with analytical results from the average false alarm probabilities from simulations over one lakh independent realizations of the Rayleigh fading channels. Fig1 shows average false alarm probability for correlation coefficient $\rho = 0.5$. Primary users can be implemented by 8-PSK. From Fig1, we can compare false alarm probability of the locally optimum detector are lower than the energy detector and also there are very small errors when simulation and analytical results are verified. Now let us take the false alarm probability as 0.05 and also average detection probabilities for different SNR's are defined. From Fig 2, the energy detector and proposed detector are compared where the proposed detector has higher detection probability and results of simulation are very closer to the analytical results. The effects of no. of samples on the detection performance are also considered. Different

correlations for energy detection and LO detection are considered.

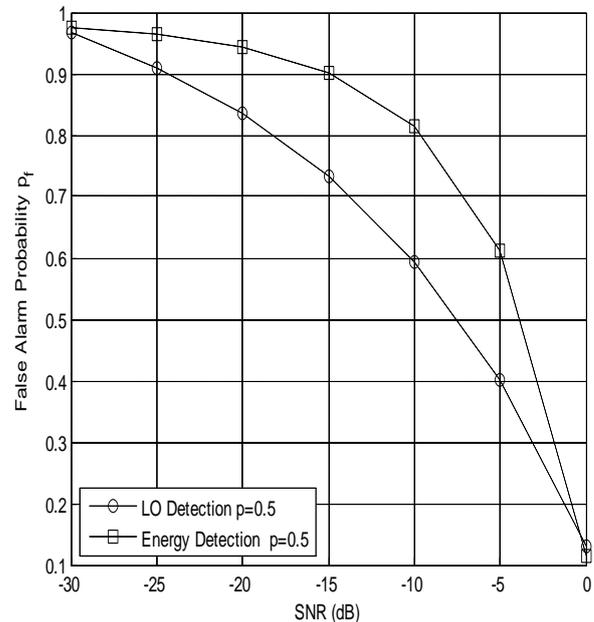


Fig.1 Average false alarm probabilities using analytical results as well as simulation results at different SNRs for detection probability of 0.95 and $\rho = 0.5$.

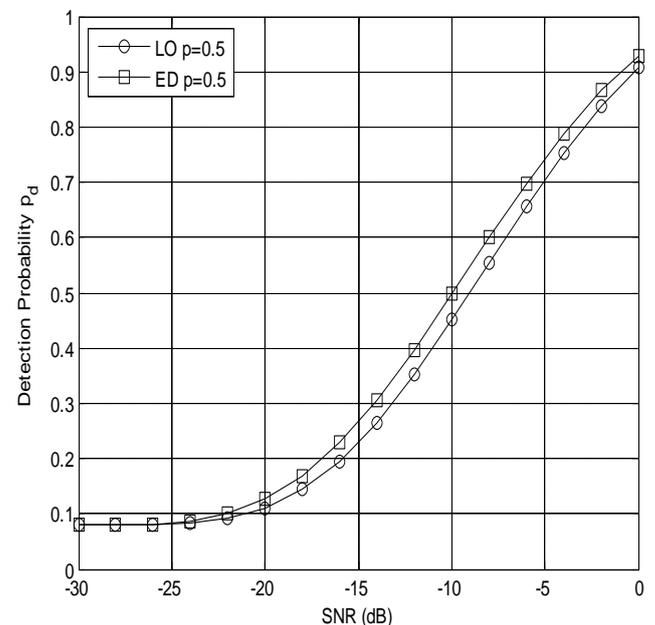


Fig 2 Average detection probabilities using analytical results as well as simulation results at different SNRs for

false alarm probability of 0.05 and $\rho = 0.5$.

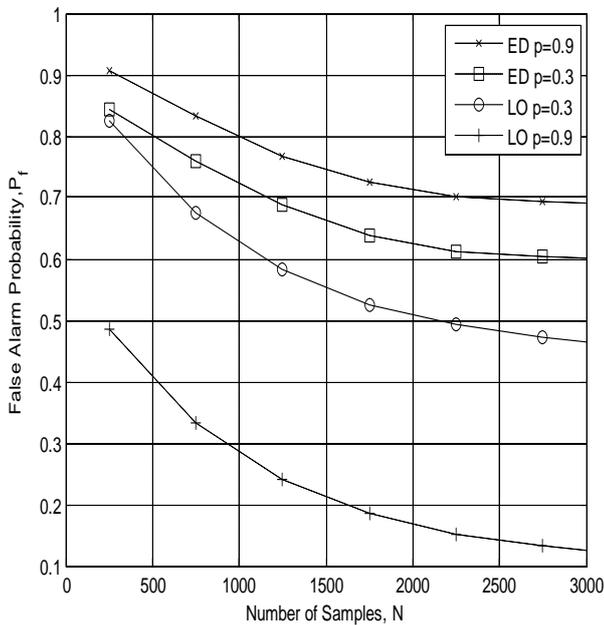


Fig 3 Average false alarm probabilities versus the number of samples for detection probability of 0.95

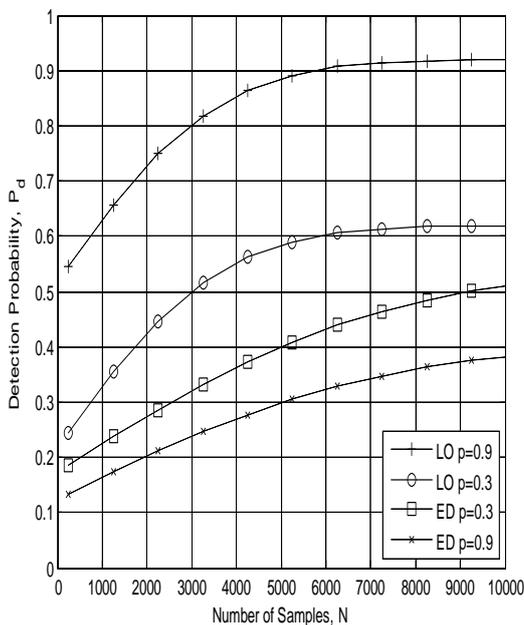


Fig 4 Average detection probabilities versus the number of samples for false alarm probability of 0.05

The figures, Fig 3 and Fig 4 shows average false alarm and detection probabilities. With the increase in number of samples the false alarm probabilities gets lower and detection probabilities gets higher. In figures, all the curves have rate of decreasing P_f with increasing P_d and is higher at the beginning with lower samples are decreased when number of samples increases. For all values of N , for

each correlation the LO detection proposed is much better than conventional energy detection. With high correlation between two methods the difference between the values of P_f are more. For each curve, we compare the simulation and analytical results and the results is the matching between them has very less errors.

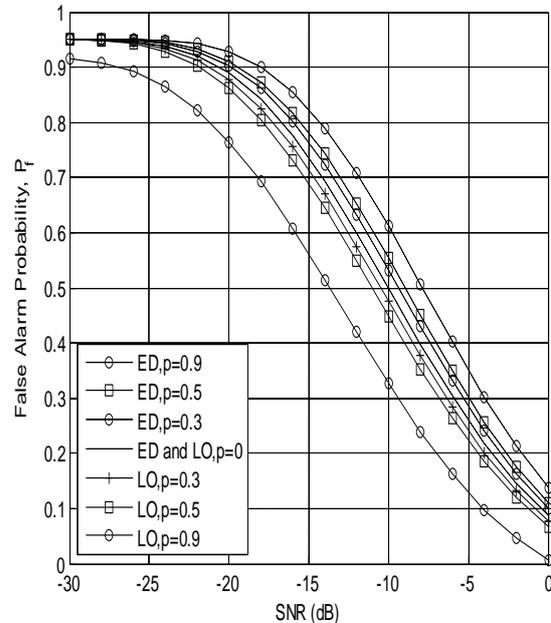


Fig 5 Average false alarm probabilities at different SNRs for detection probability of 0.95 and different correlation coefficients.

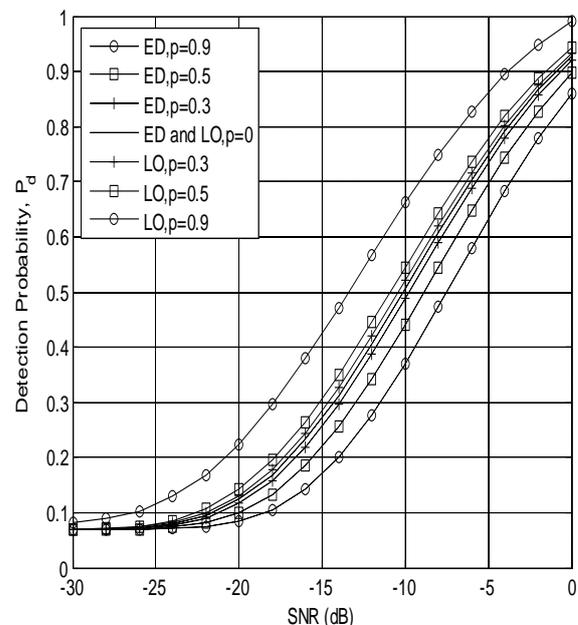


Fig 6 Average detection probabilities at different SNRs for false alarm probability of 0.05 and different correlation coefficients.

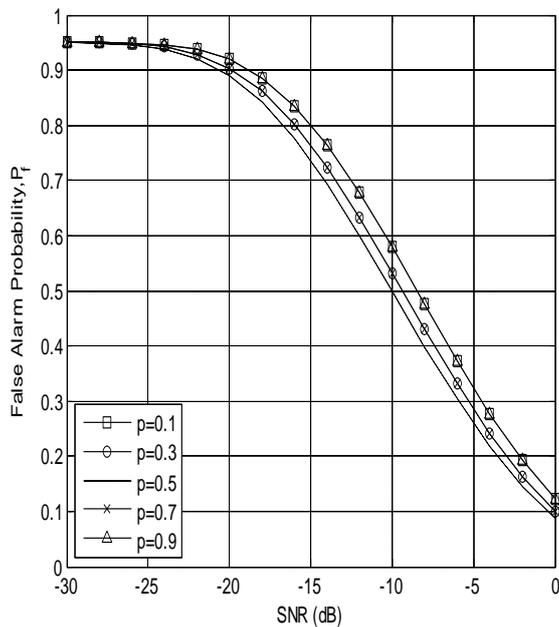


Fig7 Detection probabilities at different SNRs for false alarm probability of 0.05, for the case the estimated correlation is 0.5 and the actual correlation

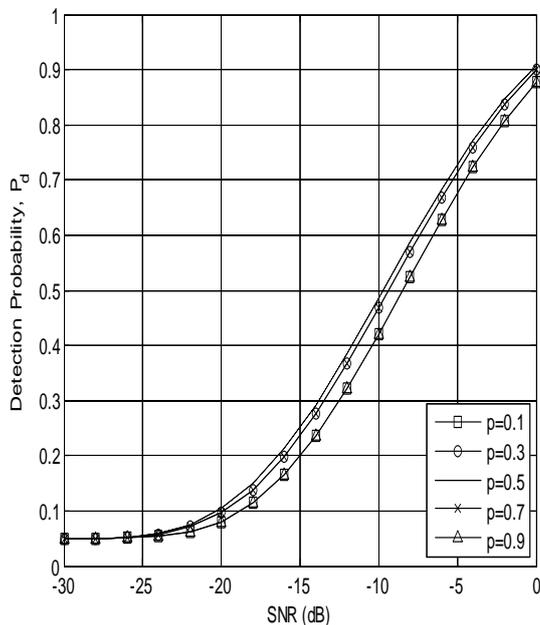


Fig 8 False alarm probabilities at different SNRs for detection probability of 0.95, for the case the estimated correlation is 0.5 and the actual correlations of [0.1:0.2:0.9].

In Fig 5 and Fig 6, different correlation coefficients ρ are taken and average probabilities of false alarm and detection are shown. In Fig 5 the gain of proposed detector becomes high with increase in correlation coefficient. Let

us take an example ,to achieve a false alarm probability of 0.2,the LO detector ,has 1.2dB,3 dB and 11 dB gain for the energy detector of correlation coefficients 0.3,0.5,0.9 respectively. From Fig 6, it is clear that gain is higher when the correlation coefficient is higher in proposed detector compared with energy detector. Let us consider an example that to have a detection probability of 0.2, the LO detector has 1.2dB, 3dB, 12dB gain for the correlation coefficients 0.3, 0.5 and 0.9 respectively over energy detector. The energy detector and proposed detector will have same false alarm and detection probabilities when the correlation coefficients are equal to zero. So far concerned all the figures are obtained from perfect knowledge about the correlation coefficients of the detector. In situations where estimated correlation coefficient $\hat{\rho}$ is different from the estimated coefficient ρ the false alarm and detection probabilities are calculated. Fig 7 and Fig 8 shows the false alarm and detection probabilities with estimated correlation is 0.5 and the actual correlations of[0.1:0.2:0.9].Fig 7 shows detection probability decreases with the increase in the difference between estimated and actual correlation coefficients. For $\hat{\rho} = \rho = 0.5$ detection probability will be maximum. Degradation in performance depends on the absolute values obtained from the difference between the estimated and actual correlation values. In situations where $\rho = 0.1$ and $\rho = 0.9$ have the similar performances.

VII. CONCLUSION

A locally optimum detector for detection of random signals under a weakly correlated noise model over fading channels has been proposed. For a correlation mismatch, normal situations where the estimated correlations between noise samples are not equal to actual correlations are considered. For various channel gains, the false alarm and detection probabilities are derived for a particular channel gain h and then average of these probabilities are derived. In order to compare the performance of proposed model over energy detector common correlated noise are taken. The numerical and simulation results obtained from the probabilities of false alarm and detection clearly explains that LO detector has less false alarm probability and high detection probability when compared with energy detector. Higher the correlation coefficient, gain also becomes higher in case of proposed detector when compared to energy detector. The performance of proposed detector and energy detector will be same when there is no correlation. When there is a mismatch of correlation, the false alarm and detection probabilities are calculated. It is clear from the results that if correlation mismatches are higher then the detection probabilities becomes lower and false alarm probabilities gets higher. The value $\hat{\rho} = \rho$ is a perfect estimation case which results highest detection probability and lowest false alarm probability. The degradation in the performance is dependent on absolute value of the variations between

estimated and actual correlated values. The correctness of analytical results over simulation results are verified based on false alarm and detection probabilities. In this paper, the most important assumption is primary user samples are independent over time. For situations where time correlation between primary user samples is more, a spectrum sensing technique could be implemented. In future, Eigen decomposition-based sensing techniques are to implement in which both primary user signal and noise are correlated to each other.

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BIOGRAPHIES



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