Autonomous HexaRotor Arial Dynamic Modeling and a Review of Control Algorithms

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Abstract - ***This paper is about modelling and control of Vertical Takeoff and Landing (VTOL) type Unmanned Aerial Vehicle (UAV) specifically, micro hexarotor. The nonlinear dynamic model of the hexarotor is formulated using the Newton-Euler method, the formulated model is detailed including aerodynamic effects and rotor dynamics that are omitted in many literature. Based on the mathematical model, several algorithms have been analyzed, varying between the classical linear Proportional-Integral-Derivative (PID) controllers to more complex nonlinear schemes as backstepping or sliding-mode controllers. Simulation based experiments were conducted to evaluate and compare the performance of the proposed control techniques in terms of dynamic performance, stability and the effect of possible disturbances. Finally, integral backstepping is augmented with FST (Frenet-Serret Theory) action and proposed as a tool to design attitude, altitude and position controllers. The conclusion of this work is a proposal of hybrid systems to be considered as they combine advantages from more than one control philosophy.***

These developments are part of the overall project initiated by the team (EAS) of the Computer Laboratory, systems and renewable energy (LISER) of the National School of Electrical and Mechanical (ENSEM).

Key Words: *Hexacopter; Nonlinear control; Newton-Euler method; PID; Backstepping; sliding-mode; Frenet-Serret Theory (FST).*

1. INTRODUCTION

Unmanned autonomous aerial vehicles have become a real center of interest. In the last few years, their utilization has significantly increased. Many research papers have been published on the topic of modeling and control strategies of autonomous multirotors. Today, they are used for multiple tasks of civil as well as military applications, such as navigation, search and rescue mission, building exploration, surveillance, security, transportation and much more. The multirotors are commonly used in dangerous and inaccessible environments.

We introduce one configuration of a multi rotor composed of six rotors. This work will focus on the modeling and control of a hexarotor type UAV. The reason for choosing the multirotor is in addition to its advantages (high agility and maneuverability, relatively better payload, vertical take-off relies on fixed pitch rotors and uses the variation in motor speed for vehicle control [1].

However, these advantages come at a price as controlling a hexarotor is not easy because of the coupled dynamics and its commonly under-actuated design configuration [2]. In addition, the dynamics of the hexarotor are highly non-linear and several uncertainties are encountered during its missions [3], thereby making its flight controls a challenging venture. This has led to several control algorithms proposed in the literature.

The contributions of this paper are: firstly, deriving an accurate and detailed mathematical model of the hexarotor UAV, developing linear and nonlinear control algorithms and applying those on the derived mathematical model in computer based simulations and to provide a valid confrontation and a comparison between five different control techniques in terms of their dynamic performance and their ability to stabilize the system under the effect of possible disturbances. The conclusion of this work is a proposal of hybrid systems to be considered as they combine advantages from more than one control philosophy.

Fig 1. A picture of the developed hexarotor (SMART\ENSEM)

The paper remainder is organized as follows. In the next Section the mathematical formulation and the dynamic model of the hexacopter are described, while section III gives a review of the popular controllers proposed for the hexarotor systems, then a integral backstepping augmented with FST action is proposed as a tool to control attitude, altitude and position controllers; In section IV, the simulation results are given to highlight the proposed method; Finally, the last section, draws the conclusions and suggests possible improvements to be carried out during future work dynamic model of the hexarotor UAV.

2. DYNAMIC MODEL OF THE HEXAROTOR

The hexarotor is an under-actuated system because it has six-degree of freedom while it has only four inputs. The collective input (or throttle input) is the sum of the thrusts of each motor. The goal of this section is to define physical Equations of Motion that describes the dynamics and aerodynamics of the UAV involved. The mathematical model of the hexacopter has to describe its attitude according to the well-known geometry of this UAV. More specifically, this aerial vehicle basically consists of six propellers located orthogonally along the body frame. Figure 1 shows this configuration. There are three movements that describe all possible combinations of attitude: Roll (rotation around the X axis) is obtained when the balance of rotors 1, 2 and 3(or 6, 5 and 4) is changed (speed increases or decreases). By changing the angle, lateral acceleration is obtained; pitch movement (rotation around the Y axis) is obtained when the balance of the speed of the rotors1 and 6 (or 3 and 4) is changed. The angle change results in a longitudinal acceleration; yaw (rotation about the Z axis) is obtained by a simultaneous change of speed of the motors (1,3,5) or (2,4,6)

2.1 Hexacopter Kinematics

Suppose that $\mathcal{R}_I$-frame denotes earth fixed frame and $\mathcal{R}_B$ -frame denotes body fixed frame which can be seen in Fig. 1, the airframe orientation is denoted by $\Re_I^B$ matrix. $\Re_I^B$ stands for the rotation from $\mathcal{R}_B$ to $\mathcal{R}_I$. The dynamic model of hexarotor is derived from Newton-Euler approach.

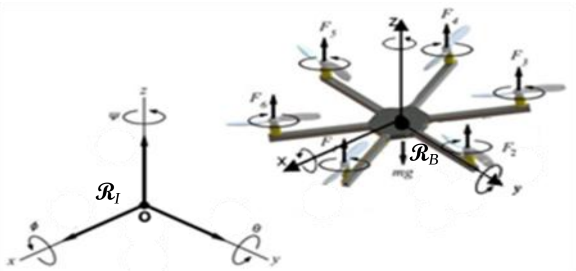

Fig 2. The structure of hexarotor and its frames

Meanwhile, the Euler angles are roll angle ϕ]- , [, pitch angle θ∈]- , [and yaw angle ψ∈]-π ,π [respectively. The rotation matrices from body frame to earth frame can be obtained as,

$$\Re_I^B = \begin{pmatrix} \cos\theta\cos\psi & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \sin\psi\cos\theta & \sin\theta\sin\psi\sin\phi + \cos\psi\cos\theta & \cos\phi\sin\psi\sin\theta - \cos\psi\sin\phi \\ -\sin\theta & \sin\phi\cos\theta & \cos\theta\cos\phi \end{pmatrix}$$

And denoting ω as the angular body rate of the airframe in body-fixed frame, angular body rate and Euler angle parameterization relationship can be given as, With $\eta = [\phi, \theta, \psi]^T$,

$$\mathcal{R}_r = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \quad \text{with } \omega = \mathcal{R}_r$$

The two main forces come from gravity and the thrust of the rotors but to make the model more realistic rotor drag and air friction is also included. The UAV rotorcraft system are quite complex. Their movements are governed by several effects either mechanical or aerodynamic.

In order to get equations of motion of entire system, the following assumptions have been made:

- The hexarotor structure is rigid and symmetrical
- The hexarotor center of mass and body-fixed frame coincides
- Thrust and drag forces are proportional to the square of the propellers' speeds
- The propellers are rigid

According to Newton-Euler equation [4]:

$$\begin{bmatrix} mI_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & J \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} \omega & \wedge mV \\ \omega & \wedge J\omega \end{bmatrix} = \begin{bmatrix} \sum F \\ \sum M \end{bmatrix} \qquad (1)$$

2.2 Hexacopter Mathematical Model

The equations of motion, that governs the translational and the rotational motion for the hexarotor with respect to the body frame are,

2.2.1 Translational Dynamics:

$$\begin{cases} \ddot{x} = 1/m\,(\cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi)U_1 - k_{ftx}\,\dot{x}/m \\ \ddot{y} = 1/m\,(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)U_1 - k_{fty}\,\dot{y}/m \\ \ddot{z} = 1/m\,(\cos\theta\cos\phi)U_1 - k_{ftz}\,\dot{z}/m - g. \end{cases} \qquad (2)$$

2.2.2 Rotational Dynamics:

$$\begin{cases} J_{xx}\,\ddot{\phi} = \dot{\theta}\,\dot{\psi}\,(J_{yy} - J_{zz}) - K_{fax}\,\dot{\phi}^2 - J_r\Omega_r\,\dot{\theta} + l\,U_\phi \\ J_{yy}\,\ddot{\theta} = \dot{\phi}\,\dot{\psi}\,(J_{zz} - J_{xx}) - K_{fay}\,\dot{\theta}^2 + J_r\Omega_r\,\dot{\phi} + lU_\theta \\ J_{zz}\,\ddot{\psi} = \dot{\phi}\,\dot{\theta}\,(J_{xx} - J_{yy}) - K_{faz}\,\dot{\psi}^2 + lU_\psi \end{cases} \qquad (3)$$

The hexacopter's total thrust force and torque control inputs U_1, U_ϕ, U_θ, U_ψ are related to the six motor's speed by the following equations: $U^T=[U_1,U_\phi,U_\theta,U_\psi]$ is the vector of (artificial) input variables[5]:

$$\begin{pmatrix} U_1 \\ U_\phi \\ U_\theta \\ U_\psi \end{pmatrix} \begin{pmatrix} B & B & b & b & b & b \\ -b/2 & -b & -b/2 & B/2 & b & B/2 \\ -b\ \ /2 & 0 & b\ \ /2 & b\ \ /2 & 0 & -b\ \ /2 \\ -d & D & -d & d & -d & d \end{pmatrix} \begin{pmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \\ \Omega_4^2 \\ \Omega_5^2 \\ \Omega_6^2 \end{pmatrix} \qquad (4)$$

If the rotor velocities are needed to be calculated from the control inputs, an inverse relationship between the control inputs and the rotors' velocities is needed, which can be acquired by inverting the matrix in (5) to give,

$$\begin{cases} = (U_1 + 2U_\phi - U_\psi) \\ = (U_1 + U_\phi - U_\theta + U_\psi) \\ = (U_1 - U_\phi - U_\theta - U_\psi) \\ = (U_1 - 2U_\phi + U_\psi) \\ = (U_1 - U_\phi + U_\theta - U_\psi) \\ = (U_1 + U_\phi + U_\theta + U_\psi) \end{cases} \quad (5)$$

The complete dynamics of the UAV is described by

$$\begin{cases} \ddot{\phi} = \frac{1}{J_{xx}} [\dot{\theta}\ \dot{\psi}\ (J_{yy} - J_{zz}) - K_{fax}\dot{\phi}^2 - J_r\Omega_r\dot{\theta} + l\,U_\phi]. \\ \ddot{\theta} = \frac{1}{J_{yy}} [\dot{\phi}\ \dot{\psi}\ (J_{zz} - J_{xx}) - K_{fay}\dot{\theta}^2 + J_r\Omega_r\dot{\phi} + lU_\theta]. \\ \ddot{\psi} = \frac{1}{J_{zz}} [\dot{\phi}\ \dot{\theta}\ (J_{xx} - J_{zz}) - K_{faz}\dot{\psi}^2 + U_\psi] \\ \ddot{x} = -\frac{k_{ftx}}{m}\ \dot{x} + \frac{1}{m} U_X U_1 \\ \ddot{y} = -\frac{k_{fty}}{m}\ \dot{y} + \frac{1}{m} U_Y U_1 \\ \ddot{z} = -\frac{k_{ftz}}{m}\ \dot{z} - g + \frac{\cos\phi\cos\theta}{m} U_1 \end{cases} \quad (6)$$

with : $\begin{cases} U_X = \cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi\ . \\ U_Y = \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi\ . \end{cases}$

3. CONTROL STRATEGY

Due to the nature of the dynamics of the hexarotor, several control algorithms have been applied to it. As to be expected, each control scheme has its advantages and disadvantages. The control schemes used could be broadly categorized as linear and non-linear control schemes. In this review a broad range of controllers within these categories are discussed.

In this section, a control strategy is based on two loops (inner loop and outer loop). The inner loop contains four control laws: roll command (ϕ), pitch command (θ), yaw control (ψ) and controlling altitude Z. The outer loop includes two control laws positions (x, y). The outer control loop generates a desired for roll movement (θ_d) and pitch (ϕ_d) through the correction block. This block corrects the rotation of roll and pitch depending on the desired yaw (ψ_d). The figure below shows the control strategy we will adopt

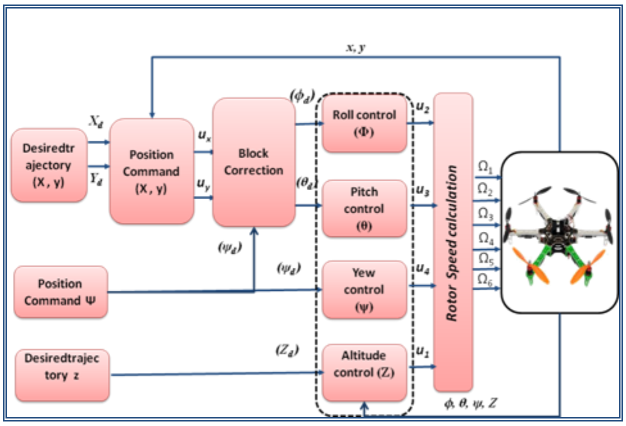

Fig 2. Synoptic scheme of the proposed control strategy

The dynamic model presented in equation set (6) can be rewritten in the state-space form $\dot{X} = f(X, U)$. X ϵ $\Re^{12}$ is the vector of state variables given as follows:

$$X^T = \begin{bmatrix} \phi & & \theta & & \psi & & x & & y & & z & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} \end{bmatrix}$$

$x_1 = \phi$	$x_7 = x$
$x_2 = \quad_1 =$	$x_8 = \quad_7 =$
$x_3 = \theta$	$x_9 = y$
$x_4 = \quad_3 =$	$x_{10} = \quad_9 =$
$x_5 = \psi$	$x_{11} = z$
$x_6 = \quad_5 =$	$x_{12} = \quad_{11} =$

$$\begin{cases} _2 = \ = a_1x_4x_6 + a_2 \quad + a_3\Omega_r x_4 + b_1U_\phi \\ _4 = \ = a_4x_2x_6 + a_5 \quad + a_6\Omega_r x_2 + b_2\,U_\theta. \\ _6 = \ = a_7x_2x_4 + a_8 \quad + b_3\,U_\psi. \\ _8 = \ = a_9x_8 + \quad U_XU_1. \\ _{10} = \ = a_{10}x_{10} + \quad U_YU_1. \\ _{12} = \ = a_{11}x_{12} + \quad U_1 - g. \end{cases} \quad (7)$$

To simplify, define,

$a_1 = (J_{yy} - J_{zz}) / J_{xx}$	$a_2 = -K_{fax} / J_{xx}$	$a_9 = -K_{ftx} / m$
$a_4 = (J_{zz} - J_{xx}) / J_{yy}$	$a_5 = -K_{fay} / J_{yy}$	$a_{10} = -K_{fty} / m$
$a_7 = (J_{xx} - J_{yy}) / J_{zz}$	$a_8 = -K_{faz} / J_{zz}$	$a_{11} = -K_{ftz} / m$
$a_3 = -J_r/J_{xx}$	$a_6 = -J_r/J_{yy}$	
$b_1 = l/J_{xx}$	$b_2 = l/J_{yy}$	$b_3 = l/J_{zz}$

Rewriting the last equation (7) to have the angular and the translational accelerations in terms of the other variables (Rotational and translational equations of motion),

$$\begin{bmatrix} _2 \\ _4 \\ _6 \end{bmatrix} = \begin{bmatrix} a_1x_4x_6 + a_2 \quad + a_3\Omega_r x_4 + b_1U_\phi \\ a_4x_2x_6 + a_5 \quad + a_6\Omega_r x_2 + b_2\,U_\theta \\ a_7x_2x_4 + a_8 \quad + b_3\,U_\psi \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} 8 \\ 10 \\ 12 \end{bmatrix} = \begin{bmatrix} a_9x_8 + (\cos x_1 \cos x_5 \sin x_3 + \sin x_1 \sin x_5)\frac{U_1}{m} \\ a_{10}x_{10} + (\cos x_1 \sin x_3 \sin x_5 - \sin x_1 \cos x_5)\frac{U_1}{m} \\ a_{11}x_{12} - g + (\cos x_1 \cos x_3)\frac{U_1}{m} \end{bmatrix}$$

3.1 PID Controller for Hexarotor

After the mathematical model of the hexarotor along with its open loop simulation is verified, a PID controller was developed. The block diagram for a PID controller is shown in Figure 3.

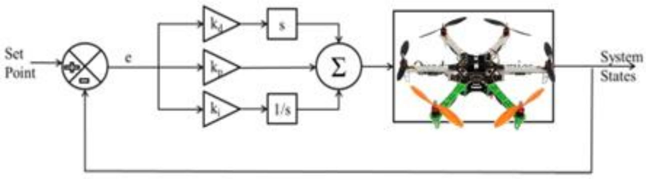

Fig 3. PID Controller Block Diagram

The purpose of the PID controller is to force the Euler angles to follow desired trajectories. The objective in PID controller design is to adjust the gains to arrive at an acceptable degree of tracking performance in Euler angles.

Position and Altitude controller:

PID controller is defined for controlling the $e_x = x_d - x$, $e_y = y_d - y$ and $e_z = z_d - z$. The control objective is to drive both values to zero (e_x, e_y, e_z)=(0,0,0). In this sense, the control laws are:

$$_d = k_{px}(x_d - x) + k_{dx}(\ _d - \ ) + k_{ix}\int(x_d - x)dt$$
$$_d = k_{py}(y_d - y) + k_{dy}(\ _d - \ ) + k_{iy}\int(y_d - y)dt$$
$$U_1 = k_p(z - z_d) + k_d(\ - \ _d) + k_i\int(z - z_d)dt$$

Attitude and heading controller:

The control objective is to maintain the hexarotor in a constant altitude (z). The PID controller for the ϕ, θ and ψ dynamics can be given as

$$U_\phi = k_{p\phi}(\phi_d - \phi) + k_{d\phi}(\ _d - \ ) + k_{i\phi}\int(\phi_d - \phi)dt \quad (Roll\ angle)$$
$$U_\theta = k_{p\theta}(\theta_d - \theta) + k_{d\theta}(\ _d - \ ) + k_{i\theta}\int(\theta_d - \theta)dt \quad (Pitch\ angle)$$
$$U_\psi = k_{p\psi}(\psi_d - \psi) + k_{d\psi}(\ _d - \ ) + k_{i\psi}\int(\psi_d - \psi)dt \quad (Yaw\ angle)$$

In order to design the PID controllers, nonlinear rotational dynamics of hexarotor are linearized around zero, which are given by (8).

$$\phi(s) = (l / s^2J_{xx})U_\phi(s);\ \theta(s) = (l / s^2J_{yy})U_\theta(s);\ \psi(s) = (1 / s^2J_{xx})\,U_\psi(s)$$

It is well known that a conventional linear control (PID) can stabilize a VTOL hexarotor in non critical conditions (e.g., without external disturbances such as wind gusts) or around a specific operating point. In real conditions the use of a classical linear control is limited to a small neighborhood around the operating point. Classical linear controllers are not applicable, and nonlinear control approaches are required

3.2 Feedbach linearization theory

The central idea of feedback linearization is to algebraically transform a given nonlinear system dynamics to dynamics of a linear one, so that the well established linear control techniques can be applied. Feedback linearization is achieved by exact state transformation and feedback, rather than by linear approximation of the dynamics. The relative degree and order of the roll, pitch and yaw subsystem is the same. Input-state feedback linearization can be applied to three subsystems. The nonlinear system is transformed into controllable canonical form after feedback linearization. The closed loop system is reduced to three double integrators after applying inputs U_ϕ, U_θ and U_ψ,

$$U_\phi = f_\phi(x_2, x_4, x_6) + U_2$$
$$U_\theta = f_\theta(x_2, x_4, x_6) + U_3 \quad (9)$$
$$U_\psi = f_\psi(x_2, x_4, x_6) + U_4$$

Where U2, U_3 and U4 are new inputs. On this basis, in order to obtain a linear model, the following equations still demand to be met:

$$a_1x_4x_6 + a_2 \quad + a_3\Omega_r x_4 + b_1 f_\phi(x_2, x_4, x_6) = K_2\, x_2$$
$$a_4x_2x_6 + a_5 \quad + a_6\Omega_r x_2 + b_2 f_\theta(x_2, x_4, x_6) = K_3\, x_4 \quad (10)$$
$$a_7x_2x_4 + a_8 \quad + b_3 f_\psi(x_2, x_4, x_6) = K_4\, x_6$$

where K_2, K_3 and K_4 are undetermined parameters. According to (9), the feedback linearization items f_ϕ, f_θ and f_ψ are given as,

$$f_\phi(x_2, x_4, x_6) = 1/b_1\,(K_2\, x_2 - a_1x_4x_6 - a_2 \quad - a_3\Omega_r x_4)$$
$$f_\theta(x_2, x_4, x_6) = 1/b_2\,(K_3\, x_4 - a_4x_4x_6 - a_5 \quad - a_6\Omega_r x_2) \quad (11)$$
$$f_\psi(x_2, x_4, x_6) = 1/b_3\,(K_4\, x_6 - a_7x_2x_4 - a_8 \quad)$$

Substituting (9) into (8), meanwhile considering (10), the linear system can be obtained as

$$_2 = K_2\, x_2 + b_1U_2.$$
$$_4 = K_3\, x_4 + b_2U_3 \quad (12)$$
$$_6 = K_4\, x_6 + b_3U_4$$

We consider the Lyapunov function

$$V(x_2, x_4, x_6) = \ (\qquad\qquad) \quad (13)$$
$$(x_2, x_4, x_6) = x_2\ \ _2 + x_4\ \ _4 + x_6\ \ _6 = K_2 \quad + K_3 \quad + K_4$$

The derivative expressed by (13) is negative definite if $K_2 < 0$, $K_3 < 0$, and $K_4 < 0$, which guarantees that the operating point of the feedback linearization system is asymptotically stable.

Considering $_1 = x_2$, $_3 = x_4$ and $_5 = x_6$, it is obvious that the feedback linearization system, that is, (12), can be described by linear decoupled differential equations of second order; namely,

$$= K_2 \quad + b_1U_2$$
$$= K_3 \quad + b_2U_3 \quad (14)$$
$$= K_4 \quad + b_3U_4$$

The Laplace transformation is applied to (14), which can transform the system from time domain to frequency domain. Further, the open-loop transfer functions of controlled object hexarotor can be obtained as,

$G_1(s) = \qquad ;\ G_2(s) = \qquad ;\ G_3(s) =$

Feedback linearization controller is able to stabilize the hexarotor even for relatively critical initial conditions. However, the feedback linearization method has some important limitations. In feedback linearization an important assumption is that the model dynamics are perfectly known and can be canceled entirely. Full state measurement is necessary in implementing the control law. Many efforts are being made to construct observers for nonlinear systems and to extend the separation principle to nonlinear systems. Moreover, no robustness is guaranteed in the presence of parameter uncertainty or unmodeled dynamics.

3.3 Backsteepping controller

In this section, a Backstepping controller is used to control the attitude, heading and altitude of the hexarotor. The Backstepping controller is based on the state space model derived in (7).Using the backstepping approach, one can synthesize the control law forcing the system to follow the desired trajectory. Refer to [6] and [7] for more details. By introducing the partial Lyapunov functions [8], to all x-coordinates results in the following backstepping controller:

$$e_i = \begin{cases} x_{id} - x_i & i \epsilon\ [3,5,7,9,11] \\ _{(i-1)d} + k_{(i-1)} e_{(i-1)} - x_i & i \epsilon\ [4,6,8,10,12] \end{cases}$$

$$with\ k_i > 0 \qquad i \epsilon\ [2,\ldots,12]$$

3.3.1Backstepping Control of the Rotations Subsystem

Using the backstepping approach, one can synthesize the control law forcing the system to follow the desired trajectory. Refer to [7] and [8] for more details. For the first step we consider the tracking-error $e_i = x_{1d} - x_1$ and we use the Lyapunov theorem by considering the Lyapunov function V_i positive definite and it's time derivative negative semi-definite:

$$V_i = \begin{cases} & i \epsilon\ [3,5,7,9,11] \\ V_{i-1} + & i \epsilon\ [4,6,8,10,12] \end{cases}$$

For the first step we consider the tracking-error:

$$\begin{cases} e_1 = x_{1d} - x_1 = \phi_d - x_1 \\ V_1 = \quad \text{and} \quad _1 = e_1 \quad _1 = e_1(\ _d - x_2) \end{cases}$$

The stabilization of $e_1$can be obtained by introducing a virtual control input x_2:

$$x_2 = \ _d + k_1 e_1 \Longrightarrow \ _1 = -k_1 \quad \leq 0.$$

For the second step we consider the augmented Lyapunov function:

$$\begin{cases} e_2 = \ _d + k_1 e_1 - x_2 \\ V_2 = \quad + \end{cases}$$

And it's time derivative is then:

$$_2 = e_1 \ _1 + e_2 \ _2$$

$$= e_1(-k_1 e_1 + e_2) + e_2(\ _d + k_1 \ _1 - a_1 x_4 x_6 - a_2 \quad - a_3 \Omega_r x_4 - b_1 u_2)$$

The control input U_ϕ is then extracted, satisfying:

$$_2 = -k_1 \quad - k_2 \quad \leq 0.$$

$$U_\phi = \quad [-a_1 x_4 x_6 - a_2 \quad - a_3 \Omega_r x_4 + \ _d + k_1(-k_1 e_1 + e_2)_+ k_2 e_2 + e_1]$$

Following exactly the same steps as the roll controller, the control input U_θ responsible of generating the pitch rotation and U_ψ responsible of generating the yaw rotation are calculated to be,

$$U_\phi = \quad [-a_1 x_4 x_6 - a_2 \quad - a_3 \Omega_r x_4 + \ _d + k_1(-k_1 e_1 + e_2)_+ k_2 e_2 + e_1]$$

$$U_\theta = \quad [-a_4 x_2 x_6 - a_5 \quad - a_6 \Omega_r x_2 + \ _d + k_3(-k_3 e_3 + e_4)_+ k_4 e_4 + e_3]$$

$$U_\psi = \quad [-a_7 x_2 x_4 - a_8 \quad + \ _d + k_5(-k_5 e_5 + e_6)_+ k_6 e_6 + e_5] \qquad (15)$$

3.3.2Backstepping Control of the Linear Translations

The altitude control U1 and the Linear (U_X,U_Y) Motion Control are obtained using the same approach described in 3.3.1.

$$U_1 = \quad [g - a_{11} x_{12} + \ _d + k_{11}(-k_{11} e_{11} + e_{12})_+ k_{12} e_{12} + e_{11}]$$

$$U_X = (\quad)[-a_9 x_8 + \ _d + k_7(-k_7 e_7 + e_8)_+ k_8 e_8 + e_7]$$

$$U_Y = (\quad) [-a_{10} x_{10} + \ _d + k_9(-k_9 e_9 + e_{10})_+ k_{10} e_{10} + e_9].$$

Backstepping control is a recursive algorithm that breaks down the controller into steps and progressively stabilizes each subsystem. Its advantage is that the algorithm converges fast leading to less computational resources and it can handle disturbances well. The main limitation with the algorithm is its robustness is not good. To increase robustness (to external disturbances) of the general backstepping algorithm, an integrator is added and the algorithm becomes Integrator backstepping control. The integral approach was shown to eliminate the steady-state errors of the system, reduce response time and restrain overshoot of the control parameters.

3.4 Sliding-mode control

The basic sliding mode controller design procedure is performed in two steps. Firstly, choice of sliding surface (S) is made according to the tracking error, while the second step consist the design of Lyapunov function which can satisfy the necessary sliding condition *(S <0)* [9],[10]. Theapplication of sliding mode control to hexarotor dynamic is presented here by obtaining the expression for control input. The sliding surface are define,

$$S_\phi = e_2 + \lambda_1 e_1 = \ _{1d} - x_2 + \lambda_1(x_{1d} - x_1)$$
$$S_\theta = e_4 + \lambda_2 e_3 = \ _{3d} - x_4 + \lambda_2(x_{3d} - x_3)$$
$$S_\psi = e_6 + \lambda_3 e_5 = \ _{5d} - x_6 + \lambda_3(x_{5d} - x_5)$$
$$S_x = e_8 + \lambda_4 e_7 = \ _{7d} - x_8 + \lambda_4(x_{7d} - x_7)$$
$$S_y = e_{10} + \lambda_5 e_9 = \ _{9d} - x_{10} + \lambda_5(x_{9d} - x_9)$$
$$S_z = e_{12} + \lambda_7 e_{11} = \ _{11d} - x_{12} + \lambda_6(x_{11d} - x_{11})$$

Such that

$$\begin{cases} e_i = x_{id} - x_i \\ e_{i+1} = \ _i & i\epsilon[1,\ldots,11] \\ \lambda_i > 0 \end{cases}$$

Assuming here that $V(S_\phi) =$ then, the necessary sliding condition is verified and lyapunov stability isguaranteed. The chosen law for the attractive surface is the time derivative of satisfying($S < 0$)

$$_\phi = -k_1 sign(S_\phi) = \ _2 + \lambda_1 \ _1 = \quad - \ _2 + \lambda_1(\ _{1d} - x_2)$$

$$= -a_1 x_4 x_6 - a_2 x_4 \Omega_r - b_1 U_\phi + \ _d + \lambda_1 (\ _d - x_2).$$

$$U_\phi = \quad [-a_1 x_4 x_6 - a_2 x_4 \Omega_r + \ _d + \lambda_1(\ _d - x_2) - k_1 sign(S_\phi)].$$

The same steeps are followed to extract U_θ, U_ψ and U_1:

$$U_\phi = \quad [-a_1 x_4 x_6 - a_2 x_4 \Omega_r + \ _d + \lambda_1 e_2 - k_1 sign(S_\phi)] \ \text{(Roll)}$$

$$U_\theta = \quad [-a_3 x_2 x_6 - a_4 x_2 \Omega_r + \ _d + \lambda_2 e_4 - k_2 sign(S_\theta)] \ \text{(Pitch)}$$

$$U_\psi = \quad [-a_5 x_2 x_4 - a_4 x_2 \Omega_r + \ _d + \lambda_3 e_6 - k_3 sign(S_\psi)] \text{(Yaw)}$$

$U_1=$ $[g+ \ _d+\lambda_4 e_8 - k_4 sign(S_z)]$ (Altitude)

$U_X= (\ )[\ _d+\lambda_5 e_{10} - k_5 sign(S_x)]$ (Linear x Motion)

$U_Y= (\ )[\ _d+\lambda_6 e_{12} - k_6 sign(S_y)]$ (Linear y Motion)

The SMC resulted in good stability and robustness of the system; but an undesirable Chattering effect of SMC was observed which was very notable in the attitude response unlike the altitude. It minimized with a continuous approximation of a predetermined "sign" function. The presence of the ''sign'' term in the SMC's control law makes it a discontinuous controller. Shows that whenever the value of the surface s is positive, the control law works to decrease the trajectory to reach the sliding surface (s = 0). Ideally it should continue sliding on the surface once hitting it, but due to the delay between the change of sign and the change in the control action, the trajectory passes the surface to the other side. The main drawbacks of chattering are that it causes the excitation of unmodeled system dynamics that yields a possible instability of the system. In addition to that it is associated with a high power consumption and possible actuator damage. These drawbacks make the SMC hard to be implemented on real systems.

3.5 Control using Backstepping+FST Technique.

To increase robustness (to external disturbances) of the general backstepping algorithm, the Backstepping \ FST control results from the merge of the Frenet-Serret Theory [11] and the integral backstepping technique.

As shown in Fig. 2, the complete system control is composed by a cascade-connection of altitude, position and attitude controllers. However, attitude control is the heart of the control system, which maintains the UAVs stable and oriented towards the desired direction. This section shows *roll*- control derivation based on hybrid backstepping and the Frenet-Serret equations previously introduced.

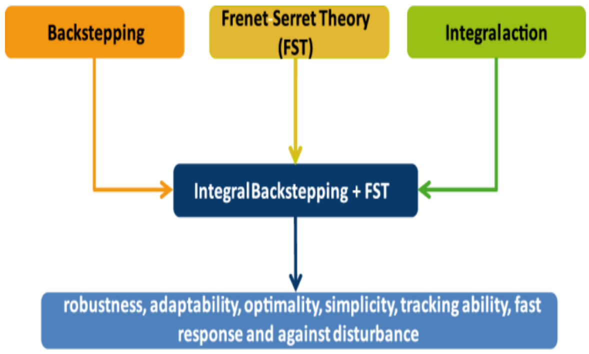

Fig 4. The Proposed control approach

Consider the *roll* tracking error e_ϕ and its dynamics:

$e_\phi = (\ _d - \phi)$ and $\ _\phi = (\ _d - \omega_x)$

A Lyapunov function:

$V(e_\phi) =$; then $(e_\phi) = e_\phi(\ _d - \omega_x)$

The angular velocity ω_x is not our control input, hence we must define a virtual control that fulfill with desired system behavior. The virtual control law for stabilizing the angular tracking error e_2 is then defined as:

$e_2 = \omega_{xd} - \omega_x$

$\omega_{xd} = \ _d + \alpha_\phi e_\phi + \lambda_\phi \int e_\phi(\tau)d\tau$ (α_ϕ and $\lambda_\phi > 0$)

$\Longrightarrow \ _2 = \alpha_\phi \ _\phi + \ _d + \lambda_\phi e_\phi - \ = \alpha_\phi(\omega_{xd} - \omega_x) + \ _d + \lambda_\phi e_\phi -$

With $\ _\phi = -\alpha_\phi e_\phi + \lambda_\phi \int e_\phi(\tau)d\tau + e_2$ **(16)**

$\Longrightarrow \ _2 = \alpha_\phi(-\alpha_\phi e_\phi + \lambda_\phi \int e_\phi(\tau)d\tau + e_2) + \ _d + \lambda_\phi e_\phi -$

3.5.1 Attitude control:

$\ _2 = \alpha_\phi(-\alpha_\phi e_\phi + \lambda_\phi \int e_\phi(\tau)d\tau + e_2) + \ _d + \lambda_\phi e_\phi - (a_1 x_4 x_6 + a_2 \ + a_3 \Omega_r x_4 + b_1 U_\phi)$.

Solving this equation which is the control law for achieving *roll* stabilization being the desirable dynamics for the angular speed tracking error $\ _2 = -e_\phi - \lambda_2 e_2$:

The control input U_ϕ is then extracted, satisfying:

$U_\phi = [\alpha_\phi(-\alpha_\phi e_\phi + \lambda_\phi \int e_\phi(\tau)d\tau + e_2) + \ _d + \lambda_\phi e_\phi - a_1 \ + - a_2 \ - a_3 \Omega_r \ + e_\phi + \lambda_2 e_2]$

$U_\phi = [e_\phi(1 + \lambda_\phi - \ ) + e_2(\alpha_\phi + \lambda_2) + \alpha_\phi \lambda_\phi \int e_\phi(\tau)d\tau + \ _d - a_1 \ + - a_2 \ - a_3 \Omega_r \ ]$.

where (α_ϕ, λ_ϕ and λ_2)> 0 are the control parameters of the backstepping +FST method.

Pitch and yaw control is derived by applying the same procedure. Control laws are:

$U_\phi = [e_\phi(1 + \lambda_\phi - \ ) + e_2(\alpha_\phi + \lambda_2) + \alpha_\phi \lambda_\phi \int e_\phi(\tau)d\tau + \ _d - a_1 \ + - a_2 \ - a_3 \Omega_r \ ]$. **(17)**

$U_\theta = [e_\theta(1+\lambda_\theta - \ ) + e_3(\alpha_\theta + \lambda_3) + \alpha_\theta \lambda_\theta \int e_\theta(\tau)d\tau + \ _d - a_4 \ - a_5 \ - + a_6 \Omega_r \ ]$.

$U_\psi = [e_\psi(1+\lambda_\psi - \ ) + e_4(\alpha_\psi + \lambda_4) + \alpha_\psi \lambda_\psi \int e_\psi(\tau)d\tau + \ _d - a_7 \ - a_8 \ ]$.

3.5.2 Altitude control:

Using the same procedure showed in the previous subsection, altitude tracking error and its dynamics are:

$e_z = \varepsilon_d - \varepsilon$

$e_5 = \alpha_z e_z + \lambda_z \int e_z(\tau)d\tau + \ _z$

$U_1 = [g - a_{11} \ + e_z(1+\lambda_z - \ ) + e_5(\alpha_z + \lambda_5) + \alpha_z \lambda_z \int e_z(\tau)d\tau]$

where (α_z, λ_z and λ_5)> 0 are the control parameters of the backstepping +FST method.

3.5.3 Position control:

$e_x = x_d - x$ $\quad e_6 = \alpha_x e_x + \lambda_x \int e_x(\tau)d\tau + \ _x$

$e_y = y_d - y$ $\quad e_7 = \alpha_y e_y + \lambda_y \int e_y(\tau)d\tau + \ _y$

Control laws are then introduced in Equation,

$U_X = (\ )[e_x(1+\lambda_x - \ ) + e_6(\alpha_x + \lambda_6) - \alpha_x \lambda_x \int e_x(\tau)d\tau]$

$U_Y = (\ )[e_y(1+\lambda_y - \ ) + e_7(\alpha_y + \lambda_7) - \alpha_y \lambda_y \int e_y(\tau)d\tau]$.

where (α_x, α_y, λ_x, λ_y, λ_6, λ_7)> 0.

Equations (17) show the Backstepping+FST methodology. The aim of addressing a new term within the single backstepping was to make the control effort more energetic in terms of angular response. This new term, called $\ _d$ corresponds to a desired acceleration function that strictly depends on the velocity and acceleration of the vehicle. As already mentioned, the Frenet Serret formulas were used to obtain that function.

4. RESULTS AND DISCUSSION

In this paper, five different controllers are presented for the attitude, altitude and heading of a hexarotor. The first technique is a PID controller, it proved to be well adapted to the hexarotor when flying near hover. When the controllers were used outside of the linear region (away from hover), the PID controller failed to stabilize the system due to the fact that PID comes out of a family of linear controllers.. The PID controller was only able to control the hexarotor in near hover and absence of large disturbances. The second one is a feedback linearization controller, the feedback controller, with simplified dynamics, was found to be very sensitive to sensor noise and not robust. Hence, feedback linearization nonlinear control shows good tracking but poor disturbance rejection. However, feedback linearization applied with another algorithm with less sensitivity to noise give good performance. The third control technique is the Backstepping, its ability to control the orientation angles in presence of relatively high perturbations is very interesting. To increase robustness (to external disturbances) of the general backstepping algorithm, an integrator is added and the algorithm becomes Integrator backstepping control. The integral approach was shown to eliminate the steady-state errors of the system, reduce response time and restrain overshoot of the control parameters. The sliding-mode technique is the fourth approach, it was well enough to stably drive the hexarotor to a desired position, but it did not provide excellent results. The SMC controller has the problem of chattering, the switching nature of the controller seems to be ill adapted to the dynamics of the hexarotor. Last but not least, the hybrid control algorithms based on a novel technique developed during this work called: hybrid Backstepping + Frenet-Serret Theory. This controller supports on existing backstepping methodology but adopts the FST formulation that allows introducing a desired attitude angle acceleration function dependent on hexarotor acceleration. Consequently, improvements on disturbance rejection and attitude tracking are achieved against other classical techniques.

As evident from the review, no single algorithm presents the best of the required features. It was found out, in recent literature, that using only one type of flight control algorithms was not sufficient to guarantee a good performance, specially when the hexarotor is not flying near its nominal condition.

It also been discussed that getting the best performance usually requires hybrid control schemes that have the best combination of robustness, adaptability, tracking ability, optimality, fast response, simplicity and disturbance rejection among other factors. However, such hybrid systems do not guarantee good performance; hence a compromise needs to be found for any control application on which of the factors would be most appropriate.

In table.3 summarizes the comparison of the various algorithms as applied to hexarotors with all things being equal. The performance of a particular algorithm depends on many factors that may not even be modeled. Hence, this table serves as guide in accordance with what is presented in this work and common knowledge ([12], [13])

TABLE.3 Comparison of hexarotor control algorithms.

Characteristic	PID	SMC	FBL	BS	BS+FST
Robust	A	A	A	LN	A
Adaptive	LN	H	A	H	H
Optimal	LN	A	LN	LN	LN
Intelligent	LN	LN	LN	LN	A
Tracking ability	A	H	H	H	H
Fast convergence	A	H	H	LN	A
Precision	A	H	H	A	A
Simplicity	H	A	A	LN	LN
Disturbance rejection	LN	H	A	H	H
Noise (signal)	H	LN	A	LN	A
Chattering	LN	H	LN	LN	LN

Legend: **LN**—low to none; **A**—average; **H**—high.

It is evident that even the best linear or nonlinear algorithms had limitations and no single controller had it all. Researchers have tackled this by combining the philosophies of one or more algorithms. Here are few examples, which are not in any way exhaustive of what is in literature. A hybrid fuzzy controller with backstepping and sliding mode control was implemented in [14] and successfully eliminated chattering effect of the sliding mode control algorithm. A feedback linearization controller was applied parallel with a high-order sliding mode controller to a hexarotor in [15]. The sliding mode controller was used as an observer and estimator of external disturbances. The system showed good disturbance rejection and robustness. The backstepping was used to achieve good tracking of desired translational positions and yaw angle whilst maintaining stability of roll and pitch angles [16]. Neural networks were used to compensate for un- modeled dynamics. The major contribution of the paper was the fact that the controller did not require the dynamic model and other parameters. This meant greater versatility and robustness.

This paper has reviewed several common control algorithms that have been used on hexarotors in literature. As evident from the review, no single algorithm presents the best of the required features. It also been discussed that getting the best performance usually requires hybrid control schemes that have the best combination of robustness, adaptability, optimality, simplicity, tracking ability, fast response and disturbance rejection among other factors.

5. CONCLUSIONS AND FUTURE WORKS

The goal of this work was to derive a mathematical model for the hexarotor Unmanned Aerial Vehicle (UAV) and develop nonlinear control algorithms to stabilize the states of the hexarotor, which include its altitude, attitude, heading and position in space and to verify the performance of these controllers with comparisons via computer simulations.The mathematical model of a hexarotor UAV was developed in details including its aerodynamic effects and rotor dynamics which we found lacking in many literatures; a review of the popular controllers proposed for the hexarotor systems is developed.

An important part of this work was dedicated to finding a good control approach for hexarotors. Five techniques were explored from theoretical development to final experiments.

As evident from the review, no single algorithm presents the best of the required features. It also been discussed that

getting the best performance usually requires hybrid control schemes that have the best combination of robustness, adaptability, optimality, simplicity, tracking ability, fast response and disturbance rejection among other factors.
The integral Backstepping + FST control was used as an approach for attitude control (Integral Backstepping + Frenet-Serret Theory). This controller supports on existing backstepping methodology but adopts the FST formulation that allows introducing a desired attitude angle acceleration function dependent on hexarotor acceleration. Consequently, improvements on disturbance rejection and attitude tracking are achieved against other classical techniques. Thus, integral backstepping+FST have been proposed for full control of hexarotors.
Our future work is to implementing the developed control techniques on real hexarotor hardware to give a more fair comparison between their performances. The development of novel control strategies and methodologies for improving the level of autonomy of miniature flying vehicles remains under current research. The research in the Computer Laboratory, systems and renewable energy (LISER) of the National School of Electrical and Mechanical (ENSEM) is continuing toward implementing these algorithms in real time.
The positive results achieved through this development enhance our knowledge of this very unstable system and encourages us to continue towards full autonomy hexarotor.

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