

Fractional PID and Sliding-Mode-Controller for Active Vehicle Suspension System

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Abstract- *The performance of vehicle suspension systems is typically rated by their ability to provide improved road handling and passenger comfort. Passenger comfort in ground vehicles depends on a combination of vertical and angular motions. The main objective of vehicle suspension system is to reduce the effect of the vibrations generated by road irregularities on the human body. The performance of vehicle suspension systems is typically rated by their ability to provide improved road handling and passenger comfort. Active suspension system is the modern system which uses control system for reducing unwanted oscillations of suspension systems. The existing system (using adaptive PID and sliding-mode-controller) used for reducing parameter variations and actuator faults in active suspension system have a main disadvantage is that, settling time of oscillations of suspension system is very high. This problem reduces the performance of active vehicle suspension system. To avoid this, in proposed system, introduce fractional order PID and sliding-mode-controller. To analyze the performance of the proposed approach, computer simulations are carried out to illustrate control performance and robustness*

Keywords— *Active suspension system; Sliding mode controller; Full-scale car suspension control and Fractional PID controller.*

1.INTRODUCTION

The main objective of vehicle suspension is to reduce the effect of the vibrations generated by road irregularities on the human body. The performance of vehicle suspension systems is rated by their ability to provide improved road

handling and passenger comfort to the passengers. In conventional vehicle suspension system only a passive suspension system is used. It contains springs and dampers. In passive suspension systems, the mass-spring-damper are normally fixed and are chosen based on the vehicle's design requirements. However, the mechanical springs and dampers are known to have the limitations of vibration isolation and lack of fine attitude control of the vehicle body. Hence, in recent years, many research efforts have been devoted to the design and control of active suspension systems.

Active suspension system shows better performance than passive and semi-active suspension systems. Active suspension systems can provide more handling capability and ride quality than passive or semi-active suspension systems. But the main problem facing by the active suspension systems are actuator faults and parameter variations. Due to these factors, its effectiveness decreases. However, suitable solutions are needed to optimize their performance and provide closed-loop stability in a full-scale car model to mitigate road disturbances such as bumps and grade changes on the passenger's ride comfort. [7].

The contribution of this paper consists of the design and the integration of control, diagnosis, and fault tolerance for a nonlinear full vehicle active suspension system. In addition, the dynamics of the four force actuators are taken into consideration. This is because force actuators are crucial elements of the system. The control law is designed using sliding mode techniques with fractional PID. A robust sliding-mode controller was employed to handle system uncertainties.

This paper is organized as follows. In Section II, the state space model of system is given. The design of the control law is then detailed in Section III. In Section IV, explain the need of second order differentiator. Section V deals with simulation results, to illustrate the proposed strategy of the control, the diagnosis, and the fault tolerance. In Section VI conclusions are discussed.

2. STATE-SPACE MODEL OF THE FULL-SCALE VEHICLE DYNAMICS SYSTEM

Designing an automotive suspension system is an interesting and challenging control problem. When the suspension system is designed, a 1/4 model (one of the four wheels) is used to simplify the problem to a 1D multiple spring-damper system. A diagram of this system is shown below. This model is for an active suspension system where an actuator is included that is able to generate the control force U to control the motion of the vehicle body.

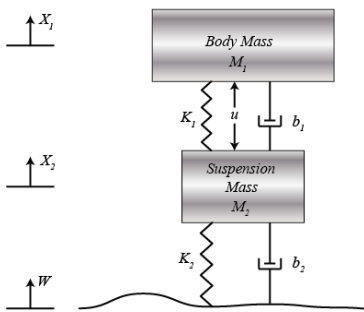


Figure 1: Model of suspension system (1/4 vehicle)

Where

- M_1 - 1/4 bus body mass
- M_2 - suspension mass
- K_1 - spring constant of suspension system
- K_2 - spring constant of wheel and tire
- b_1 - damping constant of suspension system
- b_2 - damping constant of wheel and tire
- U - control force

When the vehicle is experiencing any road disturbance (i.e. pot holes, cracks, and uneven pavement), the vehicle body should not have large oscillations. The oscillations should dissipate quickly. The deformation of the tire ($X_2 - W$) is negligible. So distance ($X_1 - W$) is very difficult to measure. So we will use the distance ($X_1 - X_2$) instead of ($X_1 - W$)

From the picture above, we can obtain the dynamic equations

$$M_1 \ddot{X}_1 = -b(\dot{X}_1 - \dot{X}_2) - K_1(X_1 - X_2) + U \quad (1)$$

$$M_2 \ddot{X}_2 = b(\dot{X}_1 - \dot{X}_2) + K_1(X_1 - X_2) + b_2(\dot{W} - \dot{X}_2) + K_2(W - X_2) - U \quad (2)$$

There for dynamic equations in state-space form are given below.

$$\begin{bmatrix} \dot{X}_1 \\ \ddot{X}_1 \\ \dot{Y}_1 \\ \ddot{Y}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-b_1 b_2}{M_1 M_2} & 0 & \frac{b_1}{M_1} \left[\frac{b_1}{M_1} + \frac{b_2}{M_1} + \frac{b_3}{M_2} \right] - \frac{k_1}{M_1} & \frac{-b_1}{M_1} \\ \frac{b_2}{M_2} & 0 & - \left[\frac{b_1}{M_1} + \frac{b_1}{M_2} + \frac{b_2}{M_2} \right] & 1 \\ \frac{k_2}{M_2} & 0 & - \left[\frac{k_1}{M_1} + \frac{k_1}{M_2} + \frac{k_2}{M_2} \right] & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ \dot{X}_1 \\ Y_1 \\ \dot{Y}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_1} \\ 0 \\ \frac{1}{M_1} + \frac{1}{M_2} \end{bmatrix} \begin{bmatrix} U \\ W \end{bmatrix} \quad (3)$$

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ \dot{X}_1 \\ Y_1 \\ \dot{Y}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} U \\ W \end{bmatrix} \quad (4)$$

Where $Y_1 = X_1 - X_2$

3. FAULT-TOLERANT CONTROL DESIGN

3.1. SMC Design

In control system, sliding mode control is a nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to "slide" along a cross-section of the system's normal behavior. The state-feedback control law is not a continuous function of time. Instead, it can switch from one continuous structure to another based on the current position in the state space. The motion of the system as it slides along the boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding surface.

The chassis part of the vehicle is described by the equations,

$$\dot{X}_{17} = X_{18} \quad (5)$$

$$\dot{X}_{18} = f_{f18} + G_Z U_Z \quad (6)$$

$$G_Z = \frac{1}{M}; u_z = A_p(X_3 + X_7 + X_{11} + X_{15}) \quad (7)$$

$$\dot{X}_{19} = X_{20} \quad (8)$$

$$\dot{X}_{20} = f_{f20} + G_{\theta}U_{\theta} \quad (9)$$

$$G_{\theta} = \frac{\cos X_{19}}{I_{yy}} \quad (10)$$

$$u_{\theta} = A_p[\alpha(X_3 + X_7) - b(X_{11} + X_{15})] \quad (11)$$

$$\dot{X}_{21} = X_{22} \quad (12)$$

$$\dot{X}_{22} = f_{f22} + G_{\varphi}U_{\varphi} \quad (13)$$

$$G_{\psi} = \frac{\cos X_{21}}{I_{xx}} \quad (14)$$

$$u_{\psi} = A_p[-c(X_3 + X_{11}) + d(X_7 + X_{15})] \quad (15)$$

Where

$X_1 = Z_{ufr}$ =front-right unsprung mass height

$X_2 = \dot{Z}_{ufr}$ =front-right unsprung mass velocity

$X_3 = A_{fr}$ =front-right actuator load pressure

$X_4 = Z_{vfr}$ =front-right spool valve position

$X_5 = Z_{ufl}$ =front-left unsprung mass height

$X_6 = \dot{Z}_{ufl}$ =front-left unsprung mass velocity

$X_7 = A_{fl}$ =front-left actuator load pressure

$X_8 = Z_{vfl}$ =front-left spool valve position

$X_9 = Z_{urr}$ =rear-right unsprung mass height

$X_{10} = \dot{Z}_{urr}$ =rear-right unsprung mass velocity

$X_{11} = A_{rr}$ =rear-right actuator load pressure

$X_{12} = Z_{vrr}$ =rear-right spool valve position

$X_{13} = Z_{url}$ =rear-left unsprung mass height

$X_{14} = \dot{Z}_{url}$ =rear-left unsprung mass velocity

$X_{15} = A_{rl}$ =rear-left actuator load pressure

$X_{16} = Z_{vrl}$ =rear-left spool valve position

$X_{17} = Z$ =heave position of the sprung mass

$X_{18} = \dot{Z}$ =heave velocity of the sprung mass

$X_{19} = \theta$ =pitch angle of the sprung mass

$X_{20} = \dot{\theta}$ =pitch angular velocity of the sprung mass

$X_{21} = \varphi$ =roll angle of the sprung mass

$X_{22} = \dot{\varphi}$ =roll angular velocity of the sprung mass

$y = [X_3, X_7, X_{11}, X_{15}, X_{17}, X_{18}, X_{19}, X_{20}, X_{21}, X_{22}]^T$

The output error E is defined as,

$$E = \begin{bmatrix} e_z \\ e_{\theta} \\ e_{\varphi} \end{bmatrix} = \begin{bmatrix} X_{d17} - X_{17} \\ X_{d19} - X_{19} \\ X_{d21} - X_{21} \end{bmatrix} \quad (16)$$

Where X_d represents corresponding desired values.

From above equation (16), sliding surface is defined as

$$S = \begin{bmatrix} S_z \\ S_{\theta} \\ S_{\varphi} \end{bmatrix} = \dot{E} + \lambda E \quad (17)$$

Where $\lambda = \text{diag} |\lambda_z, \lambda_{\theta}, \lambda_{\varphi}|$ is a positive constant.

$S_z, S_{\theta}, S_{\varphi}$ are sliding surfaces for heave, roll and pitch motions of vehicle. Take derivative of (17) and apply chassis equation to it, then we get,

$$\begin{aligned} \dot{S} = \dot{E} + \lambda \dot{E} &= \begin{bmatrix} \ddot{e}_z + \lambda_z \dot{e}_z \\ \ddot{e}_{\theta} + \lambda_{\theta} \dot{e}_{\theta} \\ \ddot{e}_{\varphi} + \lambda_{\varphi} \dot{e}_{\varphi} \end{bmatrix} \\ &= \begin{bmatrix} \ddot{X}_{d17} - f_{f18} - \Delta f_{f18} - G_z u_z - \Delta G_z u_z + \lambda_z \dot{e}_z \\ \ddot{X}_{d19} - f_{f20} - \Delta f_{f20} - G_{\theta} u_{\theta} - \Delta G_{\theta} u_{\theta} + \lambda_{\theta} \dot{e}_{\theta} \\ \ddot{X}_{d21} - f_{f22} - \Delta f_{f22} - G_{\varphi} u_{\varphi} - \Delta G_{\varphi} u_{\varphi} + \lambda_{\varphi} \dot{e}_{\varphi} \end{bmatrix} \end{aligned} \quad (18)$$

By simplification (18) become,

$$\dot{S} = \begin{bmatrix} X_z - \Delta f_{f18} - G_z u_z - \Delta G_z u_z \\ X_{\theta} - \Delta f_{f20} - G_{\theta} u_{\theta} - \Delta G_{\theta} u_{\theta} \\ X_{\varphi} - \Delta f_{f22} - G_{\varphi} u_{\varphi} - \Delta G_{\varphi} u_{\varphi} \end{bmatrix} \quad (19)$$

Where

$$\begin{bmatrix} X_z \\ X_{\theta} \\ X_{\varphi} \end{bmatrix} = \begin{bmatrix} \ddot{X}_{d17} - f_{f18} + \lambda_z \dot{e}_z \\ \ddot{X}_{d19} - f_{f20} + \lambda_{\theta} \dot{e}_{\theta} \\ \ddot{X}_{d21} - f_{f22} + \lambda_{\varphi} \dot{e}_{\varphi} \end{bmatrix} \quad (20)$$

The equivalent control is designed by imposing the dynamics of the sliding surface $\dot{S} = 0$ in the absence of uncertainties in the system dynamics. From (19) we get,

$$u_{eq} = \begin{bmatrix} u_{eqz} \\ u_{eq\theta} \\ u_{eq\varphi} \end{bmatrix} = \begin{bmatrix} G_z^{-1} X_z \\ G_{\theta}^{-1} X_{\theta} \\ G_{\varphi}^{-1} X_{\varphi} \end{bmatrix} \quad (21)$$

Where G_z, G_{θ} and G_{φ} are the input gains, get from chassis equations of the vehicle. There for the complete SMC equation is given by,

$$\begin{bmatrix} u_z \\ u_\theta \\ u_\phi \end{bmatrix} = \begin{bmatrix} u_{eqz} + G_z^{-1}u_{sz} \\ u_{eq\theta} + G_\theta^{-1}u_{s\theta} \\ u_{eq\phi} + G_\phi^{-1}u_{s\phi} \end{bmatrix} \quad (22)$$

Now we check the stability of the SMC using Lyapunov stability criterion. For that we consider a Lyapunov function,

$$V = \frac{1}{2}S_z^2 + \frac{1}{2}S_\theta^2 + \frac{1}{2}S_\phi^2 \quad (23)$$

Then found out \dot{V} from above equation. Then we get

$$\dot{V} = S_z\dot{S}_z + S_\theta\dot{S}_\theta + S_\phi\dot{S}_\phi \quad (24)$$

Then substitute (19) and (22) to (24), get the equation

$$\begin{aligned} \dot{V} = & S_zX_z - S_z\Delta f_{f18} - S_zG_zu_z - S_z\Delta G_zu_z + S_\theta X_\theta - \\ & S_\theta\Delta f_{f20} - S_\theta G_\theta u_\theta - S_\theta\Delta G_\theta u_\theta + S_\phi X_\phi - S_\phi\Delta f_{f22} - \\ & S_\phi G_\phi u_\phi - S_\phi\Delta G_\phi u_\phi \end{aligned} \quad (25)$$

Sliding mode controller is designed by the equations

$$\begin{aligned} u_{sz} &= \frac{\epsilon_z}{1-\rho_z} \frac{S_z}{\|S_z\|}; \quad u_{s\theta} = \frac{\epsilon_\theta}{1-\rho_\theta} \frac{S_\theta}{\|S_\theta\|}; \\ u_{s\phi} &= \frac{\epsilon_\phi}{1-\rho_\phi} \frac{S_\phi}{\|S_\phi\|} \end{aligned} \quad (26)$$

Apply (26) into (25) and by the simplification we get

$$\dot{V} \leq 0 \quad (27)$$

There for by the Lyapunov stability criterion, from above equation, we can say that designed SMC is stable. The chattering phenomenon is one of the actual problems in modern SMC theory [1]. In chattering mode of SMC, the control action is highly discontinues and needs infinite frequency. This is not desirable to most of the systems, so (26) is modified as,

$$\begin{aligned} u_{sz} &= \frac{\epsilon_z}{1-\rho_z} \frac{S_z}{\|S_z\|+\delta_z}; \quad u_{s\theta} = \frac{\epsilon_\theta}{1-\rho_\theta} \frac{S_\theta}{\|S_\theta\|+\delta_\theta} \\ u_{s\phi} &= \frac{\epsilon_\phi}{1-\rho_\phi} \frac{S_\phi}{\|S_\phi\|+\delta_\phi} \end{aligned} \quad (28)$$

Where δ_z, δ_θ and δ_ϕ are arbitrary small positive constants.

Now we require the values of x_3, x_7, x_{11} and x_{15}

From (5) to (15) we get

$$\begin{bmatrix} u_z \\ u_\theta \\ u_\phi \end{bmatrix} = A_p \begin{bmatrix} 1 & 1 & 1 & 1 \\ a & a & -b & -b \\ -c & d & -c & d \end{bmatrix} \begin{bmatrix} x_3 \\ x_7 \\ x_{11} \\ x_{15} \end{bmatrix} \quad (29)$$

The above equation can be written as

$$\begin{bmatrix} u_z \\ u_\theta \\ u_\phi \end{bmatrix} = A_p W \begin{bmatrix} x_3 \\ x_7 \\ x_{11} \\ x_{15} \end{bmatrix} \quad (30)$$

$$\Rightarrow \begin{bmatrix} x_3 \\ x_7 \\ x_{11} \\ x_{15} \end{bmatrix} = A_p^{-1} W^* \begin{bmatrix} u_z \\ u_\theta \\ u_\phi \end{bmatrix} \quad (31)$$

Where

$$W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ a & a & -b & -b \\ -c & d & -c & d \end{bmatrix} \text{ and}$$

$$W^* = \text{invers of } [W^T(WW^T)]$$

The states in (29) are considered the desired outputs for the four subsystems. Therefore, we define as

$$[X_3, X_7, X_{11}, X_{15}] = [X_{3d}, X_{7d}, X_{11d}, X_{15d}] \quad (32)$$

3.2. Fractional PID Design

The main problem in existing system is the large amount of oscillation in the step response of displacement. Angular motion does not have such large problem. So FPID is concentrate on the displacement motion.

The differential equation of fractional order PID controller is given by,

$$u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^\mu e(t) \quad (33)$$

Fractional PID has five tunable parameters such as K_p, K_i, K_d, λ and μ

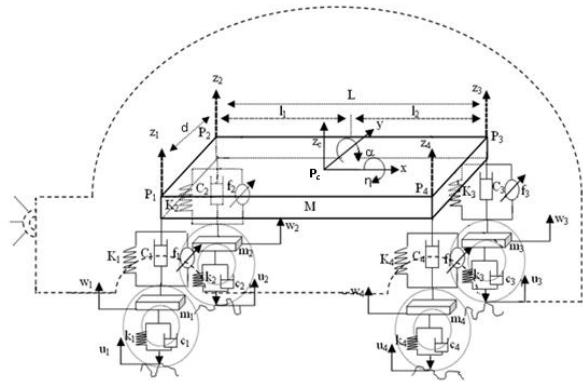


Figure 2: Full vehicle nonlinear active suspension systems

The vertical motions of the sprung mass are governed by the following equation

$$M_z \ddot{z}_c = - \sum_{i=1}^4 F_{ki} - \sum_{i=1}^4 F_{ci} + \sum_{i=1}^4 F_{pi} \quad (34)$$

Where

$$F_{ki} = K_i(Z_i - W_i) + \xi K_i(Z_i - W_i)^3 \quad (35)$$

$$F_{ci} = C_i(\dot{Z}_i - \dot{W}_i) + \xi C_i(\dot{Z}_i - \dot{W}_i)^2 \text{sgn}(\dot{Z}_i - \dot{W}_i) \quad (36)$$

Where F_{ki} and F_{ci} are nonlinear suspension spring forces and nonlinear suspension damping force, respectively.

$$F_{pi} = F_{ai} - F_{fi} \quad (37)$$

Where F_{ai} - nonlinear hydraulic force is provided by the i^{th} actuator and F_{fi} -is the nonlinear frictional force due to rubbing of piston seals with the cylinders wall inside the i^{th} actuator. Stiffness and damping coefficients denoted by K_i and C_i ; sprung mass represented by M ; unsprung masses by m_i ; The vertical displacements of unsprung masses are denoted by W_1, W_2, W_3 and W_4 ; vertical displacements at each suspension point are denoted by Z_1, Z_2, Z_3 and Z_4 ; Z_c, α and η denote the displacement at the center of gravity of the vehicle, pitch angle and roll angle, respectively.

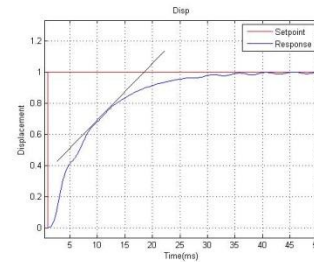


Fig 3: S-shaped unit-step response

$$K_p = 2.1187 - 3.5207L - 0.1563T + 1.5827L^2 + 0.0025T^2 + 0.1824TL$$

$$K_i = -0.5201 + 2.6643L + 0.3453T - 1.0944L^2 + 0.0002T^2 - 0.1054TL$$

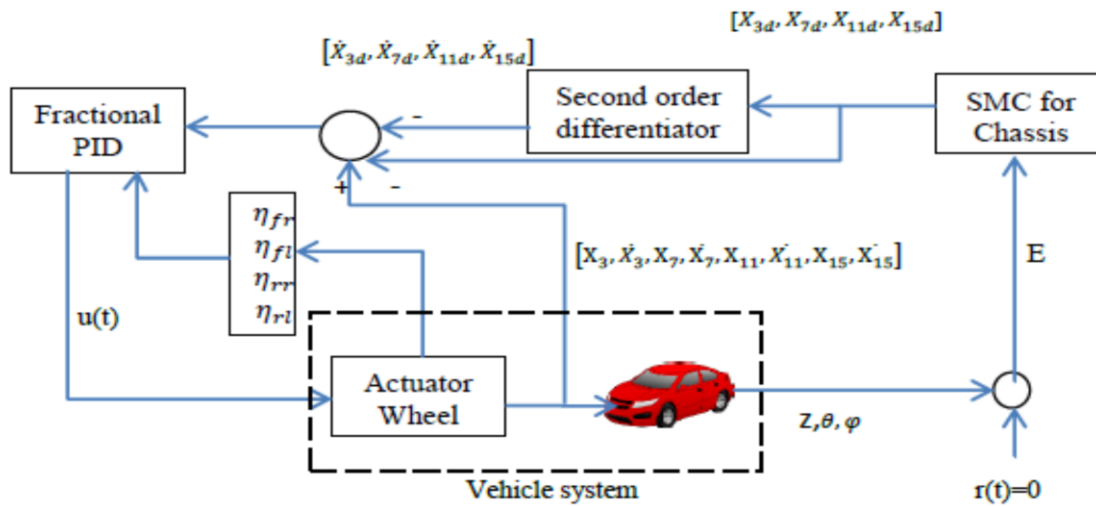


Figure 4: Block diagram of the proposed FPID-SMC

The first set of rules proposed by Ziegler and Nichols apply to systems with an S-shaped unit step response is used for fractional PID design.

From step responses (fig.3) we get $L=1, L+T=18$ and $T=17$. In the rule, it is given that if $0.1 \leq T \leq 50$ and $L \leq 2$, we can get the values of P, I, λ, D and μ from first set of rules proposed by Ziegler and Nichols as,

$$K_d = 1.1421 - 1.3707L + 0.0357T + 0.5552L^2 - 0.0002T^2 + 0.2630TL$$

$$\lambda = 1.0645 - 0.3268L - 0.0229T + 0.2018L^2 + 0.0003T^2 + 0.0028TL$$

$$\mu = 1.2902 - 0.5371L - 0.0381T + 0.2208L^2 + 0.0007T^2 - 0.0014T$$

4. SECOND-ORDER DIFFERENTIATOR

While designing SMC, we need \dot{E} . For that we need $[\dot{X}_{3d}, \dot{X}_{7d}, \dot{X}_{11d}, \dot{X}_{15d}]$. $[X_{3d}, X_{7d}, X_{11d}, X_{15d}]$ are get from (29). For finding derivative values, a second order differentiator is used. The block diagram of the overall fractional PID-sliding-mode controller (FOPID-SMC) is shown in Fig. 4. At first, the heave, pitch, and roll and their corresponding rates of change in the chassis are measured and transmitted to the SMC controller. Then, the output of the SMC and its derivatives are compared with the actuator pressure and the rate of pressure variations. These errors along with the actuator effectiveness factors are employed in the FOPID controller to produce suitable input signals.

5. SIMULATION RESULTS

The road disturbances are represented by step responses. So to evaluate how suspension system behaves in road undulations, we take step responses of displacement and angular movements of suspension system. The step response of displacement and angle of existing system (using APID and SMC) is shown in fig.5 to fig.7.

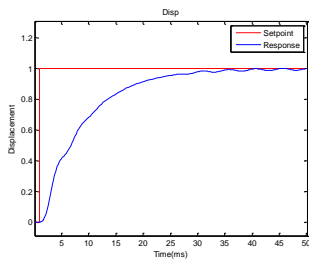


Figure 5: Step response for displacement

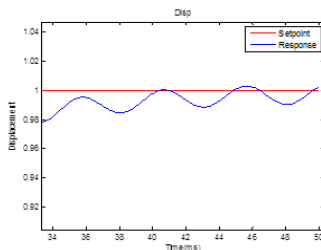


Figure 6: Step response for displacement (zoomed)

From the above step response of displacement, it is seen that, oscillation take more time to settle down. These sustaining oscillations reduce the performance of active

suspension systems. But step response of angle is good (fig.7). It shows only very negligible oscillations.

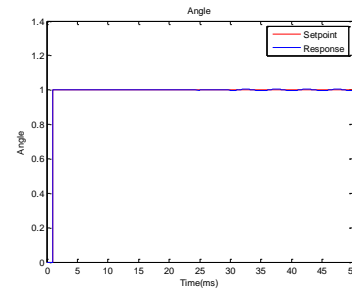


Figure 7: Step response for angle

In proposed system, we introduce a fractional PID to reduce settling time of displacement and reduce unwanted oscillations. By introducing FOPID we can also accommodate fractional gains of sliding mode controller. Thus the performance of suspension system should increase to a great extent.

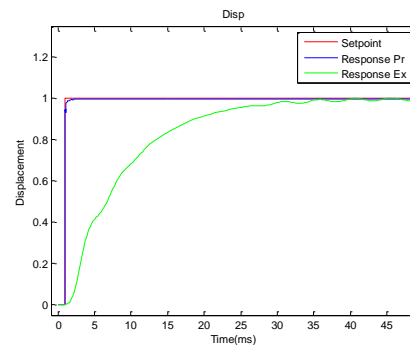


Figure 8: proposed system v/s existing system

6. CONCLUSION

In this paper, a fractional PID and a sliding-mode based fault tolerant control approach were introduced to handle the uncertainties and actuator faults in a full-scale car suspension system. Robust sliding mode controller is used to handle system uncertainties and fractional PID part helps to reduce settling time of oscillations of displacement motion (in step responses). Thus we can improve the performance of active suspension systems. Also there is a chance of occurrence of fractional gains in sliding mode controller. Fractional PID can also accommodate this. Simplicity of the overall scheme and the stabilization of the system under both faulty and fault-free conditions are the main positive features of the proposed approach.

**APPENDIX
TABLE 1
SIMULATION PARAMETERS**

Notation	Parameters	Value
a	Center of gravity to front axle distance	1.4m
b	Center of gravity to rear axle distance	1.7m
c	Center of gravity to right side	1.5m
d	Center of gravity to left side	1.5m
α	Actuator parameters	4.15e13
β	Actuator parameters	1
γ	Actuator parameters $N/m^2/kg^{\frac{1}{2}}$	1.54e9
P_s	Supply pressure(in pa)	10342500
A_p	Area of piston	3.35e-4m ²
τ	Servo valve time constant	.003s
C_{srr}, C_{srl}	Rear suspension damping	1100N/m/s
I_{xx}, I_{yy}	Roll & pitch moment of inertia	460 & 2160 kgm ²
M, m	Sprung & unsprung mass(in kg)	1500 & 59 kg
k_{sfr}, k_{sfl}	Front sprung stiffness	35000 N/m
k_{srr}, k_{srl}	Rear sprung stiffness	38000 N/m
$k_{ufr,fl,rr,rl}$	Tires stiffness	190000 N/m
$C_{sfr,fl}$	Front suspension damping	1000 N/m/s

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