Power Flow Method for Loss Allocation in Radial Distribution Networks with DGs

G.Naguraiah[1], K. Jithendra Goud[2]

[1]*M.Tech student, Electrical Department, JNTUA College of Engi*neering, Anantapur, India.
[2]*Assistant Professor, Electrical Department, JNTUA College of Engi*neering, Anantapur, India.

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Abstract: **The allocation of the system losses to suppliers and consumers is a challenging issue for the restructured electricity business. Loss allocation to sinks and sources is important task for radial distribution networks with DGs, since DGs plays an important role in restructured electricity market. In this paper, new loss allocation algorithm is proposed for radial distribution Networks with DGs. The algorithm is based on the active and reactive power flow through the distribution feeder, and loss allocation depends upon the sink nodes and DGs placement in the radial distribution Network. In this method, an algorithm is composed of three steps. In the first step, starts from source nodes, the power loss allocated to the loads connected to each node is obtained. In the next step, the total power loss is allocated to the DGs based on the results of this step. Likewise, in this step, power losses allocate to the nodes starts from sink nodes, in the final step; normalization is executed at the end to compensate. This paper proposes a power flow loss allocation method that can be applied to radial medium voltage distribution systems with DGs. Voltage at each node of the 69-node distribution network is calculated by using genetic algorithm and these voltages are compared with the previous method.**

Keywords: Distributed generation, Loss allocation, Radial distribution system, Active and Reactive power flows.

1. INTRODUCTION

Distributed generation has been increasing many parts of the world. The main reason for increase DGs is due to liberalization electricity market, constraints on building new transmission and distribution lines, and environmental concerns. DG is becoming an increasing important part of the power infrastructure. The advantages of increased power reliability, higher energy efficiency when waste heat is utilized, and the elimination of electric grid transmission and distribution losses are all driving the installation of DG. The placement of Distribution Generators are increased in distribution network, Due to this the distribution network can change from passive mode to active mode. Therefore some of the transmission issues will be considered to radial distribution network including DGs [1]. There are so many issues will take into distribution network, but in this paper will consider as loss allocation to the nodes which are connected to the network. Here loss allocation specifies the fraction of total distribution loss that each load or DG is responsible for calculations. Although there are many transmission Loss Allocation methods, but distribution Loss Allocation is still a new phenomenon and most of the distribution system operators still do not have a standard policy.

Different methods for loss allocation have been reported in the literature. Pro rata [2], loss allocated to nodes based on active power flow. Marginal method [3]-[5], loss calculated based on marginal loss coefficients, based on results of power flow. Direct loss coefficients method [3], which finds a direct relation between the losses and nodal injections, both this method and marginal method are based on the results of Newton–Raphson power flow.Z-bus method [7], which is not applicable to congaing only over head lines, since Y-bus matrix is singular for such systems, shunt admittance is negligible [8], the power summation algorithm proposed in [6] is a tracing method, in which active and reactive power of the receiving end of each branch are decomposed to the nodal injection of the system nodes and losses of downstream branches that are connected to the branch. Substitution method, where the responsibility of a participant is calculated by subtracting the total loss when the participant is not attached to the system from the loss when it is attached; this method is proved to give unfair results [3].

Reference [10] and [11] ,the following points should be considered in distribution loss allocation:
1) The slack node for distribution systems is always the node connecting transmission and distribution systems; however, in transmission LA, there are many alternatives for the slack node.
2) Unlike the transmission LA methods, in which a fraction of loss may be allocated to the slack node, in distribution LA methods, no loss is assigned to the slack node.
3) The methods used for transmission LA could be used in distribution systems; however, the loss allocated to the slack node in these methods should be redistributed among other nodes in proportion to the nodes' currents [8].
4) It is implicitly assumed that the loads and DGs have bilateral contracts with the distribution company [12].

This paper proposes a power flow loss allocation method that can be applied to radial medium voltage distribution systems with DGs. The method starts by assigning zero power losses to a particular group of nodes. Then, the power loss allocated to other nodes is calculated based on the power loss of the lines connecting the zero assigned nodes and these nodes.

2. LITERATURE SURVEY

In this Operation is presented. These concepts are derived from extensive study of the practical power system operation Chapter certain concepts related to "Power flow method for loss allocation in Radial Systems with DGs" of Indian Electrical system and various regulations stipulated by Central Electricity Authority of India.

1. P. M. Sotkiewicz and J. M. Vignolo, "Nodal pricing for distribution networks: Efficient pricing for efficiency enhancing DG," *IEEE Trans. Power Syst.*, vol. 21, no. 2, pp. 1013–1014, May 2006.

As distributed generation (DG) becomes more widely deployed distribution networks become more active and take on many of the same characteristics as transmission. We propose the use of nodal pricing that is often used in the pricing of short-term operations in transmission. As an economically efficient mechanism, nodal pricing would properly reward DG for reducing line losses through increased revenues at nodal prices, and signal prospective DG where it ought to connect with the distribution network. Applying nodal pricing to a model distribution network we show significant price differences between busses reflecting high marginal losses. Moreover, we show the contribution of a DG resource located at the end of the network to significant reductions in losses and line loading. We also show the DG resource has significantly greater revenue under nodal pricing reflecting its contribution to reduced line losses and loading.

2. M. Ilic, F. Galiana, and L. Fink, *Power Systems Restructuring: Engineering and Economics*. Norwell, MA: Kluwer, 1998.

This thesis describes the development of three decision support models for long-term investment planning in restructured power systems. The model concepts address the changing conditions for the electric power industry, with the introduction of more competitive markets, higher uncertainty and less centralized planning. Under these circumstances there is an emerging need for new planning models, also for analyses of the power system in a long-term perspective. The thesis focuses particularly on how dynamic and stochastic modelling can contribute to the improvement of decision making in a restructured power industry.

The models can also serve as decision supporting tool on regulatory approaches have become more important after the introduction of competitive power markets, due to the participants' increased exposure to price fluctuations and economic risk. Our models can be applied by individual participants in the power system to evaluate investment projects for new power generation capacity. The models can also serve as a decision support tool on a regulatory level, providing analyses of the long-term performance of the power system under different regulations and market designs.

3. POWER FLOW LOSS ALLOCATION METHOD ASSUMPTIONS

1) Loss allocated to each node connecting to the distribution network is assumed to be zero.

2)In case total generation is greater than the demand at any node, the loss allocation method allocates zero losses to all loads connected to this node, since it means that the loads are locally fed by the DGs. In case, total generation is less than demand at any node the method allocates zero losses to the DGs connected to the node.

3) Negative loss is not allocated to the nodes of the distributed network.

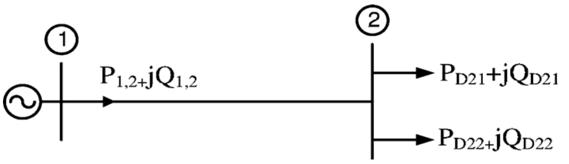

Figure-1

4) Consider the circuit shown in Figure 2, which shows two nodes of a system. The power loss of the line connecting nodes 1 and 2 can be written as

$$P_{LOSS_{1,2}} = r_{1,2}\frac{P_{1,2}{}^2+Q_{1,2}{}^2}{|V_1|^2} = k(P_{1,2}{}^2 + Q_{1,2}{}^2), \qquad (1)$$

Where $r_{1,2}$ is resistance of the line; $P_{1,2}$ and $Q_{1,2}$ are active power and reactive power through the line and V_1 is the node 1 voltage. In the above equation 1 shows as $P_{LOSS_{1,2}}$ is composed of two terms. The first term is $kP_{1,2}{}^2$ is active flows through the line and second term $kQ_{1,2}{}^2$ is reactive flows through the line. Therefore equation 1 can be written as

$$P_{LOSS_{1,2}} = P^p_{LOSS_{1,2}} + P^q_{LOSS_{1,2}} \qquad (2)$$

As $P_{1,2} = P_{D21} + P_{D22} + P_{LOSS_{1,2}}$, can be written as

$$P^p_{LOSS_{1,2}} = kP^2_{1,2} = k(P_{D21} + P_{D22} + P_{LOSS_{1,2}})^2 \qquad (3)$$

This is equal to k $(P_{D21}+P_{D22})^2$, $P_{LOSS_{1,2}}$ is small compared to P_{D21} and P_{D22}.Such that, it can be ignored.

Based on the Shapley value, which is for calculating the contribution of a player in a game played by a number of players, the contribution of load P_{D21} in $P^p_{Loss1,2}$ is equal to

$$K(P^2_{D21} + P_{D21}P_{D22}) - \qquad (4)$$

A similar formulation can be derived, if the number of loads or lines connected to node 2 increases.

4. POWER FLOW LOSS ALLOCATION METHOD

4.1 Calculating the loss allocated to the loads:

4.1.1 Loss Allocated to Loads Due to Active Power Flows:

In this step, first the loss assigned to any of the nodes due to active flows is calculated and then distributed between the loads connected to it.

The procedure is based on the fact that the loss assigned to node k is dependent on the loss assigned to all nodes that are adjacent to this node and send active power to it. Let branch $b_{n,k}$ connect node n to node k and $P^s_{n,k}$, which is the active power flow at sending node n of this branch, be positive. Presume that the loss allocated to node n due to active flows is denoted as L_n^p and is known. We want to distribute L_n^p among all the loads that are connected to node n and the branches connected to this node whose active power flow from this node is positive. Based on the Shapley value method results given in (4), the contribution $P_{n,k}^s$ in L_n^p is proportional to

$$\left\{\left(P_{n,k}^s\right)^2 + P_{n,k}^s\left(\sum_{\substack{m\in A_{n+1}\\ m\neq k}} P_{n,m}^s + \sum_{D_m\in D_n} P_{D_m}\right)\right\} \text{- } (5)$$

Where $P_{n,k}^s$ is the active power flow at the sending point of branch $b_{n,m}$, which is a positive value. P_{Dm} is the active power demand of load D_m; A_{n+1} is the set of nodes receiving active power from node n and connected to this node with branch $b_{n,m}$ and D_n is the set of loads connected to node n. Since the contribution of all loads and active power sending branches connected to node n should add up to L_n^p the following term is assigned to $P_{n,k}^s$

$$L_n^p \frac{\left(P_{n,k}^s\right)^2 + P_{n,k}^s\left(\sum_{\substack{m\in A_{n+1}\\ m\neq k}} P_{n,m}^s + \sum_{D_m\in D_n} P_{Dm}\right)}{\left(\sum_{m\in A_{n+1}} P_{n,m}^s + \sum_{D_m\in D_n} P_{Dm}\right)^2} \quad \text{- } (6)$$

Based on (7), the procedure to calculate $L^p{}_k$'s is as follows

Step 1) Assign zero L_k^p to each of the active source nodes, the connection node of the transmission and distribution systems.

Step 2) Loop over all nodes whose L_k^p is not obtained. If the loss assigned to power to this node was previously calculated, then obtain the loss assigned to this node using (7)

$$L_k^p = \sum_{n\in A_{k-1}}\left[L_n^p \frac{\left(P_{n,k}^s\right)^2 + P_{n,k}^s\left(\sum_{\substack{m\in A_{n+1}\\ m\neq k}} P_{n,m}^s + \sum_{D_m\in D_n} P_{D_m}\right)}{\left(\sum_{m\in A_{n+1}} P_{n,m}^s + \sum_{D_m\in D_n} P_{D_m}\right)^2} + P_{Loss_{n,k}}^p\right] - (7)$$

Step3) If there is a node whose $L^p{}_k$ is not obtained yet, go back to Step 2); otherwise, stop the procedure.

L_k^p is not the actual loss allocated to node k, rather it is the loss assigned to this node for calculating the loss allocated to the loads (and not the DGs) connected to this node. This will become more clear, in the following step, it is used to calculate the loss allocated to the DGs connected to node k. Similar to (6), the loss allocated to load D_i connected to node k, which has the active power demand of P_{Di}, is calculated as

$$L_{Di}^p = L_k^p \frac{(P_{Di})^2 + P_{Di}\left(\sum_{\substack{D_n\in D_k\\ n\neq i}} P_{Dn} + \sum_{n\in A_{n+1}} P_{k,n}^s\right)}{\left(\sum_{D_n\in D_k} P_{Dn} + \sum_{n\in A_{k+1}} P_{k,n}^s\right)^2} \quad \_\_(8)$$

Where D_k is the set of loads connected to node k

4.1.2 Loss Allocated to Loads Due to Reactive Power Flows:

The procedure is same as the previous section. First, the reactive source nodes, since the nodes whose reactive generation is more than their reactive load, are assigned zero reactive losses. Then, L_k^q is calculated as shown in (9)

R_{k-1} Set of nodes that sends reactive power to node k and is connected to this node with branch $b_{n,k}$;

$Q^s{}_{n,k}$ Sending end reactive power flow of branch $b_{n,k}$, which is a positive value;

$Q^s{}_{n,m}$ Sending end reactive power flow of branch $b_{n,m}$, which is a positive value;

$P^q{}_{lossn,k}$ Power loss of branch $b_{n,k}$ due to reactive power flows;

R_{n+1} Set of nodes receiving reactive power from node n and connected to this node n and connected to this node with branch $b_{n,m}$;

D_n Set of loads connected to node n;

Q_{Dm} Reactive power demand of load D_m;

$L^q{}_n$ Loss allocated to node n due to reactive flows.

Similar to the previous section, the procedure should be applied to calculate the losses to all nodes due to reactive flows. As a result, the loss allocated to load D_i is calculated similar to (8) as

$$L_k^q = \sum_{n\in R_{k-1}}\left[L_n^q \frac{\left(Q_{n,k}^s\right)^2 + Q_{n,k}^s\left(\sum_{\substack{m\in R_{n+1}\\ m\neq k}} Q_{n,m}^s + \sum_{D_m\in D_n} Q_{D_m}\right)}{\left(\sum_{m\in R_{n+1}} Q_{n,m}^s + \sum_{D_m\in D_n} Q_{D_m}\right)^2} + P_{LOSS_{n,k}}^q\right] \text{- } (9)$$

$$L_{D_i}^q = L_k^q \frac{\left(Q_{D_i}\right)^2 + Q_{D_i}\left(\sum_{\substack{D_n\in D_k\\ n\neq i}} Q_{D_n} + \sum_{n\in R_{k+1}} Q_{k,n}^s\right)}{\left(\sum_{D_n\in D_k} Q_{D_n} + \sum_{n\in R_{k+1}} Q_{k,n}^s\right)^2} - (10)$$

4.1.3 Total Loss Allocated to Loads:

The total loss allocated to load D_i is obtained by adding (8) and (10) as

$$L_{D_i} = L_{D_i}^p + L_{D_i}^q \text{- } (11)$$

4.2 Calculating the Loss Allocated to the DGs:

4.2.1 Loss Allocated to the DGs Due to Active Power Flows:

The loss allocated to node k due to active flows is calculated as given in (12),

Where

A_{k+1} Set of nodes receiving active power from node k and are connected to this Node with branch $b_{k,n}$;

$P^r{}_{k,n}$ Receiving end active power flow of branch $b_{k,n}$, which is a positive Value;

$P^r{}_{m,n}$ Receiving end active power flow of branch $b_{m,n}$, which is a positive value;

$P^p{}_{lossk,n}$ Loss of branch $b_{k,n}$ due to active flows;

A_{n-1} Set of nodes sending active power to node n and connected to this node with branch $b_{m,n}$;

G_n Set of DG's connected to node n;

P_{Gm} Active power output of DG G_m;

$L'_n{}^p$ Loss allocated to node n due to active flows.

$L'_k{}^p$ is set to zero for all of the active sink nodes, since the nodes whose active load is more than their active generation. This is due to the active generated power of all DGs connected to these nodes is consumed locally by the loads connected to

$$L_k'^{\,p} = \sum_{n\in A_{k+1}}\left[L_n'^{\,p} \frac{\left(P_{k,n}^r\right)^2 + P_{k,n}^r\left(\sum_{\substack{m\in A_{n-1}\\ m\neq k}} P_{m,n}^r + \sum_{G_m\in G_n} P_{G_m}\right)}{\left(\sum_{m\in A_{n-1}} P_{m,n}^r + \sum_{G_m\in G_n} P_{G_m}\right)^2} + P_{LOSS_{k,n}}^p\right] - (12)$$

The procedure to calculate $L'_k{}^p$s is as follows.

Step 1) Assign zero L_k^p to each of the active sink nodes, and also to the node connecting the transmission and distribution systems.

Step 2) Loop over all the nodes whose $L'_k{}^p$ is not obtained.

If $L'_k{}^p$ of all nodes that receive active power from this node is previously calculated, then calculate $L'_k{}^p$ for this node using (14).

Step 3) If there is a node whose $L'_k{}^p$ is not obtained, go back to Step 2); otherwise, stop the procedure.

Assume DG G_j with power output of P_{G_j} is connected to node k .The loss allocated to this DG, due to active flows, it can be calculated by using $L'_k{}^p$ as

$$L^p_{G_j} = L'^{\,p}_k \frac{\left(P_{G_j}\right)^2 + P_{G_j}\left(\sum_{\substack{G_n\in G_k \\ n\neq j}} P_{G_n} + \sum_{n\in A_{k-1}} P^r_{n,k}\right)}{\left(\sum_{G_n\in G_k} P_{G_n} + \sum_{n\in A_{k-1}} P^r_{n,k}\right)^2} \quad - (13)$$

Where G_k represents the set of DGs connected to node k.

4.2.2 Loss Allocated to the DGs Due to Reactive Power Flows:

The loss allocated to node k due to reactive flows is calculated as (14), where

R_{k+1} Set of nodes receiving reactive power from node k and are connected to this node with branch $b_{k,n}$;

$Q^r{}_{k,n}$ Reactive power flow at the receiving point of branch $b_{k,n}$ which is a Positive value;

$Q^r{}_{m,n}$ Reactive power flow at the receiving point of branch $b_{m,n}$ which is a Positive value

$P^q{}_{loss\,k,n}$ Power loss of branch $b_{k,n}$ due to reactive flows;

R_{n-1} Set of nodes sending reactive power to node n and connected to this node with branch $b_{m,n}$;

Q_{Gm} Reactive power output of DG G_m ;

$L'_n{}^q$ Loss allocated to node n, due to reactive flows.

$$L'^{\,q}_k = \sum_{n\in R_{k+1}} \left[L'^{\,q}_n \frac{\left(Q^r_{k,n}\right)^2 + Q^r_{k,n}\left(\sum_{\substack{m\in R_{n-1} \\ m\neq k}} Q^r_{m,n} + \sum_{G_m\in G_n} Q_{G_m}\right)}{\left(\sum_{m\in R_{n-1}} Q^r_{m,n} + \sum_{G_m\in G_n} Q_{G_m}\right)^2} + P^q_{LOSS_{k,n}} \right]^q \quad - (14)$$

The loss allocated to DG G_j due to reactive flows might be obtained as

$$L^q_{G_j} = L'^{\,q}_k \frac{\left(Q_{G_j}\right)^2 + Q_{G_j}\left(\sum_{\substack{G_n\in G_k \\ n\neq j}} Q_{G_n} + \sum_{n\in R_{k-1}} Q^r_{n,k}\right)}{\left(\sum_{G_n\in G_k} Q_{G_n} + \sum_{n\in R_{k-1}} Q^r_{n,k}\right)^2} \quad (15)$$

4.2.3 Total Loss Allocated to DGs:

The total loss allocated to DG G_j is obtained by adding (14) and (15) as

$$L_{G_j} = L^p_{G_j} + L^q_{G_j} \quad - (16)$$

4.3 Final loss allocated based on normalization:

Normalization is executed, here total loss cost is obtained, since total loss cost is sum of the amount paid by loads and DGs. The normalization factor is obtained as

$$NF = \frac{P_{LOSS}}{\sum L_{D_i} + \sum L_{G_j}} \quad - (17)$$

Hence, the loss allocated to load D_i and the loss allocated to DG G_j is normalized as

$$L^{normalized}_{D_i} = L_{D_i} NF$$

$$L^{normalized}_{G_j} = L_{G_j} NF \quad - (18)$$

Equation (18) is the final formulation for calculating the loss allocated to load D_i and DG G_j.

5. TEST RESULTS

5.1 Test system Results for 69 bus sink & source nodes:

The power flow method is tested with a 12.66-KV radial distribution system that has 69 nodes, 73 branches and 6 DGs, whose single-line diagram is shown in figure. 2 and data can be found in [13]. The results of the power flow method not only depend on the user's location, but the method also considers the users demand or generation. The placement DGs also changes the voltages of the nodes and these voltages at each node are calculated. Loss allocation results for the 69-node system are shown in table I and table II. And voltage profiles are shown in figure- 3 and figure- 4.

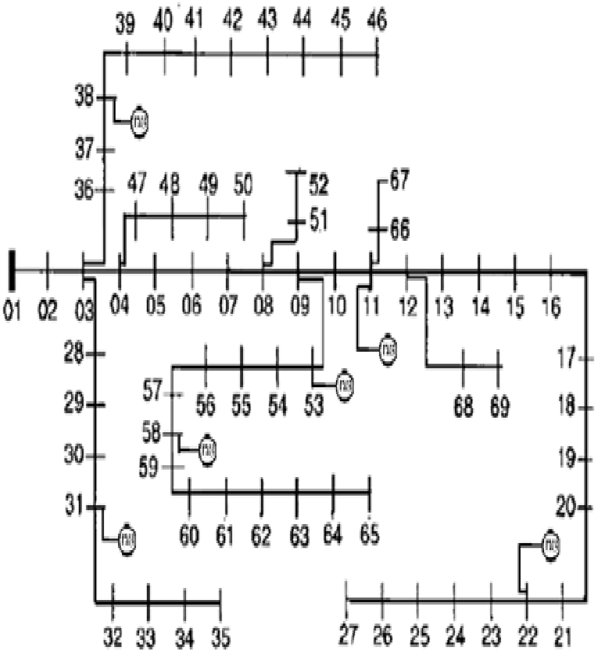

Figure-2. 12.66KV 69 node radial distribution network

The results of the power flow method not only depend on the user's location, but the method also considers the users demand or generation. The placement DGs also changes the voltages of the nodes and these voltages at each node are calculated. Loss allocation results for the 69-node system are shown in table I and table II. And voltage profiles are shown in figure- 3 and figure- 4.

TABLE-I

Loss Allocation results for the 69-node system: Source node

Generation Node number.	P (KW)	Q (KVAR)	LA in KW (Light loads)	LA in KW (Over loads)

1	500	220.0	0.7700	1.5200
22	100	44.00	0.0100	0.9500
31	200	88.00	0.0500	0.3600
38	1000	440.0	0.1000	0.5300
53	300	132.0	0.0500	0.2300
58	400	176.0	0.1300	0.4920

TABLE II

Loss Allocation for the 69-node system: Sink node

Load Node number	P (KW)	Q (KVAR)	LA in KW (Light loads)	LA in KW (Over loads)
6	20.600	12.20	0.05000	0.09
7	40.400	30.00	0.27000	0.35
8	75.000	54.00	0.97000	1.60
9	30.000	22.00	0.69000	1.35
10	28.000	19.00	1.20000	1.78
11	145.00	104.0	0.00000	0.23
12	145.00	104.0	0.33000	0.698
13	81.000	52.00	0.47000	0.98
14	93.000	72.00	1.21000	1.84
15	71.000	52.00	1.65000	2.36
16	5.5000	2.000	0.19000	0.25
17	21.000	15.00	1.63000	1.98
18	14.000	10.40	2.06000	2.85
20	1.0000	0.600	0.53000	1.0
22	56.000	31.50	0.00000	0.02
23	64.000	52.50	0.01000	0.06
24	28.000	20.00	0.01000 0	0.18
25	33.300	23.40	0.040	0.21
26	44.000	30.00	0.10000	0
27	44.000	30.00	0.14000	0
28	26.000	18.60	0.00000	0
29	26.000	18000	0.01000	0.02
30	20.300	12.70	0.02000	0.04
31	35.000	33.00	0.00000	0.0
32	65.000	52.00	0.02000	0.30
33	75.000	51.00	0.08000	0.14
34	31.000	24.50	0.06000	0.10
35	31.000	24.00	0.09000	0.14
36	26.00	18.55	0.010	0.02
37	26.00	18.55	0.000	0.1
38	35.60	22.40	0.000	0
39	105.0	87.00	0.010	0.04
40	93.00	72.00	0.010	0.05
41	139.2	96.30	0.190	0.19
42	71.00	66.00	0.180	0.18
43	25.00	13.30	0.080	0.15
44	54.00	43.70	0.350	0.38
45	39.00	26.00	0.330	0.65
46	1.200	1.000	0.010	0.09
47	51.00	43.50	0.030	0.1
48	79.00	56.40	0.110	0.25
49	284.7	174.5	0.930	1.25
50	284.7	174.5	1.370	1.65
51	40.50	28.30	0.570	1.2
52	26.60	12.70	0.410	0.8
53	87.35	63.50	0.000	0
54	96.40	79.00	0.030	0.09
55	24.00	17.20	0.010	0.01
56	125.0	85.90	0.160	0.19
57	100.0	72.00	0.130	0.25
58	12.00	7.500	0.000	0
59	29.50	20.00	0.010	0.32
60	51.97	43.20	0.060	0.45
61	44.00	28.00	0.100	0.18
62	32.00	23.00	0.120	3.5
63	13.60	9.700	0.090	0.84
64	27.00	12.00	0.300	1.23
65	59.00	42.0	1.160	2.044
66	18.00	13.0	0.000	0
67	18.00	13.0	0.000	0
68	28.00	20.0	0.070	0.25
69	28.00	20.0	0.100	0.56

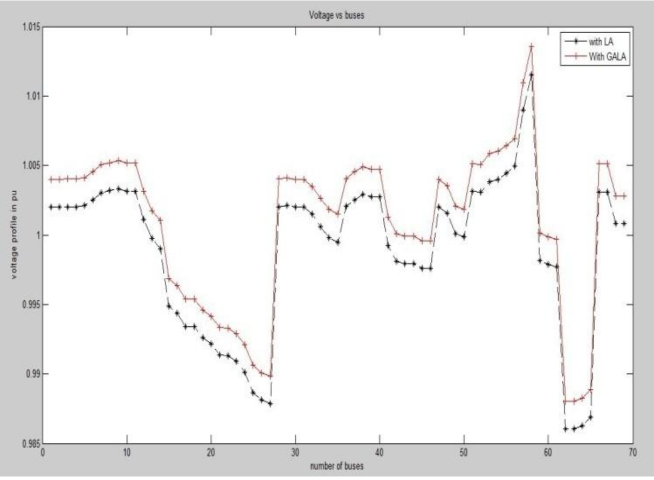

Figure-4 comparison of voltages of Genetic algorithm loss allocation method with loss allocation method for sink nodes.

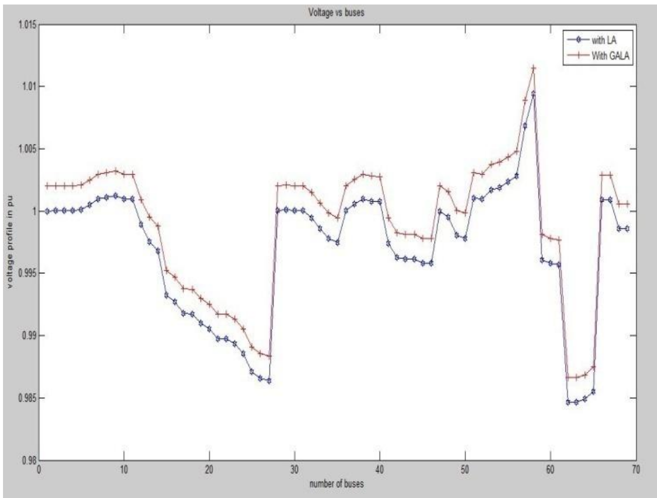

Figure-4 comparison of voltages of Genetic algorithm loss allocation method with loss allocation method for source nodes.

6. CONCLUSION

This paper presents a power flow method for radial distribution systems, in which the loss allocated to each node is dependent on adjacent nodes and the loss of the lines connected to the node. In order to allocate the loss at each node some properties are considered, which are explained in [9] to be desirable properties of every loss allocation method.

- The method is consistent with the results of power flow.
- The losses allocated to the loads or DGs depend on the amount of energy they consume or produce.
- The location of each DG and load is a key factor in the loss allocated to them.

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