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Fuzzy Inventory Optimization in a Two Stage Supply Chain with Full Information Sharing and two Backorder Costs

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Abstract

Nowadays a new spirit of cooperation requires between the buyer and the vendor in supply chain environment. We consider the case of two stage supply chain system which involves a single vendor who supplies a single product to a single buyer. In order to coordinate the replenishment policies and optimizing the operational costs, the two supply chain partners fully share their information. For this purpose we develop the proposed model by assuming linear and fixed back order costs. In addition set up cost and holding cost are taken as fuzzy triangular numbers. Signed distance method is used for defuzzification. Numerical example illustrates the procedure of the proposed model.

Keywords:*Two stage supply chain, two backorder costs, fuzzy inventory model, Triangular fuzzy numbers, defuzzification.*

1. Introduction:

The significant advances in information and communication technologies facilitated the provision and sharing of the business information necessary for efficiency improvement. Goyal and Szendrovits [1] presented a constant lot size model where the lot is produced through a fixed sequence of manufacturing stages, with a single set up and without interruption. A one vendor multi-buyer integrated inventory model was developed in [2] with the objective of minimizing the vendor's total annual cost subject to the maximum cost that the buyer may be prepared to incur.

The investigation by [3] on effect in supply chains reported that lack of information sharing can lead excessive inventory, poor customer service, lotrevenues,and unplanned capacities. The paper developed by Seliaman, a two stagesupply chain inventory model under two type of backorders cost. This model is the extension of [4], which considered a single vendor and a single buyer.

This proposed model we develop an inventory model using triangular fuzzy number for holding cost, ordering cost and setup cost. Signed distance method is used for defuzzification. Due to irregularities or physical properties of the material all the time we cannot take parameters as variables. For this situation we apply fuzzy concepts.We introduce two backorders cost and minimizing the total cost.

Theremainder of this paper is organized as,

Methodology

Notations and Assumptions,

Model development

Fuzzy sense

Numerical example illustrates the proposed model.

2. Methodology

2.1. Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions, is a generalized fuzzy number $\tilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0, 1],

(ii) $\mu_A = 0$, $-\infty < x \le a_1$

(iii) $\mu_A = L(x)$ is strictly increasing on $[a_1, a_2]$

(iv) $\mu_A = w_A$, $a_2 \le x \le a_3$

(v) $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$

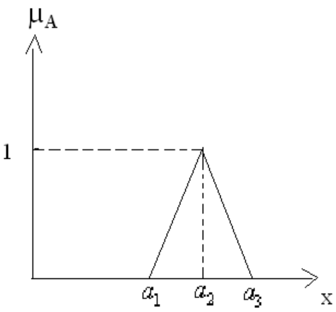
(vi) $\mu_A = 0$, $a_4 \le x \le \infty$

Where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$; when $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$.

2.2. Triangular fuzzy number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_A = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$



2.3. The Function Principle

The function principle was introduced by Chen [6] to treat fuzzy arithmetical operations. This principle is used for the operation for addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$

Where

$$T = (a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3), c_1 = \min T, c_2 = a_2 b_2, c_3 = \max T$$

if a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$.

(iii) $-\tilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(iv) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = (\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$ where b_1, b_2, b_3 are all non zero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is $\frac{\tilde{A}}{\tilde{B}} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$

(v) For any real number K,

$$K\tilde{A} = (Ka_1, Ka_2, Ka_3) if K > 0$$
$$K\tilde{A} = (Ka_3, Ka_2, Ka_1) if K < 0$$

2.4. Signed Distance Method

Defuzzification of $\tilde{A}$ can be found by signed distance method. If $\tilde{A}$ is a triangular fuzzy number and is fully determined by (a_1, a_2, a_3), the signed distance from $\tilde{A}$ to 0 is defined as

$$d(\tilde{A}, \tilde{0}) = \int_0^1 d([A_L(\alpha), A_R(\alpha)], \tilde{0}) d\alpha = \frac{(a_1 + 4a_2 + a_3)}{4}$$

3. Notations

T-basic cycle time, cycle time at the end buyer

P_v-Production rate of the vendor

K_v-integer multiplier at vendor

A_v-Setup cost of the vendor

A_b-Ordering cost of the buyer

D-Demand rate for the buyer

h_v-inventory holding cost per unit time for the vendor

h_b-inventory holding cost per unit time for the buyer

T_s-the stock- out time at the buyer

$\tilde{A}_v = (A_{v_1}, A_{v2}, A_{v_3})$ -fuzzy setup cost of the vendor

$\tilde{A}_b = (A_{b_1}, A_{b2}, A_{b_3})$ -Fuzzy ordering cost of the buyer

$\tilde{h}_v = (h_{v_1}, h_{v2}, h_{v_3})$ -Fuzzy inventory holding cost for the vendor

$\tilde{h}_b = (h_{b_1}, h_{b2}, h_{b_3})$ -Fuzzy inventory holding cost for the buyer

x-backordering cost per unit per unit time (linear) for the buyer

y-backorder cost per unit (fixed) for the buyer

$\tilde{x}$ -fuzzy backordering cost per unit per unit time (linear) for the buyer

$\tilde{y}$ -fuzzy backorder cost per unit (fixed) for the buyer

TC(T)– Total cost for the period [0, T]

$T\tilde{C}(T)$ - Fuzzy total cost for the period [0, T]

4. Assumptions

(a) The model deals with a single vendor and a single buyer for a single product.

(b) Replenishment is instantaneous

(c) Production rate and demand rate are deterministic and uniform

(d) Complete information sharing policy is adopted

(e) There are two typesof backorders costs

(f) Holding cost, ordering cost and setup cost are fuzzy in nature

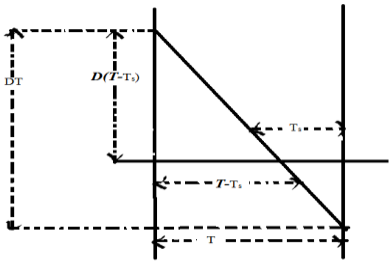


Figure 1: the buyer's inventory level behavior

5. Model Development

We consider a two stage supply chain consisting of a single vendor and a single buyer. This supply chain model is formulated for the integer multiplier inventory replenishment coordination method, where the cycle time at the vendor is an integer multiple of the inventory replenishment cycle time used by the buyer.

The total annual cost for the buyer is given by

$$TC_b = h_b \frac{TD}{2} + T_s D\left(\frac{y}{T} - h_b\right) + \frac{T_s^{\,2} D}{2T}(x + h_v) + \frac{A_b}{T} \tag{1}$$

The total cost for the vendor is,

$$TC_v = K_v \frac{TD^2}{2P_v}(h_b + h_v) + (K_v - 1)\frac{TD}{2} h_v + \frac{A_v}{K_v T} \tag{2}$$

By adding the total cost for the buyer and total cost for the vendor we get the total cost for the entire supply chain. i.e.,

TC = TC_b+ TC_v

$$TC(T) = \frac{T}{2}\left\{h_b D - h_v D + K_v\left[\frac{D^2}{P_v}(h_b + h_v) + Dh_v\right]\right\} + \frac{1}{T}\left\{A_b + \frac{A_v}{K_v}\right\} + T_s D\left(\frac{y}{T} - h_b\right) + \frac{T_s^{\,2} D}{2T}(x + h_v) \tag{3}$$

$$\frac{\partial^2 TC(T)}{\partial T^2} > 0 \quad \text{(See appendix A)}$$

Second order derivative is positive. So total cost function is convex in T. Equating the first order derivative to 0 we get the value of T^*,

$$T^* = \sqrt{\frac{2A_b + 2\frac{A_v}{K_v} + 2T_s D y + T_s^{\,2} D(x + h_v)}{h_b D - h_v D + K_v\left[\frac{D^2}{P_v}(h_b + h_v) + Dh_v\right]}} \tag{4}$$

Substituting the above T^* in (3) we get the total cost by the equation,

$$TC(T) = \frac{T^*}{2}\left\{h_b D - h_v D + K_v\left[\frac{D^2}{P_v}(h_b + h_v) + Dh_v\right]\right\} + \frac{1}{T^*}\left\{A_b + \frac{A_v}{K_v}\right\} + T_s D\left(\frac{y}{T^*} - h_b\right) + \frac{T_s^{\,2} D}{2T^*}(x + h_v) \tag{5}$$

6. Inventory model in Fuzzy sense

We consider the model in fuzzy environment. Since the ordering cost and holding cost are fuzzy in nature, we represent them by triangular fuzzy numbers. The fuzzy total cost is given by,

$$T\tilde{C}(T) = \frac{T}{2}\left\{\begin{array}{l}\tilde{h}_b D - \tilde{h}_v D + \\ K_v\left[\frac{D^2}{P_v}\left(\tilde{h}_b + \tilde{h}_v\right) + D\tilde{h}_v\right]\end{array}\right\} + \frac{1}{T}\left\{\tilde{A}_b + \frac{\tilde{A}_v}{K_v}\right\} + T_s D\left(\frac{y}{T} - \tilde{h}_b\right) + \frac{T_s^{\,2} D}{2T}\left(x + \tilde{h}_v\right)$$

(6)

$$\text{where } \tilde{T}^* = \sqrt{\frac{2\tilde{A}_b + 2\frac{\tilde{A}_v}{K_v} + 2T_s D y + T_s^{\,2} D\left(x + \tilde{h}_v\right)}{\tilde{h}_b D - \tilde{h}_v D + K_v\left[\frac{D^2}{P_v}\left(\tilde{h}_b + \tilde{h}_v\right) + D\tilde{h}_v\right]}} \quad 0$$

rdering cost, set up cost and holding cost for both buyer and vendor be triangular fuzzy numbers,

$$\tilde{A}_v = (A_{v_1}, A_{v2}, A_{v_3}),\ \tilde{A}_b = (A_{b_1}, A_{b2}, A_{b_3}),$$
$$\tilde{h}_v = (h_{v_1}, h_{v2}, h_{v_3}),\ \tilde{h}_b = (h_{b_1}, h_{b2}, h_{b_3})$$

Substituting these triangular fuzzy numbers to the total cost equation we get,

$$\tilde{T}C(T) = \frac{T}{2}\left\{\begin{array}{l}(h_{1b}, h_{2b}, h_{3b})D - (h_{1v}, h_{2v}, h_{3v})D + \\ K_v\left(\frac{D^2}{P_v}[(h_{1b}, h_{2b}, h_{3b}) + (h_{1v}, h_{2v}, h_{3v})]\right) + \\ D(h_{1v}, h_{2v}, h_{3v})\end{array}\right\} + \frac{1}{T}\left\{(A_{1b}, A_{2b}, A_{3b}) + \frac{(A_{1v}, A_{2v}, A_{3v})}{K_v}\right\} + T_s D\left[\frac{y}{T} - (h_{1b}, h_{2b}, h_{3b})\right] + \frac{T_s^{\,2} D}{2T}[x + (h_{1v}, h_{2v}, h_{3v})]$$

(7)

By using arithmetic function principles in (7) and simplifying we get,

$$\tilde{T}C(T) = \left[\begin{array}{l} h_{1b}\frac{TD}{2} - h_{1v}\frac{TD}{2} + h_{1b}\frac{TK_v D^2}{2P_v} + \\ h_{1v}\frac{TK_v D^2}{2P_v} + \frac{TK_v D}{2}h_{1v} + \frac{A_{1b}}{T} + \\ \frac{A_{1v}}{TK_v} + \frac{T_s\ Dy}{T} - T_s\ Dh_{1b} + \frac{T_s^{\,2} Dx}{2T} + \frac{T_s^{\,2} D}{2T}h_{1v} \end{array}\right], \left[\begin{array}{l} h_{2b}\frac{TD}{2} - h_{2v}\frac{TD}{2} + h_{2b}\frac{TK_v D^2}{2P_v} + \\ h_{2v}\frac{TK_v D^2}{2P_v} + \frac{TK_v D}{2}h_{2v} + \frac{A_{2b}}{T} + \frac{A_{2v}}{TK_v} + \\ \frac{T_s\ Dy}{T} - T_s\ Dh_{2b} + \frac{T_s^{\,2} Dx}{2T} + \frac{T_s^{\,2} D}{2T}h_{2v} \end{array}\right], \left[\begin{array}{l} h_{3b}\frac{TD}{2} - h_{3v}\frac{TD}{2} + h_{3b}\frac{TK_v D^2}{2P_v} + \\ h_{3v}\frac{TK_v D^2}{2P_v} + \frac{TK_v D}{2}h_{3v} + \frac{A_{3b}}{T} + \frac{A_{3v}}{TK_v} + \\ \frac{T_s\ Dy}{T} - T_s\ Dh_{3b} + \frac{T_s^{\,2} Dx}{2T} + \frac{T_s^{\,2} D}{2T}h_{3v} \end{array}\right]$$

=[a, b, c,] (say)

(8)

Finding left and right α-cuts for the fuzzy total cost equation we get,

Left α-cut $A_L(\alpha) = a + (b - a)\,\alpha$

$$
= \left[\begin{array}{l}
h_{1b}\frac{TD}{2} - h_{1v}\frac{TD}{2} + h_{1b}\frac{TK_vD^2}{2P_v} \\
+ h_{1v}\frac{TK_vD^2}{2P_v} + \frac{TK_vD}{2}h_{1v} + \frac{A_{1b}}{T} + \frac{A_{1v}}{TK_v} + \frac{T_s\ Dy}{T} - T_s\ Dh_{1b} \\
+ \frac{T_s^{\,2}Dx}{2T} + \frac{T_s^{\,2}D}{2T}h + \\
h_{2b}\frac{TD}{2}\alpha - h_{2v}\frac{TD}{2}\alpha + h_{2b}\frac{TK_vD^2}{2P_v}\alpha \\
+ h_{2v}\frac{TK_vD^2}{2P_v}\alpha + \frac{TK_vD}{2}h_{2v}\alpha + \frac{A_{2b}}{T}\alpha + \frac{A_{2v}}{TK_v}\alpha \\
+ \frac{T_s\ Dy}{T}\alpha - T_s\ Dh_{2b}\alpha + \\
\frac{T_s^{\,2}Dx}{2T}\alpha + \frac{T_s^{\,2}D}{2T}h_{2v}\alpha - h_{1b}\frac{TD}{2}\alpha + \\
h_{1v}\frac{TD}{2}\alpha - h_{1b}\frac{TK_vD^2}{2P_v}\alpha - h_{1v}\frac{TK_vD^2}{2P_v}\alpha - \frac{TK_vD}{2}h_{1v}\alpha - \\
\frac{A_{1b}}{T}\alpha - \frac{A_{1v}}{TK_v}\alpha - \frac{T_s\ Dy}{T}\alpha + \\
T_s\ Dh_{1b}\alpha - \frac{T_s^{\,2}Dx}{2T}\alpha - \frac{T_s^{\,2}D}{2T}h_{1v}\alpha
\end{array} \right]
$$

(9)

Right α-cut $A_R(\alpha) = c - (c - b)\alpha$

$$
= \left[\begin{array}{l}
h_{3b}\frac{TD}{2} - h_{3v}\frac{TD}{2} + h_{3b}\frac{TK_vD^2}{2P_v} + h_{3v}\frac{TK_vD^2}{2P_v} + \\
\frac{TK_vD}{2}h_{3v} + \frac{A_{3b}}{T} + \frac{A_{3v}}{TK_v} + \frac{T_s\ Dy}{T} - T_s\ Dh_{3b} + \\
\frac{T_s^{\,2}Dx}{2T} + \frac{T_s^{\,2}D}{2T}h_{3v} - \\
h_{3b}\frac{TD}{2}\alpha + h_{3v}\frac{TD}{2}\alpha - h_{3b}\frac{TK_vD^2}{2P_v}\alpha - \\
h_{3v}\frac{TK_vD^2}{2P_v}\alpha - \frac{TK_vD}{2}h_{3v}\alpha - \frac{A_{3b}}{T}\alpha - \frac{A_{3v}}{TK_v}\alpha - \\
\frac{T_s\ Dy}{T}\alpha + \\
T_s\ Dh_{3b}\alpha - \frac{T_s^{\,2}Dx}{2T}\alpha - \frac{T_s^{\,2}D}{2T}h_{3v}\alpha + \\
h_{2b}\frac{TD}{2}\alpha + h_{2v}\frac{TD}{2}\alpha - h_{2b}\frac{TK_vD^2}{2P_v}\alpha - \\
h_{2v}\frac{TK_vD^2}{2P_v}\alpha - \frac{TK_vD}{2}h_{2v}\alpha - \\
\frac{A_{2b}}{T}\alpha - \frac{A_{2v}}{TK_v}\alpha - \frac{T_s\ Dy}{T}\alpha + T_s\ Dh_{2b}\alpha - \\
\frac{T_s^{\,2}Dx}{2T}\alpha - \frac{T_s^{\,2}D}{2T}h_{2v}\alpha
\end{array} \right]
$$

(10)

Defuzzifying the fuzzy total cost $T\tilde{C}(T)$ by using signed distance method we get,

$$
d(T\tilde{C}(\tilde{A}, \tilde{H}), 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]d\alpha
$$

$$
=\int_0^1\left\{\begin{array}{l}
\left[\begin{array}{l}
h_{1b}\frac{TD}{2}-h_{1v}\frac{TD}{2}+h_{1b}\frac{TK_vD^2}{2P_v}+h_{1v}\frac{TK_vD^2}{2P_v}+\\
\frac{TK_vD}{2}h_{1v}+\frac{A_{1b}}{T}+\frac{A_{1v}}{TK_v}+\frac{T_s\ Dy}{T}-T_s\ Dh_{1b}+\\
\frac{T_s{}^2Dx}{2T}+\frac{T_s{}^2D}{2T}h+\\
h_{2b}\frac{TD}{2}\alpha-h_{2v}\frac{TD}{2}\alpha+h_{2b}\frac{TK_vD^2}{2P_v}\alpha+h_{2v}\frac{TK_vD^2}{2P_v}\alpha+\\
\frac{TK_vD}{2}h_{2v}\alpha+\frac{A_{2b}}{T}\alpha+\frac{A_{2v}}{TK_v}\alpha+\frac{T_s\ Dy}{T}\alpha-T_s\ Dh_{2b}\alpha+\\
\frac{T_s{}^2Dx}{2T}\alpha+\frac{T_s{}^2D}{2T}h_{2v}\alpha-h_{1b}\frac{TD}{2}\alpha+\\
h_{1v}\frac{TD}{2}\alpha-h_{1b}\frac{TK_vD^2}{2P_v}\alpha-h_{1v}\frac{TK_vD^2}{2P_v}\alpha-\frac{TK_vD}{2}h_{1v}\alpha-\\
\frac{A_{1b}}{T}\alpha-\frac{A_{1v}}{TK_v}\alpha-\frac{T_s\ Dy}{T}\alpha+\\
T_s\ Dh_{1b}\alpha-\frac{T_s{}^2Dx}{2T}\alpha-\frac{T_s{}^2D}{2T}h_{1v}\alpha
\end{array}\right]+\\
\left[\begin{array}{l}
h_{3b}\frac{TD}{2}-h_{3v}\frac{TD}{2}+h_{3b}\frac{TK_vD^2}{2P_v}+\\
h_{3v}\frac{TK_vD^2}{2P_v}+\frac{TK_vD}{2}h_{3v}+\frac{A_{3b}}{T}\\
+\frac{A_{3v}}{TK_v}+\frac{T_s\ Dy}{T}-T_s\ Dh_{3b}+\frac{T_s{}^2Dx}{2T}+\frac{T_s{}^2D}{2T}h_{3v}-\\
h_{3b}\frac{TD}{2}\alpha+h_{3v}\frac{TD}{2}\alpha-h_{3b}\frac{TK_vD^2}{2P_v}\alpha-\\
h_{3v}\frac{TK_vD^2}{2P_v}\alpha-\frac{TK_vD}{2}h_{3v}\alpha-\\
\frac{A_{3b}}{T}\alpha-\frac{A_{3v}}{TK_v}\alpha-\frac{T_s\ Dy}{T}\alpha+\\
T_s\ Dh_{3b}\alpha-\frac{T_s{}^2Dx}{2T}\alpha-\frac{T_s{}^2D}{2T}h_{3v}\alpha+\\
h_{2b}\frac{TD}{2}\alpha+h_{2v}\frac{TD}{2}\alpha-h_{2b}\frac{TK_vD^2}{2P_v}\alpha-\\
h_{2v}\frac{TK_vD^2}{2P_v}\alpha-\frac{TK_vD}{2}h_{2v}\alpha-\\
\frac{A_{2b}}{T}\alpha-\frac{A_{2v}}{TK_v}\alpha-\frac{T_s\ Dy}{T}\alpha+\\
T_s\ Dh_{2b}\alpha-\frac{T_s{}^2Dx}{2T}\alpha-\frac{T_s{}^2D}{2T}h_{2v}\alpha
\end{array}\right]
\end{array}\right\}
$$

Integrating with respect to α and simplifying we get defuzzified values for total cost. i.e.,

$$
\begin{aligned}
TC(T)=&\frac{TD}{4}\left[h_{1b}-h_{1v}+K_vh_{1v}+2h_{2b}+h_{3b}-h_{3v}+K_vh_{3v}\right]\\
&+\frac{TD^2}{4}\frac{K_v}{P_v}\left[h_{1b}+h_{1v}+h_{3b}+h_{3v}\right]+\\
&\frac{1}{2T}\left[A_{1b}+\frac{A_{1v}}{K_v}+A_{3b}+\frac{A_{3v}}{K_v}\right]+\\
&\frac{T_s{}^2D}{2T}\left[2y\right]+\frac{T_sD}{2T}\left[-h_{1b}-h_{3b}\right]+\\
&\frac{T_sD}{4T}\left[x+h_{1v}\right]
\end{aligned}
\qquad (11)
$$

Finding first and second order differentiation we get

$$\frac{\partial^2 TC(T)}{\partial T^2}>0$$ which is postive (See appendix B)

Total cost TC(T) with respect to T is convex.

By equating first order derivative equals to 0, we get the optimum time T we get

$$
T^*=\sqrt{\frac{\frac{1}{2}\left[A_{1b}+\frac{A_{1v}}{K_v}+A_{3b}+\frac{A_{3v}}{K_v}\right]-\frac{T_s{}^2D}{4}\left[2y\right]-\frac{T_sD}{2}\left[-h_{1b}-h_{3b}\right]-\frac{T_sD}{8}\left[x+h_{1v}\right]}{\frac{D}{4}\left[\begin{array}{l}h_{1b}-h_{1v}+K_vh_{1v}+\\2h_{2b}+h_{3b}-h_{3v}+K_vh_{3v}\end{array}\right]+\frac{D^2}{4}\frac{K_v}{P_v}\left[h_{1b}+h_{1v}+h_{3b}+h_{3v}\right]}}
\qquad (12)
$$

Substituting T* in total cost equation (11) we get

$$TC(T)=\frac{T^*D}{4}\begin{bmatrix}h_{1b}-h_{1v}+K_vh_{1v}+\\2h_{2b}+h_{3b}-h_{3v}+K_vh_{3v}\end{bmatrix}+$$
$$\frac{T^*D^2}{4}\frac{K_v}{P_v}\left[h_{1b}+h_{1v}+h_{3b}+h_{3v}\right]+$$
$$\frac{1}{2T^*}\left[A_{1b}+\frac{A_{1v}}{K_v}+A_{3b}+\frac{A_{3v}}{K_v}\right]+$$
$$\frac{T_s{}^2D}{2T^*}[2y]+\frac{T_sD}{2T^*}\left[-h_{1b}-h_{3b}\right]+$$
$$\frac{T_sD}{4T^*}\left[x+h_{1v}\right] \tag{13}$$

7. Algorithm

Step 1. Input D, P_v, h_b, h_v, A_b, A_v, K_v, T_s, x, y in Equation 4.

Step 2. Find T*.

Step 3. Substitute T* in equation 5 we get Total Cost for crisp values.

Step 4. Input fuzzy values for ordering cost, set up cost and holding cost in equation (12)

Step 5. Find T*.

Step 6. Substitute T* in equation (13) we get total cost for fuzzy values.

Step 7. Find difference between cycle time and total cost found in steps 6 and step 3.

Step 8. Repeat these steps by changing different demand rate to give sensitivity analysis for the proposed model

8. Conclusion

Two stage supply chain with vendor, buyer coordination and cooperation is considered in this model. Information sharing between vendor and buyer is necessary for the smooth running of a trade and is suitable for the real life situations. Here holding cost for both buyer and vendor, set up cost for the vendor and ordering cost for the buyer are taken as triangular fuzzy numbers. By taking this consideration we minimize the total cost of the supply chain much better than in our base paper. This is illustrated by the numerical example.

9. Numerical examples: Numerical examples are given to mention the workability of the proposed model. Computational works are carried out using **MATLAB 7.0.**

For given P_v=1500; h_b = 1.5; h_v = 0.2; A_b=29; A_v = 300; K_v = 3; T_s = 0.113; x = 0.2; y = 20; Then T* and Total cost are shown in Table 1.

For given P_v=1500; h_b = (1, 1.5, 2); h_v = (0.1, 0.2, 0.3); A_b=(28, 29, 30); A_v = (250, 300, 350); K_v = 3; T_s = 0.113; x = 0.2; y = 20; $\tilde{x}=(0.1,0.2,0.3)$; $\tilde{y}=(15,20,25)$ Then defuzzified T* and Total cost are shown in Table 2.

From table 1 we see that as demand increases buyer's cycle time decreases but the total cost increases. Since the cycle time decreases, buyer place more order frequently and hence buyer get more profit. If buyer gets more profit obviously vendor also vendor also gets more profit.

Even though the total increases with demand the buyer can improve his business by placing more orders to the vendor frequently. Consequently the vendor also can develop his trade and hence the supply chain will work strongly. This situation is the same for various holding cost of the buyer.

More over in fuzzy environment the total cost is decreased verily for the same changes in demand and also for holding cost of the buyer. So we can observe that fuzzy environment is natural and beneficial for both vendor and the buyer.

Nutshell of the paper:

a. When demand increases gradually, then the cycle time for the process decreases.
b. When demand increases and the cycle time decreases then the total cost increases.
c. When demand increases difference between the total cost for crisp and fuzzy inputs also increases.
d. Increase of holding cost for buyer leads to increase of total cost.
e. Decrease of holding cost for buyer leads to decrease of total cost.

Table 1: Crisp model with different demand rate

Demand	T^*	TC
400	1.26	1574.8
450	1.22	1806.0
500	1.18	2045.7
550	1.15	2293.7
600	1.12	2549.7
650	1.09	2813.7
700	1.07	3085.2
750	1.05	3364.2
800	1.02	3650.5
850	1.01	3943.9

Table 2: Fuzzy model with different demand rate(substituting triangular fuzzy numbers of ordering cost, holding cost, setup cost, backorder costs (fixed & linear)

Demand	T^*	TC
400	0.389	798.9
450	0.363	874.9
500	0.340	951.6
550	0.321	1028.7
600	0.365	1104.2
650	0.277	1192.2
700	0.277	1264.5
750	0.265	1345.1
800	0.255	1426.1
850	0.245	1508.8

Table 2a: Fuzzy model with different demand rate (substituting defuzzified values of ordering cost, holding cost, setup cost, backorder costs (fixed & linear)

Demand	T^*	TC
400	1.26	1943.73
450	1.22	2202.87
500	1.18	2463.2
550	1.15	2738.04
600	1.12	3015.2
650	1.09	3293.6
700	1.07	3593.7
750	1.05	3897.6
800	1.02	4177.9
850	1.01	4514.2

Table 3: Difference between crisp and fuzzy model

Demand	T^*	TC
400	0.871	775.9
450	0.857	931.1
500	0.84	1094.1
550	0.83	1265.0
600	0.76	1445.5
650	0.81	1621.5
700	0.79	1820.7
750	0.79	2019.1
800	0.77	2224.4

850	0.77	2435.1

Table 4: Different holding cost for buyer crisp

Demand	Setup cost	Oedering cost	Holding cost buyer	Holding cost vendor	T^*	TC
400	29	300	0.8	0.2	1.61	1122.3
450	29	300	1.0	0.2	1.43	1421.4
500	29	300	1.2	0.2	1.29	1748.3
550	29	300	1.4	0.2	1.18	2102.8
600	29	300	1.5	0.2	1.12	2415.3
650	29	300	1.6	0.2	1.06	2749.1
700	29	300	1.8	0.2	0.99	3181.6
750	29	300	1.9	0.2	0.95	3559.9
800	29	300	2.0	0.2	0.91	3958.9
850	29	300	2.2	0.2	0.85	4470.9

Table 5: different holding cost buyer fuzzy

Demand	Setup cost	Ordering cost	Holding cost buyer	Holding cost vendor	T^*	TC
400	28,29,30	250,300,350	0.6,1.2,1.6	0.1,0.2,0.3	0.41	757.47
450	28,29,30	250,300,350	0.8,1.4,1.8	0.1,0.2,0.3	0.37	853.51
500	28,29,30	250,300,350	0.8,1.3,1.9	0.1,0.2,0.3	0.35	921.87
550	28,29,30	250,300,350	0.9,1.4,1.9	0.1,0.2,0.3	0.32	1015.6
600	28,29,30	250,300,350	1,1.5,2	0.1,0.2,0.3	0.31	1121.8
650	28,29,30	250,300,350	1.1,1.6,2	0.1,0.2,0.3	0.29	1231.4
700	28,29,30	250,300,350	1.2,1.6,2	0.1,0.2,0.3	0.28	1334.7

750	28,29,30	250,300,350	1.3,1.6,1.9	0.1,0.2,0.3	0.26	1435.2
800	28,29,30	250,300,350	1.3,1.9,2.2	0.1,0.2,0.3	0.25	1619.5
850	28,29,30	250,300,350	1.5,2,2.5	0.1,0.2,0.3	0.25	1811.5

Table 6: Difference between crisp and fuzzy model (varying holding cost for buyer)

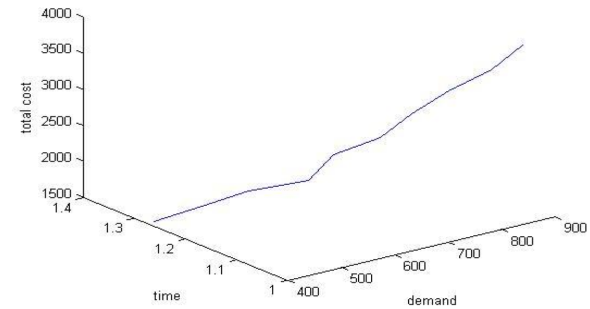
Demand	T^*	TC
400	1.2	364.8
450	1.06	567.8
500	0.94	826.4
550	0.86	1087.2
600	0.81	1293.5
650	0.77	1517.7
700	0.71	1846.9
750	0.69	2124.7
800	0.66	2339.4
850	0.6	2659.4

Figures:

Figure 1: crisp inputs

Figure 2: Input defuzzified values

Figure 3: Input fuzzy values and after defuzzification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time
400
500
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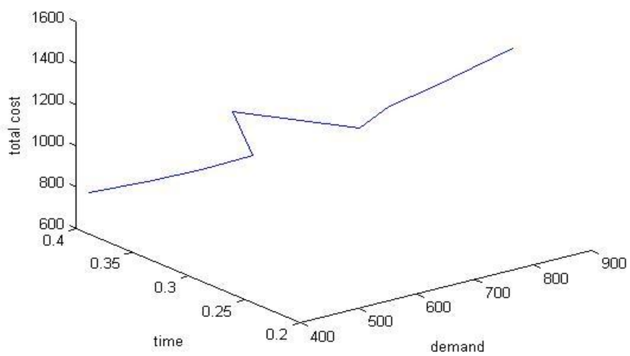
demand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time
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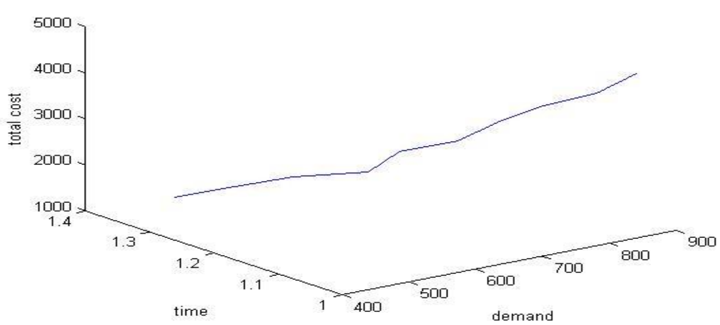
800
900
demand


5000
4000
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2000
1000
total cost
1.4
1.3
1.2
1.1
1
time
400
500
600
700
800
900
demand

Appendix A

$$TC(T)=\frac{T}{2}\left\{h_bD-h_vD+K_v\left[\frac{D^2}{P_v}(h_b+h_v)+Dh_v\right]\right\}+\frac{1}{T}\left\{A_b+\frac{A_v}{K_v}\right\}+T_sD\left(\frac{y}{T}-h_b\right)+\frac{T_s{}^2D}{2T}(x+h_v)$$

$$\frac{\partial TC(T)}{\partial T}=\frac{1}{2}\left\{h_bD-h_vD+K_v\left[\frac{D^2}{P_v}(h_b+h_v)+Dh_v\right]\right\}-\frac{1}{T^2}\left\{A_b+\frac{A_v}{K_v}\right\}-T_sD\left(\frac{y}{T^2}\right)-\frac{T_s{}^2D}{2T^2}(x+h_v)$$

$$\frac{\partial^2 TC(T)}{\partial T^2}=\frac{2}{T^3}\left\{A_b+\frac{A_v}{K_v}\right\}+T_sD\left(\frac{2y}{T^3}\right)+\frac{2T_s{}^2D}{2T^3}(x+h_v)$$

$$\frac{\partial^2 TC(T)}{\partial T^2}>0$$

Appendix B

$$TC(T)=\frac{TD}{4}\left[h_{1b}-h_{1v}+K_vh_{1v}+2h_{2b}+h_{3b}-h_{3v}+K_vh_{3v}\right]+\frac{TD^2}{4}\frac{K_v}{P_v}\left[h_{1b}+h_{1v}+h_{3b}+h_{3v}\right]+$$
$$\frac{1}{2T}\left[A_{1b}+\frac{A_{1v}}{K_v}+A_{3b}+\frac{A_{3v}}{K_v}\right]+\frac{T_s{}^2D}{2T}[2y]+\frac{T_sD}{2T}\left[-h_{1b}-h_{3b}\right]+\frac{T_sD}{4T}\left[x+h_{1v}\right]$$

$$\frac{\partial TC(T)}{\partial T}=\frac{D}{4}\left[h_{1b}-h_{1v}+K_vh_{1v}+2h_{2b}+h_{3b}-h_{3v}+K_vh_{3v}\right]+\frac{D^2}{4}\frac{K_v}{P_v}\left[h_{1b}+h_{1v}+h_{3b}+h_{3v}\right]-$$
$$\frac{1}{2T^2}\left[A_{1b}+\frac{A_{1v}}{K_v}+A_{3b}+\frac{A_{3v}}{K_v}\right]-\frac{T_s{}^2D}{2T^2}[2y]-\frac{T_sD}{2T^2}\left[-h_{1b}-h_{3b}\right]-\frac{T_sD}{4T^2}\left[x+h_{1v}\right]$$

$$\frac{\partial^2 TC(T)}{\partial T^2}=\frac{1}{T^3}\left[A_{1b}+\frac{A_{1v}}{K_v}+A_{3b}+\frac{A_{3v}}{K_v}\right]+\frac{T_s{}^2D}{T^3}[2y]+\frac{T_sD}{T^3}\left[-h_{1b}-h_{3b}\right]+\frac{T_sD}{2T^3}\left[x+h_{1v}\right]$$

$$\frac{\partial^2 TC(T)}{\partial T^2}>0$$

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