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Mixing, Weakly Mixing and Transitive Sets

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Abstract - ***Topological δ-type transitive function has been introduced by Mohammed Nokhas Murad [1]. The aim of this paper is to investigate some properties of δ-type transitive sets in a given topological space (X, f). In the present paper, we studied some new class of topological transitive set called topological δ-type transitive set and investigate some of its topological dynamical properties.***

Key Words: *Topological δ-type transitive function,* topological dynamical properties.

CHAPTER FOUR

4. TOPOLOGICAL δ-TYPE TRANSITIVE FUNCTION

4.1 INTRODUCTION

Topological δ-type transitive function has been introduced by Dr. Mohammed Nokhas Murad **[1].** The aim of this paper is to investigate some properties of δ-type transitive sets in a given topological space (X, f). Further, we study and investigate some properties of δ-type minimal mapping Moreover, the relationships among δ-type transitive conjugated functions and the related classes of transitive functions are investigated.. The collection of all δ-open sets in a topological space (X,τ) forms a topology τ_s on X , called the semi-regularization topology of τ_s, weaker than τ and the class of all regular open sets in τ forms an open basis for τ_s. Similarly, the collection of all θ-open sets in a topological space (X,τ) forms a topology τ_θ on X , weaker than τ. Some types of sets play an important role in the study of various properties in topological spaces. Many authors introduced and studied various generalized properties and conditions containing some forms of sets in topological spaces, In the present paper, we studied some new class of topological transitive set called topological δ-type transitive set and investigate some of its topological dynamical properties. Let A be a subset of a topological space (X, τ). The closure and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A of a topological space (X, τ) is said to be regular open **[2]** (resp. preopen **[3]**) if A = Int(Cl(A)) (resp. A ⊂ Int(Cl(A))). A set A ⊂ X is said to be δ-open **[4]** if it is the union of regular open sets of a space X. The complement of a regular open (resp. δ-open) set is called regular closed (resp. δ-closed). The intersection of all δ-closed sets of (X, τ) containing A is called the δ-closure **[4]** of A and is denoted by $Cl_\delta(A)$.. Recall that a subset S in a space X is called regular closed if $S = Cl(Int(S))$. A point x ε X is called a δ-cluster point **[4]** of S if $S \cap U \neq \phi$ for each regular open set U containing x. The set of all δ-cluster points of S is called the δ-closure of S and is denoted by $Cl_\delta(S)$. A subset S is called δ-closed if δCl(S) = S. The complement of a δ-closed set is called δ-open. The family of all δ-open sets of a space X is denoted by $\delta O(X,\tau)$..

The δ-interior of S is denoted by $\mathrm{Int}_\delta(S)$ and it is defined as follows

$Int_\delta(S) = \{x \in X : x \in U \subseteq Int(Cl(U)) \subseteq S, ,U \in \tau\}$

A subset S of a topological space (X, τ) is called δ-β-open **[5]** if $S = Cl(Int(Cl_\delta(S)))$. The complement of a δ-β-open set is called δ-β-closed **[5].** The intersection of all δ-β-closed sets containing S is called the δ-β-closure of S and is denoted by $\beta Cl_\delta(S)$. The δ-β-interior of S is defined by the union of all δ-β-open sets contained in S and is denoted by β $Int_\delta(S)$. The set of all δ-β-open sets of (X, τ) is denoted by δβO(X). The set of all δ-β-open sets of (X, τ) containing a point x∈ X is denoted by δβO(X, x).

Functions and of course irresolute functions stand among the most important notions in the whole of mathematical science. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences. In 1972, Crossley and Hildebrand **[6]** introduced the notion of irresoluteness. . Many different forms of irresolute functions have been introduced over the years. In the present paper, we define and introduce some new class of topological transitive functions called topological δ-type transitive and study some of its properties. let as denote the collection τ^{δ} of all *δ*–sets of a space (*X; τ*), recall that *τ*= τ^{δ} if and only if every open set is closed and therefore δ-type transitive and transitive functions are coincide.

We observe that for any topological space (X, τ) the relation $\tau^{\theta} \subseteq \tau^{\delta} \subseteq \tau$ always holds. We also have $A \subseteq Cl(A) \subseteq Cl^{\delta}(A) \subseteq Cl^{\theta}(A)$ for any subset A of X.

4.2. PRELIMINARIES AND DEFINITIONS

Definition4.2.1. Let A be a subset of a space X. A point x ∈ A is said to be a δ- Limit point of A if for each δ – open set U containing x, $U \cap (A-\{x\}) \neq \Phi$. The set of all δ- limit points of A is called the δ -derived set of A and is denoted by D_δ (A)

Definition4.2.2. [7] Let *X* be a topological space, a subset *S* of *X* is said to be regular open (respectively regular closed) if *Int.*(*cl.S*) = *S* (respectively *Cl.*(*int.S*) = *S*). A point *x* ∈ *S* is said to be a *δ*-cluster point of *S* if $U \cap S \neq \phi$, for every regular open set *U* containing *x*. The set of all *δ*-cluster point of *S* is called the *δ*-closure of *S* and is denoted by Cl_δ (S). If Cl_δ (S) = *S*, then *S* is said to be δ-closed. The complement of a *δ*-closed set is called an *δ*-open set.

For every topological space (*X, τ*), the collection of all *δ*-open sets form a topology for *X* which is weaker than *τ*. This topology τ^{δ} has a base consisting of all regular open sets in (*X, τ*).

Definition4.2.3. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be β-irresolute if $f^{-1}(V)$ is β-open in X for each β-open set V of Y (see **[3]**)

Definition4.2.4. Let (X, τ) be a topological space. A subset A is called a locally closed set (briefly LC-set) **[11]** , if $A = U \cap F$, where U is open and F is closed

Lemma4.2.5. [4] Let D be a subset of X. Then:

(1) D is a δ-open set if and only if $\mathrm{Int}_{\delta}(D) = D$.

(2) $Cl_{\delta}(D^{c}) = (Int_{\delta}(D))^{c}$ *and* $Int_{\delta}(D^{c}) = (Cl_{\delta}(D))^{c}$.

(3) $Cl(D) \subseteq Cl_{\delta}(D)$ (*resp.* $Int_{\delta}(D) \subseteq Int(D)$ *for any subset D of X*.

(4) *for an open* (*resp. closed*) *subset D of X*, $Cl(D) = Cl_{\delta}(D)$ (*resp.* $Int_{\delta}(D) = Int(D)$.

Lemma4.2.6. [4] *If X is a regular space, then:*

(1) $Cl(D) = Cl_{\delta}(D)$ *for any subset D of X*;

(2) Every closed subset of X is δ-closed and hence for any subset D, $Cl_{\delta}(D)$ *is δ-closed.*

Definition4.2.7. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be *δ*-continuous **[7]** if for every $A \in \sigma$, $f^{-1}(A) \in \delta O(X,\tau)$.or, equivalently, *f* is *δ*-continuous) if and only if for every *δ*-closed set *A* of (Y,σ), $f^{-1}(A) \in \delta C(X,\tau)$.

Definition4.2.8. A topological space (X,τ) is said to be *δ*-*T*1 **[8]** if for each pair of distinct points *x*, *y* of *X* there exists a *δ*-open set *A* containing *x* but not *y* and a *δ*-open set *B* containing *y* but not *x*, or equivalently, (*X, τ*) is a *δ*-*T*1-space if and only if every singleton is δ-closed (**[8]** , Theorem 2.5).

Example4.2.9. Let (X, τ) be a topological space such that X = {a; b; c; d} and τ ={ϕ, X, {c}, {c, d},{a, b}, {a, b, c}. Clearly, δO(X, τ)={ϕ, X, {a, b}, {c, d}

Theorem4.2.9. If f and g are completely δ-β-irresolute, then g ∘f is completely δ-β- irresolute

Theorem4.2.10. A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be δ-β-irresolute **[5]** (resp. δ-β-continuous **[9]**) if $f^{-1}(V)$ is δ-β-open (resp. δ-β-open) in X for every δ-β-open (resp. open) subset V of Y.

Theorem4.2.11. For subsets A, B of X, the following statements hold:

(1) $D(A) \subset D_{\theta}(A)$ *where D(A) is the derived set of A*

(2) *If* $A \subset B$, *then* $D_{\theta}(A) \subset D_{\theta}(B)$

(3) $D_{\theta}(A) \cup D_{\theta}(B) = D_{\theta}(A \cup B)$ *and* $D_{\theta}(A \cap B) \subset D_{\theta}(A) \cap D_{\theta}(B)$

Note that the family τ^{θ} of θ –open sets in (X,τ) always forms a topology on X denoted θ-topology and that θ-topology coarser than τ

We observe that for any topological space (X,τ) the relation $\tau^{\theta} \subseteq \tau^{\delta} \subseteq \tau$ always holds. Note that θ-closed ⇒ δ-closed ⇒ closed ⇒ α-closed.

4.3. δ-TYPES TRANSITIVE FUNCTIONS AND δ-MINIMAL SYSTEMS

By a topological system (X,f) we mean a topological space X together with a continuous function. $f:X\rightarrow X$. A set $A \subseteq X$ is called f-invariant if $f(A) \subseteq A$.

A dense orbit of a topological system on a set X is an orbit whose points form a dense subset of X. Topologically transitive and existence of a dense orbit are two notions that play an important role in every definition of chaos.

Proposition4.3.1. (1) Let (X,τ) be a topological space, and $f: X \to X$ a continuous function, then f is a topologically transitive function if and only if for every pair of open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \phi$

(2) A function $f: X \to X$ is topologically transitive if $\omega_f(x) = X$ for some $x \in X$.

Proof: Suppose that $\omega_f(x) = X$ for some $x \in X$. Then for every pair of non-empty, open $U, V \subset X$ there are positive integers $n > m$ such that $f^m(x) \in U$ *and* $f^n(x) \in V$. Hence $f^{n-m}(U) \cap V \neq \phi$ and f is topologically transitive.

Recall that a topological system (X, f) is said to be minimal if X has no closed f-invariant subset. As well known, If X is a metric space then the topological system (X, f) is topologically transitive if and only if it is minimal.

If X is a metric space, it is easy to see that if a topological system has a dense orbit, then it is transitive, since this orbit comes arbitrarily close to all points. The converse is more difficult to prove. For related works see **[11]** Recall that a system (X, f) is said to be *minimal* if X does not contain any non-empty, proper, closed f-invariant subset. In such a case we also say that the function f itself is minimal If X is a metric space, it is easy to see that if a topological system has a dense orbit, then it is transitive, since this orbit comes arbitrarily close to all points. The converse is more difficult to prove. For related works see **[11]** Recall that a system (X, f) is said to be *minimal* if X does not contain any non-empty, proper, closed f-invariant subset. In such a case we also say that the function f itself is minimal

Given a point x in a system, (X, f), $O_f(x) = \{x, f(x), f^2(x), \ldots\}$, denotes its forward orbit (by an orbit we mean a forward orbit, and $\omega_f(x)$ denotes its ω-limit set, i.e. the set of limit points of the sequence $x, f(x), f^2(x), \ldots$

Definition4.3.2. [13] (δ-type wandering points)**:** Let (X, f) be a topological system. A point $x \in X$ is δ-type wandering for a function f if it belongs to an δ-open set U disjoint from $f^n(U)$ *for all* $n > 0$ The set of all δ-type wandering points is δ-open invariant set : its complement , the set of all non-wandering points is a δ-closed invariant set. Let me introduce a new definition on minimal functions called δ-minimal and we study some new theorems associated with this new definition.

A system (X, f) is called δ-*minimal* if X does not contain any non-empty, proper, δ-closed f-invariant subset. In such a case we also say that the function f itself is δ-minimal. Another definition of minimal function is that if the orbit of every point x in X is dense in X then the function f is said to be minima. Let us introduce and study an equivalent new definition. **Definition4.3.3.[13]** (δ-minimal) Let X be a topological space and f be δ-irresolute function on X. Then (X, f) is called δ-minimal system (or f is called δ-minimal function on X) if one of the three equivalent conditions hold:

(1) The orbit of each point in X is δ-dense in X

(2) $Cl_\delta(O_f(x)) = X$ for each x ϵ X.

(3) Given x ϵ X and a nonempty δ-open U in X, there exists n ϵ N such tha $f^n(x) \in U$.

Theorem4.3.4.[13] For (X, f) the following statements are equivalent:

(1) f is an δ-minimal function.

(2) If E is an δ-closed subset of X with $f(E) \subset E$, we say E is invariant. Then E= ϕ or E=X.

(3) If U is a nonempty δ-open subset of X, then $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

Proof:

(1) $\Rightarrow$ (2): If A ≠ ϕ, let x ϵ A. Since A is invariant and δ-closed, i.e. $Cl_\delta(A) = A$ so $Cl_\delta(O_f(x)) \subset A$..

On other hand $Cl_\delta(O_f(x)) = X$. Therefore A = X.

(2) $\Rightarrow$ (3) Let A=X\ $\bigcup_{n=0}^{\infty} f^{-n}(U)$. Since U is nonempty, A ≠ X and Since U is δ-open and f is δ-continuous, A is δ-closed. Also $f(A) \subset A$, so A must be ϕ.

f is δ-continuous, A is δ-closed. Also $f(A) \subset A$, so A must be an empty set.

(3) $\Rightarrow$ (1): Let x ϵ X and U be a nonempty δ-open subset of X. Since x ϵ X = $\bigcup_{n=0}^{\infty} f^{-n}(U)$ .Therefore x ϵ $f^{-n}(U)$ for some $n > 0$. So $f^{n}(x) \in U$

Proposition 4.3.5[13] Let X be a δ-compact space without isolated point, if there exists a δ-dense orbit, that is there exists x_0 ∈X such that the set $O_f(x_0)$ is δ- dense then f is topologically δ-transitive .

Proof .Let x_0 be such that $O_F(x_0)$ is δ-dense. Given any pair U, V of δ-open sets, by δ-density there exists n such that $f^{n}(x_0) \in U$, but $O_F(x_0)$ is δ- dense implies that $O_F(f^{n}(x_0))$ is δ-dense, there exists m such that $f^{m}(f^{n}(x_0)) \in V$. Therefore $f^{m+n}(x_0) \in f^{m}(U) \cap V$ That is $f^{m}(U) \cap V \neq \varphi$. So f is topological δ-transitive.

Definition4.3.6. A system (X, f) is called *topologically δ-mixing if* for any *non-empty* δ-open set U, there exists $N \in \mathbb{N}$ such that $\bigcup_{n \geq N} f^{n}(U)$ is dense in X.

As well known if every point x in the space X has a dense orbit, and then X is mixing.

If two topological spaces are homeomorphic, then not only their respective sets of points, but also their collections of open sets are in a one-to-one correspondence. Homeomorphisms show us when two topological spaces should be considered to be the same in the eye of topology Topologically δ-conjugating: One usual way to relate two topological systems $f: X \to X$ *and* $g: Y \to Y$ is with the topological notion of Conjugacy of Two topological systems (X, f) and (Y, g) are topologically δ-conjugated if there exist a δr-homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$, where f and g are called the dynamic functions that act on the spaces X and Y respectively, Conjugacy show us when two topological systems should be considered to be the same in the eye of topological dynamics. the orbit of two topological systems (X, f) and (Y, g) are topological conjugate or conjugate if there exists a homeomorphism h X→Y such that h(f(x))=g(h(x)). for each x in X. Topological conjugacy requires that orbits be put into one-to-one correspondence and preserves many topological dynamical properties. For example, If $f: X \to X$ and $g: Y \to Y$ are topologically conjugated by the homeomorphism ψ : Y → X, then for all y ∈ Y the orbit $O_g(y)$ is dense in Y if and only if the orbit $O_f(\psi(y))$ of ψ(y) is dense in X. Thus, if one finds a topological Conjugacy of a function f with a simpler function g, one can analyze the simpler function g to obtain information about dynamical properties of the original function f. For related works see **[10]**

Definition 4.3.7 Two topological systems $f: X \to X$ *and* $g: Y \to Y$ are said to be topologically δr-conjugate if there is a δr-homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$. We will call h topological δ- Conjugacy. Thus, the two topological systems with their respective function acting on them share the same dynamics

Now, we proceed to prove an important proposition:

Proposition4.3.8. (A) if $f: X \to X$ *and* $g: Y \to Y$ are topologically δr-conjugated by δr-homeomorphism $h: X \to Y$. Then

(1) T is δ- transitive set in X if and only if h(T) is δ-transitive set in Y;

(2) T is δ –mixing set in X if and only if h(T) is δ –mixing set in Y.

(B) *If h is not δr -homeomorphism but only δ -irresolute surjection (a semi- δ r -Conjugacy),* then if Tis topological δ-transitive set in X then h(T) is topological δ- transitive set in Y.

Definition4.3.9. Let $f: X \to X$ be δ-irresolute self-function of a topological space X. A fundamental δ-type domain for f is δ-open subset $D \subset X$ such that every orbit of f intersect D in at most one point and intersect $Cl_\delta(D)$ in at least one point.

Proposition4.3.10. Let $f: X \to X$ *and* $g: Y \to Y$ be two δ-irresolute self-functions. Assume that there are a fundamental δ-type domain $D_f \subset X$ for f, a fundamental δ-type domain $D_g \subset Y$ for g and a δr-homeomorphism $h: Cl_\delta(D_f) \to Cl_\delta(D_g)$ such that $g \circ h = h \circ f$ on $f^{-1}(Cl_\delta(D_f)) \cap Cl_\delta(D_f)$. Then f and g are topologically δr-conjugated.

For the sake of comparison, let us introduce and define topological notions of δ-type recurrence. A point x ∈ X is δ-type non-wandering if for any δ-open $U \subset X$ containing x, there exists $N > 0$ such that $f^{N}(U) \cap U \neq \phi$.. The function f is δ-type non-wandering if every point of X is δ-type non-wandering. A function f is δ-type transitive if for any nonempty δ- open sets $U, V \subset X$, there is an $N \geq 0$ such that $f^N(U) \cap V \neq \phi$. A function f is topologically δ-type mixing if for any nonempty δ- open sets $U, V \subset X$, there is an $N > 0$ such that $f^n(U) \cap V \neq \phi$. for all n ≥ N.

Definition4.3.11. Let (X, f) be a topological system. A point x ϵ X is δ-type non-wandering if for any δ-open set U containing x there is N > 0 such that $f^N(U) \cap U \neq \phi$. The set of all δ-type non-wandering point is denoted by NW$_\delta$(f). A point which is not δ-type non-wandering is called δ-type wandering.

We can easily prove the following Proposition:

Proposition4.3.12. Let (X, f) be a topological system on δ-Hausdorff space X. Then:

(i) $NW_\delta(f)$ *is* $\delta-closed$.

(ii) $NW_\delta(f)$ *is* $f-$invariant.

(iii) If f is invertible, then $NW_\delta(f^{-1}) = NW_\delta(f)$.

(iv) If X is $\delta-$compact then $NW_\delta(f) \neq \phi$.

(v) If x is $\delta-$type non-wandering point in X, then for every $\delta-$open set U containing x and n_0 ϵ N there is $n > n_0$ such that $f^n(U) \cap U \neq \phi$.

Remark 4.3.13 Any δ-dense subset in X intersects any δ-open set in X.

Proof: Let A be an δ-dense subset in X, then by definition, $Cl_\delta(A) = X$, and let U be a nonempty δ-open set in X. Suppose that A∩U=ϕ. Therefore $B = U^c$ is δ-closed and $A \subset U^c$ =B. So $Cl_\delta(A) \subset Cl_\delta(B)$, i.e. $Cl_\delta(A) \subset$B, but $Cl_\delta(A) = X$, so X⊂B, this contradicts that U ≠ φ

Associated with the new definition of topologically δ-type transitive we can prove the following new theorem.

Theorem4.3.14. Let (X, τ) be a regular space and $f : X \to X$ be δ-irresolute function. Then the following statements are equivalent:

(1) f is δ-type transitive function

(2) For every nonempty δ-open set U in X, $\bigcup_{n=0}^{\infty} f^n(U)$ is δ-dense in X

(3) For every nonempty δ-open set U in X, $\bigcup_{n=0}^{\infty} f^{-n}(U)$ is δ-dense in X

(4) If $B \subset X$ is δ-closed and B is forward f - invariant i.e. $f(B) \subset B$. then B=X or B is nowhere δ-dense

(5) If U is δ-open and $f^{-1}(U) \subset U$ then U=ϕ or U is δ-dense in X. We have to prove this theorem:

Proof:

$(1) \Rightarrow (2)$

Assume that $\bigcup_{n=0}^{\infty} f^n(U)$ is not δ-dense. Then there exists a non empty δ-open set V such that $\bigcup_{n=0}^{\infty} f^n(U) \cap V = \phi$. This implies that $f^n(U) \cap V = \phi$ for all n ϵ N .This is a contradiction to the δ-type-transitivity of f. Hence $\bigcup_{n=0}^{\infty} f^n(U)$ is δ-dense in X.

$(2) \Rightarrow (1)$

Let U and V be two nonempty δ-open sets in X, and let $\bigcup_{n=0}^{\infty} f^n(U)$ be δ-dense in X, this implies that $\bigcup_{n=0}^{\infty} f^n(U) \cap V \neq \phi$ by Remark 4.3.13. This implies that there exists m ϵ N such that $f^m(U) \cap V \neq \phi$. Hence f is topologically δ-tye transitive function.

$(1) \Rightarrow (3)$

It is obvious that $\bigcup_{n=0}^{\infty} f^{-n}(U)$ is δ-open and since f is δ-type transitive function, it has to meet every δ-open set in X, and hence it is δ-dense.

$(3) \Rightarrow (1)$

Let E and F be two δ-open subsets in X. Then $\bigcup_{n=0}^{\infty} f^{-n}(F)$ is δ-dense, this implies that $\bigcup_{n=0}^{\infty} f^{-n}(F) \cap E \neq \phi$, by Remark 4.3.13.This implies that there exists m ϵ N such that $f^{-m}(F) \cap E \neq \phi$.Therefore, $f^m(f^{-m}(F) \cap E) = F \cap f^m(E) \neq \phi$. So f is δ-type transitive.

$(1) \Rightarrow (4)$

Suppose f is δ-type transitive function, $E \subset X$ is δ-closed and $f(E) \subset E$. Assume that E ≠ ϕ and E has a nonempty δ-interior (i.e. $\text{int}_\delta(E) \neq \phi$). If we define V=X\E so V is δ-open because V is the compliment of δ-closed. Let $F \subset E$ be δ-open since $\text{int}_\delta(E) \neq \phi$. We have $f^n(F) \subset E$ since E is invariant, therefore $f^n(F) \cap V = \phi$, for all n ϵ N. This is a contradiction to topological δ-type transitive. Hence E=X or E is nowhere δ-dense .

(4) $\Rightarrow$ (1)

Let V be a nonempty δ-open set in X. Suppose f is not a topological δ-type transitive function, from (3) of this theorem $\bigcup_{n=0}^{\infty} f^{-n}(V)$ is not δ-dense, but δ-open. Define $E = X \setminus \bigcup_{n=0}^{\infty} f^{-n}(V)$ which is δ-closed, because it is the complement of δ-open, and E≠X. Clearly $f(E) \subset E$. Since $\bigcup_{n=0}^{\infty} f^{-n}(V)$ is not δ-dense so by Remark 4.3.13., there exists a non-empty δ-open W in X such that $\bigcup_{n=0}^{\infty} f^{-n}(V) \cap W = \phi$. This implies that $W \subset E$. This is a contradiction to the fact that E is nowhere δ-dense. Hence f is a topological δ-type transitive function.

(1) $\Rightarrow$ (5)

Suppose that the function f is δ-type transitive, $U \subset X$ is δ-open and $f^{-1}(U) \subset U$. Assume that $U \neq \phi$ and U is not δ-dense in X (i.e. $Cl_\delta(U) \neq X$). Then there exists a non-empty δ-open $V = X \setminus Cl_\delta(U)$ since $Cl_\delta(U)$ is δ-closed, such that $U \cap V = \phi$. Further $f^{-n}(U) \cap V = \phi$ for all n ϵ N. This implies $U \cap f^n(V) = \phi$ for all n ϵ N, a contradiction to f being δ-type transitive function. Therefore $U = \phi$ or U is δ-dense in X.

Proposition 4.3.15 if $f: X \to X$ *and* $g: Y \to Y$ are topologically δr-conjugated by the δr-homeomorphism $h: Y \to X$. Then *for all* y ∈ Y *the orbit* $O_g(y)$ *is δ-dense in* Y *if and only if the orbit* $O_f(h(y))$ *of* h(y) *is δ-dense in* X.

Proof:

Let $h: Y \to X$ be the δr- Conjugacy. Assume that $O_g(y)$ is δ-dense and let us show that $O_f(h(y))$ is δ-dense. For any U ⊂ X non-empty δ-open set, $h^{-1}(U)$ is a δ-open set in Y since h^{-1} is δ-irresolute because h is a δr-homeomorphism and it is non-empty since h is surjective. By δ-density of $O_g(y)$, there exists k ∈ N such that $g^k(y) \in h^{-1}(U) \Leftrightarrow h^{-1}(g^k(y)) \in U$ .Since h is a δr-conjugation then $f^k \circ h = h \circ g^k$ so $f^k(h(y)) = h(g^k(y)) \in U$, therefore $O_f(h(y))$ intersects U. This holds for any non-empty δ- open set U and thus shows that $O_f(h(y))$ is δ-dense. The other implication follows by exchanging the role of f and g.

Theorem4.3.16. Any *two δ-minimal sets must have empty intersection.*

Proof: Let M_1 and M_2 be two distinct δ-minimal sets, and suppose that $A = M_1 \cap M_2 \neq \phi$.Then A is δ-closed, and for every a ∈ A and every n ∈ N, $f^n(a) \in M_1 \cap M_2$, so A is invariant. But then A is a proper subset of both M_1 and M_2 which is δ-closed, invariant and non-empty, contradicting the fact that M_1 and M_2 are δ-minimal.

Lemma4.3.17. If (X, f) and (Y, g) are tow topological systems. The set of periodic points of $f \times g$ is δ-dense in $X \times Y$ if and only if, for both of f and g , the sets of periodic points in X and Y are δ-dense in X, respectively Y.

Lemma 4.3.18 Let $f: X \to X$ *and* $g: Y \to Y$ be functions and assume that the product $f \times g$ is topological δ-type transitive on $X \times Y$. Then the functions f and g are both topological δ-type transitive on X and Y respectively.

Definition4.3.19. Let $f: X \to X$ be a function on the topological space X and A is a closed invariant subset of X. If for every nonempty δ-open sets $U, V \subset X$, such that $A \cap U \neq \phi$ *and* $A \cap V \neq \phi$, there exists a positive integer n_0 such that for every $n \geq n_0$, $f^n(U,) \cap V \neq \phi$ then A is called topologically δ-type mixing set.

It is clear that topological δ-type mixing implies topological δ-type transitive set but not conversely.

There is an even stronger notion that implies topological δ-type mixing.

Definition4.3.20. Let $f: X \to X$ be a function on the topological space X. If for every nonempty δ -open subset $U \subset X$ there exist a positive integer n_0 such that for every $n \geq n_0$, $f^n(U) = X$, then f is called locally δ-type eventually onto. Note that locally δ-type eventually onto $\Rightarrow$ δ-- type mixing $\Rightarrow$ weakly δ--mixing $\Rightarrow$ δ--type transitivity

Lemma4.3.21. The product of two topologically δ-type mixing functions is topologically δ-type mixing.

Proof: Let (X, f) , (Y, g) be topological systems and *f, g* be topologically δ-type mixing functions. Given

$W_1, W_2 \subset X \times Y$, there exists δ-open sets $U_1, U_2 \subset X$ and $V_1, V_2 \subset Y$, such that $U_1 \times V_1 \subset W_1$ and $U_2 \times V_2 \subset W_2$. By assumption there exist n_1 and n_2 such that

$f^k(U_1) \cap U_2 \neq \phi$ for $n \geq n_1$ and $g^k(V_1) \cap V_2 \neq \phi$ for $n \geq n_{12}$. For $n \geq n_0 = \max\{n_1, n_2\}$. We get

$$[(f\ x\ g)^k(U_1\ x\ V_1)] \cap (U_2\ x\ V_2) = [f^k(U_1)\ x\ g^k(V_1)] \cap (U_2\ x\ V_2)$$
$$= [f^k(U_1) \cap U_2]\ x\ [g^k(V_1) \cap V_2] \neq \phi$$

Which means that $f \times g$ is topologically δ-type mixing.

The term *weak incompressibility* seems to have appeared first in [12] and we adopt this term in this section:

Definition4.3.22.[13] (Weakly Incompressible) a set $A \subset X$ is *weakly incompressible* if for any proper non-empty subset $U \subset A$ which is open in A, $Cl(f(U)) \cap (A \setminus U) \neq \phi$. Equivalently we can say that for any non-empty closed subset $D \subset A$ we have that $D \cap Cl(f(A \setminus D)) \neq \phi$.

Lemma4.3.23. [13] *If* $A = \omega_f(x_0)$ *for* $f: X \to X$ *and* $x_0 \in X$ *then A is weakly incompressible.*

Proof: Assume that for some closed $M \subset A$ we have that $M \cap Cl(f(A \setminus M)) \neq \phi$, then by normality there are open sets U and V such that $Cl(U) \cap Cl(V) = \phi$, $M \subset U$ and , $Cl(f(A \setminus M)) \subset V$. Thus $(A \setminus M) \subset f^{-1}(V) = W$, where W is open by continuity. $f(Cl(W)) = Cl(f(W)) = Cl(V)$, so $f(Cl(W)) \cap Cl(U) = \phi$. Since $A = \omega_f(x_0) \subset (W \cup U)$, there is an integer $k_0 > 0$ such that $f^n(x_0) \in W \cup U$ for every $n \geq k_0$. Moreover, $f^n(x_0) \in W$ for infinitely many $n \geq k_0$. and $f^m(x_0) \in U$ for infinitely many $m \geq k_0$, so for infinitely many $n \geq k_0$, $f^n(x_0) \in W$ and $f^{n+1}(x_0) \in U$. Thus there is a sequence $\{n_i\}_{i \in N}$ such that $f^{n_i}(x_0) \in W$, $f^{n_i+1}(x_0) \in U$ and $y = \lim_{i \to \infty} f^{n_i}(x_0) \in Cl(W)$.

Then $f(y) = f(\lim_{i \to \infty} f^{n_i}(x_0)) = \lim_{i \to \infty} f^{n_i+1}(x_0) \in Cl(U)$;

i.e. $f(y) = f(Cl(W)) \cap Cl(U)$, which contradicts the fact that $f(Cl(W)) \cap Cl(U) = \phi$. Hence $M \cap Cl(f(A \setminus M)) \neq \phi$.

We define new definitions of stable subsets of topological spaces.

Definition4.3.24. Let $A \subseteq X$ be δ- compact invariant set.

1- A is δ-type stable if every δ-open set U containing A contains δ-open set V containing A such that $f^n(V) \subset U$ for all n ≥ 0.

2- The δ -closed, nonempty invariant set A is said to be δ **-type** *attractor* if and only if there is δ -open set $U \supset A$ such that

(i) $Cl_\delta(f(U)) \subseteq U$,

(ii) $\omega_f(x, \delta) \subseteq A$ for every $x \in U$.

where $\omega_f(x, \delta) = \bigcap_{n=0}^{\infty} Cl_\delta\{f^k(x) : k \geq n\}$.

Definition4.3.25. A set $A \subset X$ is *weakly* δ-type *incompressible* (or has *weak* δ-type *incompressibility*) if $D \cap Cl(f(A \setminus D)) \neq \phi$. whenever D is a nonempty, δ-closed, proper subset of A. Clearly A is weakly δ-type incompressible if and only if $Cl(f(U)) \cap (A \setminus U) \neq \phi$ for any proper, nonempty subset $U \subset A$ which is δ-open in A.

Lemma4.3.26. Every weakly incompressible is *weakly* δ-type *incompressible* but not conversely.

4.4. NEW TYPES OF TRANSITIVE FONCTIONS ON SEMI-REGULAR SPACES

Definition4.4.1. Let X be a set, and **B** a collection of subsets of X. Recall that **B** is a basis for a topology on X, and each B_n in **B** is called a basis element if **B** satisfies the following axioms:

B1. For every $x \in X$, there is a set $B \in$ **B** such that $x \in B$, and

B2. If B_1 and B_2 are in **B** and $x \in B_1 \cap B_2$, then there exists a B_3 in **B** such that $x \in B_3 \subseteq B_1 \cap B_2$.

Definition4.4.2. Let X be a topological space and let $x \in X$. A neighborhood of x is an open set N such that $x \in N$.

Proposition4.4.3. Let X is a set and B a basis over X. Let τ be a collection of subsets of X which includes ϕ and which is closed with respect to arbitrary unions of basis elements. Then τ is a topology over X.

Definition4.4.4. Let X be a set and **B** a basis over X. The topology τ generated by **B** is created by defining the open sets of τ to be ϕ and all sets that are a union of basis elements.

Given a space (X, τ), denote by σ the topology on X whose basis consists of regular open subsets of (X, τ) . The space (X, σ) is said to be the semi-generalization of (X, τ) . It is easy to see that $\sigma \subseteq \tau$ and that the spaces (X,τ) and (X,σ) have the same regular open subsets.

Definition4.4.5. A space whose regular open subsets form a base for its topology is called semi-regular. The set of all open subsets of semi-regular space are denoted by $O_{sr}(X)$, whose elements are called sr-open sets.

Definition4.4.6.. *Let (X,τ) and (Y,σ) be two topological spaces, A function is called sr-irresolute if for every sr-open subset H of Y, $f^{-1}(H)$ is sr-open in X.*

Definition4.4.7. *Let (X,τ) be a topological space, $f:X\to X$ be sr- irresolute function, then the map f is called semi-regular transitive (in short: sr-transitive) if for every pair of non-empty sr-open sets U and V in X there is a positive integer n such that $f^n(U)\cap V\neq\phi$. In this case, we can say that the system (X, f) is sr-transitive . It is sr-minimal if every orbit is sr-dense.*

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Definition 4.4.8 (1) A point $x\in X$ is sr-recurrent if, for every sr-open set U containing x, infinitely many $n\in\mathbf{N}$ satisfy $f^n(x)\in U$.

(2) Let (X, τ) be a topological space, $f:X\to X$ be sr-irresolute map, then the **function** f is called topologically sr- strongly mixing if, given any nonempty sr-open subsets $U,V\subseteq X$ $\exists N\geq 1$ such that $f^n(U)\cap V\neq\phi$. for all $n>N$. A subset B of X is f-*invariant* if $f(B)\subset B$. A non-empty sr-closed invariant subset B of X is sr- *minimal*, if $Cl_{sr}(O_f(x))=B$ for every x ∈ B. A point x ∈ X is sr-*minimal* if it is contained in some sr-minimal subset of X

Clearly if f is topologically sr- strongly mixing then it is also sr-transitive but not conversely.

(3) The function f is sr-exact if, for every nonempty sr-open set $U\subset X$, there exists some $n\in\mathbf{N}$ such that $f^n(U)=X$. Note that topological sr-exactness implies sr-mixing implies weakly sr-mixing implies sr-transitivity.

(4) The function f is (topological) semi-regular transitive (resp., sr-mixing) if for any two nonempty sr- open sets U, $V\subset X$, there exists some $n\in\mathbf{N}$ such that $f^n(U)\cap V\neq\phi$ (resp. $f^m(U)\cap V\neq\phi$, for all $m\geq n$).

(5) The function f is weak sr-mixing if $f\times f$ is sr–type transitive on $X\times X$.

(6) The sr-mixing function $f:X\to X$ is pure sr-mixing if and only if there exists sr- open set $U\subset X$ such that $f^n(U)\neq X$. for all n 0.

(6) The **function** $f:X\to X$ is sr-type chaotic if f is sr-transitive on X and the set of periodic points of f is sr-dense in X.

(7) The **function** f is called sr-type exact chaos (resp., sr-type mixing chaos and weakly sr-type mixing chaos) if f is sr-exact (resp., sr-mixing and weakly sr-mixing) and sr-type chaotic function on the space X.

(8) Let (X,τ) be a topological space, $f:X\to X$ be sr-irresolute **function**, then the set $A\subseteq X$ is called semi-regular transitive set (in short sr-transitive set) if for every pair of non-empty sr-open sets U and V in X with $A\cap U\neq\phi$ and $A\cap V\neq\phi$ there is a positive integer n such that $f^n(U)\cap V\neq\phi$..

(9) Let (X,τ) be a topological space, $f:X\to X$ be sr-irresolute function, then the set $A\subseteq X$ is called topologically sr-mixing set if, given any nonempty sr-open subsets $U,V\subseteq X$ with $A\cap U\neq\phi$ and $A\cap V\neq\phi$ then $\exists\ N>0$ such that $f^n(U)\cap V\neq\phi$ for all $n>N$.

(10) *The sr-closed set* $A\subseteq X$ is called a weakly sr-mixing set of (X,f) if for any choice of nonempty sr -open subsets V_1,V_2 of A and nonempty sr –open subsets $U_{,1}U_2$ of X with $A\cap U_1\neq\phi$ *and* $A\cap U_2\neq\phi$ there exists n ∈ N such that $f^n(V_1)\cap U_1\neq\phi$ *and* $f^n(V_1)\cap U_2\neq\phi$

Note: If A is a weakly sr- mixing set of (X,f), then A is a semi-regular transitive set of (X,f).

Definition 4.4.9 Two topological dynamical systems (X, f) and (Y, g) are said to be $semi-regular$-conjugated (in short sr-conjugate) if there is a $semi-regular-$homeomorphism (in short sr-homeomorphism) $h:X\to Y$ such that $h\circ f=g\circ h$.

4.5. S-REGULAR-CHAOS AND TOPOLOGICAL S-REGULAR-CONJUGACY

I introduce and define sr-type transitive functions and sr –type minimal functions . I will study some of their properties and prove some results associated with these new definitions. I investigate some properties and characterizations of such functions. Let (X, f) be a topological system. A function $f:X\to X$ is called semi-regular chaotic, if it is topological sr-type transitive and, its periodic points are sr-dense in X , i.e. every non-empty sr-open subset of X contains a periodic point. (A point x X is called periodic if there exists $n\geq 1$ *with* $f^n(x)=x$. The set of all periodic points of f denoted by $Per(f)$.

Definition 4.5.1 Recall that a subset A of a space X is called sr-dense in X if $Cl_{sr}(A)$, we can define equivalent definition that a subset A is said to be sr-dense if for any x in X either x in A or it is a sr-limit point for A.

Remark 4.5.2 any sr-dense subset in X intersects any sr-open set in X.

Definition 4.5.3 Recall that a subset A of a topological space (X, τ) is said to be nowhere sr-dense, if its sr-closure has an empty sr-interior, that is, $Int_{sr}(Cl_{sr}(A)) = \phi$.

Definition 4.5.4 if for $x \in X$ the set $\{f^n(x) : n \in \mathbf{N}\}$ is sr-dense in X then x is said to have sr-dense orbit. If there exists such an $x \in X$, then *f* is said to have sr-dense orbit.

Definition 4.5.5. A function $f : X \to X$ is called s-regular-homeomorphism if *f* is sr-irresolute bijective and $f^{-1} : X \to X$ is sr-irresolute.

Definition 4.5.6 Two topological systems $f : X \to X$, $x_{n+1} = f(x_n)$ and $g : Y \to Y$, $y_{n+1} = g(y_n)$ are said to be topologically s-*regular* -conjugate if there is s-*regular* -homeomorphism $h : X \to Y$ such that $h \circ f = g \circ h$. We will call h a topological semi-*regular* –Conjugacy (in short sr-Conjugacy). When the two systems are topologically s-*regular* -conjugate they have the same dynamics.

Then I have proved some of the following statements:

1. $h^{-1} : Y \to X$ is a topological s-regular-Conjugacy.

2. $h \circ f^n = g^n \circ h \ \ \forall n \in \mathbf{N}$

3. $x \in X$ is a periodic point of *f* if and only if h(x) is a periodic point of *g* .

4. If x is a periodic point of the function *f* with stable set $W_f(x)$, then the stable set of h(x) is $h(W_f(x))$.

5. The periodic points of *f* are dense in X if and only if the periodic points of *g* are dense in Y.

6. The function *f* is sr-exact if and only if *g* is sr-exact

7. The function *f* is sr-mixing if and only if *g* is sr-mixing

8. The function *f* is semi- regular chaotic if and only if *g* is semi-regular chaotic

9. *The function f is weakly sr-mixing if and only if g is weakly sr- mixing.*

Remark 4.5.7

If $\{x_0, x_1, x_2, \ldots\}$ denotes an orbit of $x_{n+1} = f(x_n)$ then $\{ y_0 = h(x_0), \ y_1 = h(x_1), \ y_2 = h(x_2), \ldots \}$ yields an orbit of g since $y_{n+1} = h(x_{n+1}) = h(f(x_n)) = g(h(x_n)) = g(y_n)$. i.e. *f* and g have the same kind of dynamics.

I introduced and defined the new type of transitive called topologically semi-regular transitive (in short sr-transitive), in such a way that it is preserved under semi-regular conjugation.

Theorem 4.5.8 For sr -irresolute function f : X → X, where X is a topological space, the following are equivalent:.

1. f is semi-regular transitive;
2. Any Proper $sr-$closed subset $A \subset X \ni f(A) \subseteq A$ is nowhere $sr-$dense;
3. $\forall \ A \subseteq X \ni f(A) \subseteq A$, A is either $sr-$dense or nowhere $sr-$dense;
4. *Any subset* $A \subseteq X \ni f^{-1}(A) \subseteq A$ with non-empty $sr-$interior is $sr-$dense.

Proposition 4.5.9 if $f : X \to X$ *and* $g : Y \to Y$ are sr -conjugated by the $sr-$homeomorphism $h : Y \to X$. Then *for all* $y \in Y$ *the orbit* $O_g(y)$ *is sr-dense in* Y *if and only if the orbit* $O_f(h(y))$ *of* h(y) *is* sr *-dense in* X.

Proposition 4.5.10 Let X be a sr-compact space without isolated point, if there exists a sr-dense orbit, that is there exists $x_0 \in X$ such that the set $O_f(x_0)$ is sr- dense then the function f is semi-regular transitive .

Proposition 4.5.11 if $f : X \to X$ *and* $g : Y \to Y$ are sr -conjugated by $h : X \to Y$.

Then:

(1) *f* is semi-regular transitive if and only if *g* is semi-regular transitive

(2) *f* is sr -minimal if and only if *g* is sr -minimal.

(3) f is topologically sr -mixing if and only if *g* is topologically sr -mixing.

Proposition 4.5.12 If the two functions $f : X \to X$ *and* $g : Y \to Y$ are topologically semi-regolar-conjugated by h from X to Y. Then

(1) The function *f* is sr -exact if and only if *g* is sr -exact

(2) *f* is weakly sr -mixing if and only if *g* is weakly sr -mixing.

(3) f is sr -type chaotic on X if and only if *g* is sr -type chaotic in Y.

Proof (1) $\Rightarrow$) given any nonempty sr -open set V in Y, let us take $U = h^{-1}(V)$. Clearly, *U* is a nonempty sr -open set in *X*. Since *f* is sr -exact, there exists $n \in \mathbf{N}$ such that $f^n(U) = X$. Noting that *f* and *g* are sr -conjugate functions and that h is a sr-homeomorphism, it follows that $g^n(V) = g^n(h(U)) = h(f^n(U)) = h(X) = Y$. Since V is an arbitrary sr -open set, this implies that *g* is sr -exact.

$\Leftarrow$) It can be proved similarly.

Proof(2)

$\Rightarrow$) For any two nonempty sr-open sets $V, W \subset Y \times Y$, according to the construction of product topology, it follows that there exist nonempty sr-open sets $W_1, W_2, V_1, V_2 \subset Y$ such that $W_1 \times W_2 \subset W$ and $V_1 \times V_2 \subset V$. Since f is weakly sr-mixing, t9\here exists $n \in N$ such that

$$(f \times f)^n (h^{-1}(W_1) \times h^{-1}(W_2)) \cap (h^{-1}(V_1) \times h^{-1}(V_2)) \neq \phi,$$

This implies that

$$\begin{aligned}(g \times g)^n (W) \cap V &\supset (g \times g)^n (W_1 \times W_2) \cap (V_1 \times V_2) \\ &= (g \times g)^n [(h \times h)(h^{-1}(W_1) \times h^{-1}(W_2))] \cap [(h \times h)(h^{-1}(V_1) \times h^{-1}(V_2))] \\ &= (h \times h)(f^n (h^{-1}(W_1)) \times f^n (h^{-1}(W_2))) \cap [(h \times h)(h^{-1}(V_1) \times h^{-1}(V_2))] \\ &\supset (h \times h)([f^n (h^{-1}(W_1)) \times f^n (h^{-1}(W_2))] \cap (h^{-1}(V_1) \times h^{-1}(V_2))) \neq \phi\end{aligned}$$

Therefore, g is weakly sr-mixing.

$\Leftarrow$) It can be proved similarly.

Proof (3)
Necessity: Similarly to the proof of Proposition 4.5.11 part (1), it can be verified that g is semi-regular transitive on Y, as f is semi-regular transitive on X. According to the definition of periodic points, it is easy to check that $Per(g) \supset h(Per(f))$. Applying this, one has

$Cl_{sr}(Per(g)) \supset Cl_{sr}[h(Per(f))] \supset h[Cl_{sr}(Per(f)] = h(X) = Y$.

This implies that $Cl_{sr}[Per(g)] = Y$. Therefore g is sr-type chaotic function on Y and also $Cl_{sr}[Per(g)] = Y$, it follows that g is sr-type chaotic. Sufficiency can be proved similarly.

4.6. SEMI –REGULAR-CHAOS IN PRODUCT TOPOLOGICAL SPACES

Given two topological systems $(X, f), (Y, g)$. Consider the product of the functions f and g as $f \times g : X \times Y \to X \times Y$, defined by $(f \times g)(x, y) = (f(x), g(y))$, with product topology on $X \times Y$

Lemma 4.6.1 Let $(X, f), (Y, g)$ be topological systems. The set of periodic points of $f \times g$ is sr-dense in $X \times Y$ if and only if, for both of f and g , the sets of periodic points in X and Y are sr-dense in X, respectively Y.

Proof: Assume that the set of periodic points of f is sr-dense in X (i.e. $Cl_{sr}(Per(f)) = X$) and the set of periodic points of g is sr-dense in Y (i.e. $Cl_{sr}(Per(g)) = Y$). We have to prove that the set of periodic points of $f \times g$ is sr-dense in $X \times Y$. Let $W \subset X \times Y$ be any non-empty sr-open set. Then there exist non-empty sr-open sets $U \subset X$ and $V \subset Y$ with $U \times V \subset W$. By assumption, there exists a point $x \in U$ such that $f^n(x) = x$ with $n \geq 1$. Similarly, there exists $y \in V$ such that $g^m(y) = y$ with $m \geq 1$.For $p = (x, y) \in W$ and $k = mn$ we get

$$\begin{aligned}(f \times g)^k (p) &= (f \times g)^k (x, y) \\ &= ((f^k(x), g^k(y)) = (x, y) = p\end{aligned}$$

Therefore W contains a periodic point and thus the set of periodic
points of $f \times g$ is sr-dense in $X \times Y$.

$\Leftarrow$) Conversely let $U \subset X$ and $V \subset Y$ be non-empty sr - open subsets. Then $U \times V$ is a non-empty sr-open subset of $X \times Y$. As the set of the periodic points of f ×g is sr-dense in $X \times Y$, there exists a point $p = (x, y) \in U \times V$ such that $(f \times g)^n (x, y) = ((f^n(x), g^n(y)) = (x, y)$ for some n. From the last equality we obtain $f^n(x) = x$ for some $x \in U$ and $g^n(y) = y$ for $y \in V$ .The semi-regular-denseness of periodic points carries over from factors to products. But, topological semi-regular transitivity may not carry over to products. The converse of this situation is however true:

Lemma 4.6.2 Let $f : X \to X$ and $g : Y \to Y$ be functions and assume that the product $f \times g$ is semi-regular transitive on $X \times Y$. Then the functions f and g are both topological semi-regular transitive on X and Y respectively.

Proof. We have to prove the semi-regular transitivity of f ; the semi-regular transitivity of g can be proved similarly. Let U_1, U_2 be non-empty sr-open sets in X. Then the sets $U = U_1 \times Y$ and $V = V_1 \times Y$ are sr-open in $X \times Y$. As $f \times g$ is semi-regular transitive, there exists a positive integer n such that $(f \ x \ g)^n (U) \cap V \neq \phi$. From the equalities:

$$\begin{aligned}(f \times g)^n (U) \cap V &= [f^n(U_1) \times g^n(Y)] \cap [V_1 \times Y] \\ &= [f^n(U_1) \cap V_1] \times [g^n(Y) \cap Y] \neq \phi,\end{aligned}$$

Thus f is semi-regular transitive function.

Definition 4.6.3 Let $f : X \to X$ be a function on the topological space X. If for every nonempty sr-open subsets $U, V \subset X$ there exists a positive integer n_0 such that for every $n \geq n_0$, $f^n(U,) \cap V \neq \phi$ then f is called topologically sr- mixing.

It is clear that topological sr-mixing implies semi-regular transitive. There is an even stronger notion that implies topological sr- mixing.

Definition 4.6.4 Let $f: X \to X$ be a function on the space X. If for every nonempty sr-open subset $U \subset X$ there exists $n_0 \in \mathbf{N} \setminus \{0\}$ such that for every , $n \geq n_0$, $f^n(U) = X$, then f is called semi-regular exact.

Lemma 4.6.5 The product of two topologically sr-mixing functions is topologically sr-mixing.

Proof. Let (X, f), (Y, g) be topological dynamical systems and f, g be topologically sr- mixing functions. Given $W_1, W_2 \subset X \times Y$, there exists sr-open sets $U_1, U_2 \subset X$ and $V_1, V_2 \subset Y$, such that $U_1 \times V_1 \subset W_1$ and $U_2 \times V_2 \subset W_2$. By assumption there exist n_1 and n_2 such that

$$f^k(U_1) \cap U_2 \neq \phi \text{ for } k \geq n_1 \text{ and } g^k(V_1) \cap V_2 \neq \phi \text{ for } k \geq n_2$$

For $k \geq n_0 = \max\{n_1, n_2\}$

we get

$$[(f\ x\ g)^k(U_1\ x\ V_1)] \cap (U_2\ x\ V_2) = [f^k(U_1)\ x\ g^k(V_1)] \cap (U_2\ x\ V_2)$$
$$= [f^k(U_1) \cap U_2]\ x\ [g^k(V_1) \cap V_2] \neq \phi$$

Which means that $f \times g$ is sr- mixing.

We give some sufficient conditions for a product function to be semi-regular chaotic.

Theorem 4.6.6 Let $f: X \to X$ and $g: Y \to Y$ be semi-regular chaotic and topologically sr- mixing functions on topological spaces X and Y. Then $f \times g: X \times Y \to X \times Y$, is semi-regular chaotic.

Proof.: The function $f \times g$ has sr-dense periodic points by Lemma 4.6.1 and it is topologically sr- mixing by Lemma 4.6.5 and hence semi-regular transitive. Thus the two conditions of semi-regular chaos are satisfied.

4.7 CONCLUSION :

There are the following results:

Proposition 4.7.1

Semi - regular exact chaos ⇒ *semi-regular-mixing chaos* ⇒ *weak semi-regular - mixing chaos* ⇒ semi-regular *chaos.*

Proposition 4.7.2 if *f* is topologically sr-mixing then it is also semi-regular transitive but not conversely.

We can easily prove the following Proposition.

Proposition 4.7.3

If $f: X \to X$ and $g: Y \to Y$ are topologically *semi − regular* -conjugate. Then

(1) The function *f* is semi-regular exact if and only if *g* is semi-regular exact

(2) *f* is weakly sr-mixing if and only if *g* is weakly sr-mixing.

(3) f is semi-regular chaotic on X if and only if *g* is semi-regular chaotic in Y.

Lemma 4.7.4 Let (X, f), (Y, g) be topological systems. The set of periodic points of $f \times g$ is sr-dense in $X \times Y$ if and only if, for both of *f* and *g* , the sets of periodic points in X and Y are sr-dense in X, respectively Y.

Lemma 4.7.5 Let $f: X \to X$ and $g: Y \to Y$ be functions and assume that the product $f \times g$ is semi-regular transitive on $X \times Y$. Then the functions *f* and g are both semi-regular transitive on X and Y respectively.

Lemma 4.7.6 The product of two topologically sr-mixing functions is topologically sr- mixing.

Theorem4.7.7. Let $f: X \to X$ and $g: Y \to Y$ be semi-regular chaotic and topologically sr-mixing functions on topological spaces X and Y. Then $f \times g: X \times Y \to X \times Y$, is semi-regular chaotic.

Proposition4.7.8. Every transitive set is a δ-type transitive set, but not conversely, unless, the space is regular.

Proposition4.7.9. Every topologically mixing set is a δ-mixing set but not conversely, unless the space is regular.

Proposition4.7.10. Every topologically alpha-transitive set is transitive set bot not conversely.

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