International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395 -0056
Volume: 03 Issue: 02 | Feb-2016 www.irjet.net p-ISSN: 2395-0072

Numerical Analysis of MHD Flow of Fluid with One Porous Bounding Wall

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Abstract:

In this paper we have study the laminar flow of fluid in magnetic, viscous incompressible fluid flow between two parallel plates of a channel in which on is porous bounding wall and second is rigid bounding wall in the presence of magnetic field when the fluid is being withdrawn through both walls of a channel at same rate. A solution for the case of low Reynolds number Re (suction Reynolds number) and Magnetic field M is discussed. Expressions for the velocity components are obtained. The governing non-linear differential equations are solved using perturbation method analytically. Using matlab for the calculating the data of the graph. The graph of axial and radial velocity profiles have been drawn in different magnetic field and different height of the channel.

Key Words: - Channel flow, Porous medium, Permeable walls, Magnetic field and Slip coefficient

Nomenclature

h Height of the channel;
x The axial distance from the channel entrance;
y The coordinate axis perpendicular to the channel walls measured from the Non-porous wall;
u Velocity component in the x-direction;
u_0 Average velocity over the channel at channel inlet;
Re_w Wall Reynolds number, $\frac{v_w h}{v}$;
Re_{ent} Reynolds Number for flow entering the channel, $u0\ h/v$;
v Velocity component in the y- direction;
v_w Velocity of the fluid through the membrane;
V v/v_w;

Greek Symbols

ρ Solution density;
η Dimensionless variable in the $y-$ direction, y/h;
μ Viscosity;
B_0 The electromagnetic induction $B_0 = \mu_e H_0$
μ_e Magnetic permeability
H_0 The intensity of Magnetic field
M Magnetic Field
ν Kinematic viscosity;
α Surface characteristic of the membrane;
Δp Dimensionless pressure drop, $2[p(0,\eta) - p(x,\eta)]/(\rho {u_0}^2)$;
ϕ Slip coefficient, $\sqrt{k}/\alpha h$;
ψ Stream Function;

1. Introduction

The flow of fluid in porous medium is very important prevalent in nature and therefore the study of flow of fluid through porous medium has become of principle interest in many and engineering applications. Many research workers have investigated the steady, incompressible laminar flow of fluid in a channel with uniform porous bounding walls. Cox and King [4], studied the asymptotic solution of High order non-linear ordinary differential equation. In the earlier analysis majority have used no slip boundary conditions, but the experimental investigation reveals the existence of slip velocity at the porous bounding wall and is connected with presence of a thin layer of stream wise moving fluid just below the surface of the porous medium. Beavers et al [2], has studied the experiments on coupled parallel flow of fluid in a channel and a bounding porous medium.

Abou Zeid and Mohamed Y. [1], has studied Numerical treatment of heat and mass transfer of MHD flow of carreau fluid with Diffusion and chemical reaction through a Non-Darcy porous medium. Bujurke et al [3], has studied analysis of laminar flow in a channel with one porous bounding wall and find out the velocity of fluid for large Reynolds Number. M Hasseini et al [5], has studied Non-Newtonian fluid flow in an axisymmetric channel with porous wall and find out that increment of Reynolds number has similar effects on velocity components, both of them increases with increase of Reynolds number. At higher Reynolds number the maximum velocity point is shift to the solid wall where shear stress becomes larger as the Reynolds number grows.

O.D. Makinde & E Osalusi [6], has studied MHD steady flow in a channel with slip at the permeable boundaries, Robinson [7], Terrile [9, 10], Zuturka et al [12], have extended Berman's problem and Obtained solution for both small and large Reynolds number, Y. Takatsu and T. Musuka [11], have studied Turbulent phenomena in flow through porous media. In this paper we have studied the variation of magnetic field and height of channel of the flow of fluid with porous bounding wall.

1.1 Mathematical Formulation

Let us consider the coordinate system of fluid flow through porous medium with one porous bounding wall and one Rigid bounding wall in the presence of magnetic field. The density of the fluid be ρ, kinematic viscosity ν, channel length L and height of the channel be h, dimensionless variable η and M is magnetic field of the medium. The description of the problem is shown in Fig 1. The axial coordinate measured from the channel entrance is denoted by x *and* y the coordinate axis perpendicular to the channel walls measured from the non-porous wall, the value of u and v represent the velocity component in x and y −directions respectively.

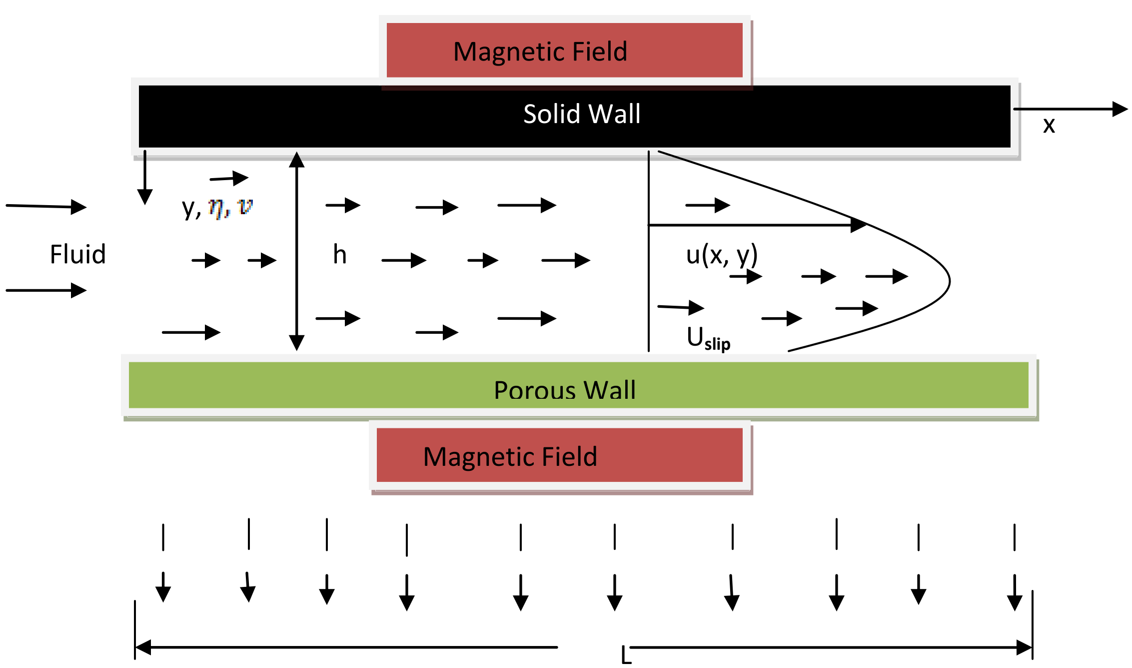


Fig. 1. The coordinate system used in the solution of the two dimensional steady State Navier-Stokes equation in Presence of Magnetic field

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \textbf{(1)}$$

The equation of momentum is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B^2}{\rho}u \quad \textbf{(2)}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma B^2}{\rho}v \quad \textbf{(3)}$$

Let the dimensionless variable be $\eta=\frac{y}{h}$ and then the equation 1 -3 becomes

$$\frac{\partial u}{\partial x}+\frac{1}{h}\frac{\partial v}{\partial \eta}=0 \quad \textbf{(4)}$$

$$u\frac{\partial u}{\partial x}+\frac{v}{h}\frac{\partial u}{\partial \eta}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \eta^2}\right)-\frac{\sigma B^2}{\rho}u \quad \textbf{(5)}$$

$$u\frac{\partial v}{\partial x}+\frac{v}{h}\frac{\partial v}{\partial \eta}=-\frac{1}{\rho h}\frac{\partial p}{\partial \eta}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \eta^2}\right)-\frac{\sigma B^2}{\rho}v \quad \textbf{(6)}$$

Where υ is the kinematic viscosity, σ the density of the fluid, μ the coefficient of viscosity and p the pressure.

We introducing the stream function $\psi(x,y)$. We know that

$$u=\frac{\partial \psi}{\partial y} \quad \& \quad v=-\frac{\partial \psi}{\partial x} \quad \textbf{(7)}$$

Introducing the dimensionless variable in equation (7) we get

$$u=\frac{1}{h}\frac{\partial \psi}{\partial \eta} \quad \& \quad v=-\frac{\partial \psi}{\partial x} \quad \textbf{(8)}$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi(x,\ \eta)=(h\,u_0-v_w\ x)\ f(\eta) \quad \textbf{(9)}$$

From Navier Stokes equation (5) & (6) eliminating pressure 'p' and then using equation (8) & (9) we get the result

$$\frac{\partial}{\partial \eta}\left[-\frac{v_w}{h}\,(f')^2+\frac{v_w}{h}\,f\,f''+\frac{\sigma B^2}{\rho}-\frac{\upsilon}{h^2}\,f'''\right]=0 \quad \textbf{(10)}$$

Or

Where, σ is the electric conductivity and $B=\mu_e\,H_0$, μ_e being the magnetic permeability.

The boundary conditions are

$u(x,h)\ =\ 0,$

$u(x,-h)=0, v(x,h)=\ V\ and\ v(x,-h)=-V$

Where V is the velocity of suction at the walls of the channel.

$$\left[\frac{v_w}{h}\left(f'f''-f\,f'''-\frac{B^2\sigma\,h}{\rho\ v_w}f''\right)+\frac{\upsilon}{h^2}\,f^{IV}\right]=0 \quad \textbf{(11)}$$

Or

$$Re\left[f'f''-\,f\,f'''-\frac{M^2}{h}f''\right]+\ f^{IV}=0 \quad \textbf{(12)}$$

Equation (12) is one of the Falkner – Skan family of equations.

New set of boundary conditions are

$$f\ =0, \quad f'=0 \quad at \quad \eta=\ 0 \quad \textbf{(13)}$$

$$f\ =1, \quad f'\ =\ -\phi\,f'' \quad at \quad \eta=1 \quad \textbf{(14)}$$

1.2 Method of Solution

The non-linear ordinary differential equation (12), we find the analytical solution of the differential equation subjection to the condition (13) & (14) must in general be integrated numerically. However for special case when Re and h are small, the approximate analytical results can be obtained by use of a regular perturbation approach. In this situation f may be expanded in the form

$$\text{Let} \quad f(\eta)=\sum_{n=0}^{\infty}Re^n f_n(\eta) \quad \textbf{(15)}$$

Or

$$f(\eta)\ =f_0(\eta)+\mathrm{Re}f_1(\eta)+\mathrm{Re}^2f_2(\eta)+\mathrm{Re}^3f_3(\eta)+ \quad \textbf{(16)}$$

$$f_n^{IV} = \sum_{r=0}^{n-1}[f_r f'''_{n-1-r} - f'_r f''_{n-1-r}], \quad n = 1, 2, 3, \ldots \tag{17}$$

With boundary conditions are

$$f_n = 0 \;\; at \;\; \eta = 0 \;\; \forall \; n \geq 0$$

$$f'_n = 0 \;\; at \;\; \eta = 0 \;\; \forall \; n \geq 0 \tag{18}$$

$$f_0 = 1, \; f_n = 0 \;\; at \;\; \eta = 1 \;\; \forall \; n \geq 1$$

$$f'_n + \phi f''_n = 0 \quad at \quad \eta = 1 \;\; \forall \; n \geq 0$$

From equation (12) & (16) using the above same boundary conditions of equation (18)

We get

$$f_0 = -\frac{2(1+\phi)}{(1+4\phi)}\eta^3 + \frac{3(1+2\phi)}{(1+4\phi)}\eta^2 \tag{19}$$

$$f_1 = -\frac{2(1+\phi)^2}{35(1+4\phi)^2}\eta^7 + \frac{(1+\phi)(1+2\phi)}{5(1+4\phi)^2}\eta^6 - \frac{3(1+2\phi)^2}{10(1+4\phi)^2}\eta^5 - \frac{M^2(1+\phi)}{10h(1+4\phi)}\eta^5 + \frac{M^2(1+2\phi)}{4h(1+4\phi)}\eta^4 - \frac{\left\{\left(\frac{35}{h}M^2-49\right)+\left(\frac{378}{h}M^2-458\right)\phi+\left(\frac{1400}{h}M^2-1336\right)\phi^2+\left(\frac{1792}{h}M^2-1368\right)\phi^3\right\}}{210(1+4\phi)^3}\eta^3 - \frac{\left\{\left(\frac{7}{h}M^2-32\right)+\left(\frac{84}{h}M^2-352\right)\phi+\left(\frac{448}{h}M^2-1112\right)\phi^2+\left(\frac{896}{h}M^2-1296\right)\phi^3\right\}}{140(1+4\phi)^3}\eta^2 \tag{20}$$

$$f_2 = \frac{4(1+\phi)^3}{5775(1+4\phi)^3}\eta^{11} - \frac{11(1+\phi)^2(1+2\phi)}{1050(1+4\phi)^3}\eta^{10} + \frac{\left\{\left(\frac{18}{h}M^2+72\right)+\left(\frac{108}{h}M^2+360\right)\phi+\left(\frac{162}{h}M^2+576\right)\phi^2+\left(\frac{72}{h}M^2+288\right)\phi^3\right\}}{15120(1+4\phi)^3}\eta^9 + \frac{\left\{\left(\frac{18}{h}M^2-9\right)+\left(\frac{126}{h}M^2-54\right)\phi+\left(\frac{324}{h}M^2-108\right)\phi^2+\left(\frac{432}{h}M^2-72\right)\phi^3\right\}}{1680(1+4\phi)^3}\eta^8 -$$

$$\frac{\left\{\left(\frac{1400}{h}M^2+392\right)+\left(\frac{1736}{h}M^2+4056\right)\phi+\left(\frac{7616}{h}M^2+14352\right)\phi^2+\left(\frac{14784}{h}M^2+21632\right)\phi^3+\left(\frac{12544}{h}M^2+10944\right)\right\}}{29400(1+4\phi)^4}\eta^7$$

$$-\frac{M^4(1+\phi)}{420h^2(1+4\phi)}\eta^7 + \frac{M^4(1+2\phi)}{120h^2(+4\phi)}\eta^6$$

$$+\frac{\left\{\left(\frac{168}{h}M^2-102\right)+\left(\frac{2142}{h}M^2-1032\right)\phi+\left(\frac{9744}{h}M^2-4728\right)\phi^2+\left(\frac{19488}{h}M^2-9792\right)\phi^3+\left(\frac{16128}{h}M^2-8640\right)\phi^4\right\}}{12600(1+4\phi)^4}\eta^6 +$$

$$\frac{6\left\{\left(\frac{7}{h}M^2-32\right)+\left(\frac{98}{h}M^2-416\right)\phi+\left(\frac{616}{h}M^2-1816\right)\phi^2+\left(\frac{1792}{h}M^2-3520\right)\phi^3+\left(\frac{1792}{h}M^2-2592\right)\phi^4\right\}}{700(1+4\phi)^4}\eta^5 -$$

$$\frac{M^2\left\{\left(\frac{36}{h}M^2-49\right)+\left(\frac{378}{h}M^2-458\right)\phi+\left(\frac{1400}{h}M^2-1336\right)\phi^2+\left(\frac{1792}{h}M^2-1368\right)\phi^3\right\}}{8400h(1+4\phi)^3}\eta^5 -$$

$$\frac{M^2\left\{\left(\frac{7}{h}M^2-32\right)+\left(\frac{84}{h}M^2-352\right)\phi+\left(\frac{448}{h}M^2-1112\right)\phi^2+\left(\frac{896}{h}M^2-1296\right)\phi^3\right\}}{1680h\,(1+4\phi)^3}\eta^4 \quad +$$

$$\frac{1}{6}\left\{\frac{\left(280+4200\phi+14440\phi^2+17400\phi^3+6880\phi^4\right)}{1925(1+4\phi)^4}-\right.$$

$$\frac{\left\{\left(\frac{7140}{h}M^2-13503\right)+\left(\frac{14145}{h}M^2-250413\right)\phi+\left(\frac{107206}{h}M^2-1744898\right)\phi^2+\left(\frac{4050648}{h}M^2-5777028\right)\phi^3+\left(\frac{7928526}{h}M^2-9439032\right)\phi^4+\left(\frac{6613376}{h}M^2-6063584\right)\phi^5\right\}}{9800(1+4\phi)^5}-\frac{3M^4(3+27\phi+\ldots}{70h^2(1+4\phi\ldots}$$

$$\left.\frac{M^2\left\{\left(\frac{175}{h}M^2-467\right)+\left(\frac{2954}{h}M^2-7376\right)\phi+\left(\frac{18684}{h}M^2-40972\right)\phi^2+\left(\frac{61936}{h}M^2-96712\right)\phi^3+\left(\frac{77056}{h}M^2-89424\right)\phi^4\right\}}{1400h\,(1+4\phi)^4}\right\}\eta^3+\frac{1}{2}\left\{-\frac{\left(244+3904\phi+13636\phi^2+16536\phi^3+7200\phi^4\right)}{5775(1+4\phi)^4}\right.-$$

$$\frac{\left\{\left(\frac{35574}{h}M^2-113995\right)+\left(\frac{743946}{h}M^2-2246442\right)\phi+\left(\frac{6162828}{h}M^2-16235264\right)\phi^2+\left(\frac{26313084}{h}M^2-54936480\right)\phi^3+\left(\frac{57636768}{h}M^2-91658448\right)\phi^4+\left(\frac{49936320}{h}M^2-61212960\right)\phi^5\right\}}{88200(1+4\phi)^5}+$$

$$\left.\frac{M^4(13+106\phi+48\phi^2)}{420\,h^2(1+4\phi)^2}+\frac{M^2\left\{\left(\frac{105}{h}M^2-258\right)+\left(\frac{1876}{h}M^2-4322\right)\phi+\left(\frac{12852}{h}M^2-25204\right)\phi^2+\left(\frac{41104}{h}M^2-61280\right)\phi^3+\left(\frac{51968}{h}M^2-58032\right)\phi^4\right\}}{4200h(1+4\phi)^4}\right\}\eta^2+\cdots$$

(21)

Now putting these values in equations (16) we get

$$f(\eta) = -\frac{2(1+\phi)}{(1+4\phi)}\eta^3 + \frac{3(1+2\phi)}{(1+4\phi)}\eta^2 +$$

$$Re\left[-\frac{2(1+\phi)^2}{35(1+4\phi)^2}\eta^7+\frac{(1+\phi)(1+2\phi)}{5(1+4\phi)^2}\eta^6-\frac{3(1+2\phi)^2}{10(1+4\phi)^2}\eta^5-\frac{M^2(1+\phi)}{10h(1+4\phi)}\eta^5+\frac{M^2(1+2\phi)}{4h(1+4\phi)}\eta^4-\right.$$
$$\frac{\left\{\left(\frac{35}{h}M^2-49\right)+\left(\frac{378}{h}M^2-458\right)\phi+\left(\frac{1400}{h}M^2-1336\right)\phi^2+\left(\frac{1792}{h}M^2-1368\right)\phi^3\right\}}{210(1+4\phi)^3}\eta^3-\frac{\left\{\left(\frac{7}{h}M^2-32\right)+\left(\frac{84}{h}M^2-352\right)\phi+\left(\frac{448}{h}M^2-1112\right)\phi^2+\left(\frac{896}{h}M^2-1296\right)\phi^3\right\}}{140(1+4\phi)^3}+$$

$$Re^2\left[\frac{4(1+\phi)^3}{5775(1+4\phi)^3}\eta^{11}-\frac{4(1+\phi)^2(1+2\phi)}{1050(1+4\phi)^3}\eta^{10}+\frac{\left\{\left(\frac{18}{h}M^2+72\right)+\left(\frac{108}{h}M^2+360\right)\phi+\left(\frac{162}{h}M^2+576\right)\phi^2+\left(\frac{72}{h}M^2+288\right)\phi^3\right\}}{15120(1+4\phi)^3}\eta^9+\right.$$
$$\frac{\left\{\left(\frac{18}{h}M^2-9\right)+\left(\frac{126}{h}M^2-54\right)\phi+\left(\frac{324}{h}M^2-108\right)\phi^2+\left(\frac{432}{h}M^2-72\right)\phi^3\right\}}{1680(1+4\phi)^3}\eta^8-$$
$$\frac{\left\{\left(\frac{1400}{h}M^2+392\right)+\left(\frac{1786}{h}M^2+4056\right)\phi+\left(\frac{7616}{h}M^2+14352\right)\phi^2+\left(\frac{14784}{h}M^2+21632\right)\phi^3+\left(\frac{12544}{h}M^2+10944\right)\right\}}{29400(1+4\phi)^4}\eta^7-\frac{M^4(1+\phi)}{420h^2(1+4\phi)}\eta^7+\frac{M^4(1+2\phi)}{120h^2(+4\phi)}\eta^6$$
$$\frac{6\left\{\left(\frac{7}{h}M^2-32\right)+\left(\frac{98}{h}M^2-416\right)\phi+\left(\frac{616}{h}M^2-1816\right)\phi^2+\left(\frac{1792}{h}M^2-3520\right)\phi^3+\left(\frac{1792}{h}M^2-2592\right)\phi^4\right\}}{700(1+4\phi)^4}\eta^5-$$
$$\frac{M^2\left\{\left(\frac{7}{h}M^2-32\right)+\left(\frac{84}{h}M^2-352\right)\phi+\left(\frac{448}{h}M^2-1112\right)\phi^2+\left(\frac{896}{h}M^2-1296\right)\phi^3\right\}}{1680h(1+4\phi)^3}\eta^4+$$
$$\frac{1}{6}\left\{\frac{(280+4200\phi+14440\phi^2+17400\phi^3+6880\phi^4)}{1925(1+4\phi)^4}-\right.$$
$$\frac{\left\{\left(\frac{7140}{h}M^2-13503\right)+\left(\frac{14145}{h}M^2-250413\right)\phi+\left(\frac{107206}{h}M^2-1744898\right)\phi^2+\left(\frac{4050648}{h}M^2-5777028\right)\phi^3+\left(\frac{7928526}{h}M^2-9439032\right)\phi^4+\left(\frac{6613376}{h}M^2-6063584\right)\phi^5\right\}}{9800(1+4\phi)^5}-\frac{3M^4(3+27\phi}{70h^2(1+}$$
$$\left.\frac{M^2\left\{\left(\frac{105}{h}M^2-258\right)+\left(\frac{1876}{h}M^2-4322\right)\phi+\left(\frac{12852}{h}M^2-25204\right)\phi^2+\left(\frac{41104}{h}M^2-61280\right)\phi^3+\left(\frac{51968}{h}M^2-58032\right)\phi^4\right\}}{4200h(1+4\phi)^4}\right\}\eta^3+\frac{1}{2}\left\{-\frac{(244+3904\phi+13636\phi^2+16536\phi^3+7200\phi^4)}{5775(1+4\phi)^4}+\right.$$
$$\frac{\left\{\left(\frac{35574}{h}M^2-113995\right)+\left(\frac{743946}{h}M^2-2246442\right)\phi+\left(\frac{6162828}{h}M^2-16235264\right)\phi^2+\left(\frac{26313084}{h}M^2-54936480\right)\phi^3+\left(\frac{57636768}{h}M^2-91658448\right)\phi^4+\left(\frac{49936320}{h}M^2-61212960\right)\phi^5\right\}}{88200(1+4\phi)^5}+$$
$$\left.\left.\frac{M^4(13+106\phi+48\phi^2)}{420h^2(1+4\phi)^2}+\frac{M^2\left\{\left(\frac{105}{h}M^2-258\right)+\left(\frac{1876}{h}M^2-4322\right)\phi+\left(\frac{12852}{h}M^2-25204\right)\phi^2+\left(\frac{41104}{h}M^2-61280\right)\phi^3+\left(\frac{51968}{h}M^2-58032\right)\phi^4\right\}}{4200h(1+4\phi)^4}\right\}\eta^2\right]+\ldots \quad (22)$$

Let us assume the height of the channel is unity then by putting h = 1 in the above equation (22) we get the differential equation (22) in the form of

$$f(\eta)=-\frac{2(1+\phi)}{(1+4\phi)}\eta^3+\frac{3(1+2\phi)}{(1+4\phi)}\eta^2+$$

$$Re\left[-\frac{2(1+\phi)^2}{35(1+4\phi)^2}\eta^7+\frac{(1+\phi)(1+2\phi)}{5(1+4\phi)^2}\eta^6-\frac{3(1+2\phi)^2}{10(1+4\phi)^2}\eta^5-\frac{M^2(1+\phi)}{10(1+4\phi)}\eta^5+\frac{M^2(1+2\phi)}{4(1+4\phi)}\eta^4-\frac{\{(35M^2-49)+(378M^2-458)\phi+(1400M^2-1336)\phi^2+(1792M^2-1368)\phi^3\}}{210(1+4\phi)^3}\eta^3-\frac{\{(7M^2-32)+(84M^2-352)\phi+(448M^2-1112)\phi^2+(896M^2-1296)\phi^3\}}{140(1+4\phi)^3}\eta^2\right]+$$

$$Re^2\left[\frac{4(1+\phi)^3}{5775(1+4\phi)^3}\eta^{11}-\frac{4(1+\phi)^2(1+2\phi)}{1050(1+4\phi)^3}\eta^{10}+\frac{\{(18M^2+72)+(108M^2+360)\phi+(162M^2+576)\phi^2+(72M^2+288)\phi^3\}}{15120(1+4\phi)^3}\eta^9+\frac{\{(18M^2-9)+(126M^2-54)\phi+(324M^2-108)\phi^2+(432M}{1680(1+4\phi)^3}\right.$$

$$\frac{\{(140M^2+392)+(1736M^2+4056)\phi+(7616M^2+14352)\phi^2+(14784M^2+21632)\phi^3+(12544M^2+10944)\}}{29400(1+4\phi)^4}\eta^7-\frac{M^4(1+\phi)}{420(1+4\phi)}\eta^7+\frac{M^4(1+2\phi)}{120(+4\phi)}\eta^6+$$

$$\frac{\{(168M^2-102)+(2142M^2-1032)\phi+(9744M^2-4728)\phi^2+(19488M^2-9792)\phi^3+(16128M^2-8640)\phi^4\}}{12600(1+4\phi)^4}\eta^6+$$

$$\frac{6\{(7M^2-32)+(98M^2-416)\phi+(616M^2-1816)\phi^2+(1792M^2-3520)\phi^3+(1792M^2-2592)\phi^4\}}{700(1+4\phi)^4}\eta^5-\frac{M^2\{(35M^2-49)+(378M^2-458)\phi+(1400M^2-1336)\phi^2+(1792M^2-1368)\phi^3\}}{8400(1+4\phi)^3}\eta^5-$$

$$\frac{M^2\{(7M^2-32)+(84M^2-352)\phi+(448M^2-1112)\phi^2+(896M^2-1296)\phi^3\}}{1680(1+4\phi)^3}\eta^4+$$

$$\frac{1}{6}\left\{\frac{(280+4200\phi+14440\phi^2+17400\phi^3+6880\phi^4)}{1925(1+4\phi)^4}-\right.$$

$$\frac{\{(7140M^2-13503)+(14145M^2-250413)\phi+(107206M^2-1744898)\phi^2+(4050648M^2-5777028)\phi^3+(7928526M^2-9439032)\phi^4+(6613376M^2-6063584)\phi^5\}}{9800(1+4\phi)^5}-\frac{3M^4(3+27\phi+52\phi^2}{70(1+4\phi)^3}$$

$$\left.\frac{M^2\{(175M^2-467)+(2954M^2-7376)\phi+(18684M^2-40972)\phi^2+(61936M^2-96712)\phi^3+(77056M^2-89424)\phi^4\}}{1400(1+4\phi)^4}\right\}\eta^3+$$

$$\frac{1}{2}\left\{-\frac{(244+3904\phi+13636\phi^2+16536\phi^3+7200\phi^4)}{5775(1+4\phi)^5}+\right.$$

$$\frac{\{(35574M^2-113995)+(743946M^2-2246442)\phi+(6162828M^2-16235264)\phi^2+(26313084M^2-54936480)\phi^3+(57636768M^2-91658448)\phi^4+(49936320M^2-61212960)\phi^5\}}{88200(1+4\phi)^5}+\frac{M^4(1}{4}$$

$$\left.\left.\frac{M^2\{(105M^2-258)+(1876M^2-4322)\phi+(12852M^2-25204)\phi^2+(41104M^2-61280)\phi^3+(51968M^2-58032)\phi^4\}}{4200(1+4\phi)^4}\right\}\eta^2\right]+\cdots$$

2. Result and discussion

The non-linear differential equation (12) subject to (13) must in be integrated by analytic procedure to use of a perturbation approach which is consider as power series in the form of (15, 16). However for the special case when Reynolds number Re and slip coefficient Ø are small, approximate analytical results can be obtained, which is solution of the differential equation (12) is equation (22), for unit channel height (h = 1) the solution of the differential equation is (23). From equation (22, 23) we form the Table 1, Table 2 and Table 3, which gives the result that increase of magnetic field the velocity of fluid flow is increases with increase of magnetic field, whereas decreases with increase of channel height at constant Reynolds number, slip coefficient.

Since the fluid is incompressible and viscous, the numerical analysis is suitable for fluid flow. It is important to note the increase of magnetic field represent an increase in the fluid injection. Reynolds number for viscous fluid flow shown in the figures below gave the graph between dimensionless variable and velocity of fluid flow in the different magnetic field. From above graph (Fig. 2 a, b, c, d) we find the result that increase of magnetic field, the velocity of fluid flow is increases slowly then sharply at different constant Reynolds number, slip coefficient and height of channel. The same results are in the figure 3 a & b, in this we take the channel height is 2 unit, whereas at high magnetic field (M>5), velocity of fluid flow is decreases. From graph (Fig. 2 b) we get that velocity of fluid flow is increases with increase of magnetic field but when we increase the magnetic field more than 5 the nature of fluid flow is changes it is decreases with increase of magnetic field.

From graph (Fig. 4 a, b, c, d) we found that the increase of channel height h, the velocity of fluid flow is decreases at constant slip coefficient Reynolds number & magnetic field. It is approximate decreases as one time of the previous velocity of fluid in magnetic field.

Table 2.1: - Velocity profile for constant Reynolds Number, slip coefficient & height of the channel

η		0	.4	.8	1.2	1.6	2.0	2.4
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 0	0	-0.2561	0.3683	1.8170	3.7558	5.3711	4.5916
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 0.5	0	-0.2562	0.3790	1.8275	3.8039	5.5419	5.1529
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 1	0	-0.2565	0.3806	1.8547	3.9357	6.0290	6.7895
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 1.5	0	-0.2562	0.3811	1.8849	4.1140	6.7572	9.3591
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 2	0	-0.2545	0.3771	1.8955	4.2766	7.6007	12.6248
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 2.5	0	-0.2498	0.3641	1.8545	4.3365	8.3838	16.2549
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 3	0	-0.2404	0.3359	1.7214	4.1819	8.8803	19.8228
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 3.5	0	-0.2241	0.2853	1.4460	3.6761	8.8139	22.8074
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 4	0	-0.1983	0.2037	0.9695	2.6575	7.8580	24.8074
$f(\eta)$	$Re = 0.4\ \phi = 0.05$ & $h = 1$ & M = 4.5	0	-0.1598	0.0811	0.2238	0.9399	5.6357	24.4673

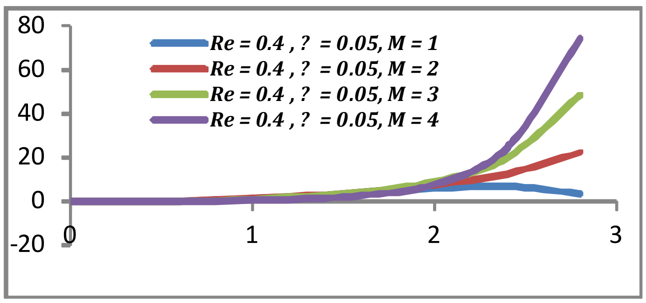
Table 2.2: - Velocity profile for constant Reynolds Number, slip coefficient & height of the channel

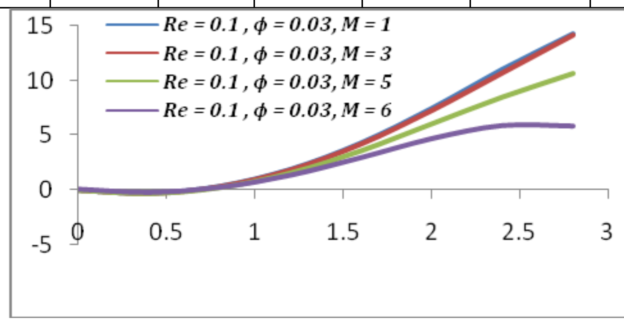
η		0	0.4	0.8	1.2	1.6	2.0	2.4	2.8
$f(\eta)$	$Re = 0.1\ \phi = 0.03$ & $h = 1$ & M = 0	0	-0.2776	0.3619	1.9063	4.3127	7.4582	11.0099	14.1711
$f(\eta)$	$Re = 0.1\ \phi = 0.03$ & $h = 1$ & M = 1	0	-0.2787	0.3584	1.9017	4.3083	7.4508	11.0029	14.2296
$f(\eta)$	$Re = 0.1\ \phi = 0.03$ & $h = 1$ & M = 2	0	-0.2816	0.3471	1.8836	4.2830	7.4044	10.9349	14.3050
$f(\eta)$	$Re = 0.1\ \phi = 0.03$ & $h = 1$ & M = 3	0	-0.2857	0.3262	1.8386	4.2006	7.2452	10.6646	14.0968
$f(\eta)$	$Re = 0.1\ \phi = 0.03$ & $h = 1$ & M = 4	0	-0.2902	0.2925	1.7446	4.0006	6.8506	9.9563	13.1044
$f(\eta)$	$Re = 0.1\ \phi = 0.03$ & $h = 1$ & M = 5	0	-0.2934	0.2414	1.5709	3.5984	6.0490	8.4804	10.6266
$f(\eta)$	$Re = 0.1\ \phi = 0.03$ & $h = 1$ & M = 6	0	-0.2938	0.1673	1.2776	2.8854	4.6197	5.8128	5.7621
$f(\eta)$	$Re = 0.1\ \phi = 0.03$ & $h = 1$ & M = 7	0	-0.2889	0.0630	0.8161	1.7286	2.2928	1.4354	-2.5906

Table 2.3: - Velocity profile for constant Reynolds Number, slip coefficient & Variable Magnetic field and variable height of the channel

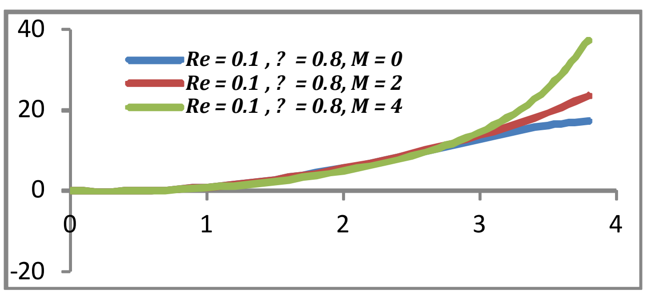
η		0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6
$f(\eta)$	$Re = 0.1, \phi = 0.03$, M = 1, h = 1	0	-0.2787	0.3584	1.9017	4.3083	7.4508	11.0290	14.2296	15.6873	12.9141
$f(\eta)$	$Re = 0.1, \phi = 0.03$, M = 1, h = 2	0	-0.2781	0.3602	1.9041	4.3107	7.4550	11.0074	14.0024	15.4913	12.1458
$f(\eta)$	$Re = 0.1, \phi = 0.03$, M = 1, h = 3	0	-0.2780	0.3608	1.9049	4.3114	7.4562	11.0084	14.1925	15.4238	11.8844
$f(\eta)$	$Re = 0.1, \phi = 0.03$, M = 1, h = 4	0	-0.2779	0.3610	1.9053	4.3118	7.4567	11.0089	14.1873	15.3896	11.7527
$f(\eta)$	$Re = 0.1, \phi = 0.03$, M = 1, h = 5	0	-0.2778	0.3612	1.9055	4.3120	7.4570	11.0091	14.1841	15.3690	11.6734
$f(\eta)$	$Re = 0.1, \phi = 0.03$, M = 1, h = 10	0	-0.2777	0.3616	1.9059	5.3123	7.4576	11.0095	14.1777	15.3274	11.5140
$f(\eta)$	$Re = 0.1, \phi = 0.03$, M = 1, h = 20	0	-0.2777	0.3617	1.9061	4.3125	7.4579	11.0097	14.1744	15.3065	11.4339
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 2, h = 1	0	-0.0472	0.4674	1.5530	3.3139	6.1123	11.0162	20.7681	41.6336	86.6757
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 2, h = 2	0	-0.0454	0.5015	1.6396	3.3631	5.7029	8.8715	13.5964	21.7856	37.7852
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 2, h = 3	0	-0.0445	0.5095	1.6510	3.3327	5.4733	8.0011	10.9687	14.8093	20.8986
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 2, h = 4	0	-0.0440	0.5129	1.6535	3.3086	5.3410	7.5369	9.6104	11.2535	12.3447
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 2, h = 5	0	-0.0437	0.5148	1.6539	3.2914	5.2561	7.2490	8.7811	9.0984	7.1770
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 2, h = 10	0	-0.0430	0.5180	1.6524	3.2506	5.0736	6.6522	7.0907	4.7396	-3.2381
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 2, h = 20	0	-0.0426	0.5194	1.6505	3.2271	4.9761	6.3433	6.2295	2.5359	-8.4854
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 3, h = 1	0	-0.0462	0.3174	0.9939	2.2679	5.3022	13.3191	34.0289	84.1588	197.2908
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 3, h = 2	0	-0.0474	0.4566	1.5191	3.2686	6.1491	11.4431	22.3945	46.3423	98.4836
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 3, h = 3	0	-0.0464	0.4863	1.6061	3.3649	5.9600	10.0312	17.3156	31.9124	62.5622
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 3, h = 4	0	-0.0457	0.4980	1.6330	3.3685	5.7770	9.1777	14.5512	24.3553	44.0416
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 3, h = 5	0	-0.0451	0.5041	1.6439	3.3565	5.6389	8.6185	12.8205	19.7117	32.7502
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 3, h = 10	0	-0.0438	0.5139	1.6538	3.3003	5.2991	7.3938	9.1971	10.1780	9.7642
$f(\eta)$	$Re = 0.4, \phi = 0.8$, M = 3, h = 20	0	-0.0431	0.5176	1.6528	3.2562	5.0973	6.7283	7.3043	5.2880	-1.9304

f(η)	Re = 0.4, ϕ = 0.8, M = 4 , h = 2	0	-0.0470	0.3548	1.1449	2.5826	5.6738	13.2081	31.9103	76.4646	176.4948
f(η)	Re = 0.4, ϕ = 0.8, M = 4 , h = 3	0	-0.0477	0.4365	1.4518	3.1639	6.1524	12.0576	24.9564	53.9647	117.7949
f(η)	Re = 0.4, ϕ = 0.8, M = 4 , h = 4	0	-0.0472	0.4674	1.5530	3.3139	6.1123	11.0162	20.7681	41.6336	86.6757
f(η)	Re = 0.4, ϕ = 0.8, M = 4 , h = 5	0	-0.0466	0.4828	1.5970	3.3589	5.9988	10.2422	18.0274	33.8891	67.4386
f(η)	Re = 0.4, ϕ = 0.8, M = 4, h =10	0	-0.0448	0.5065	1.6475	3.3477	5.5707	8.3586	12.0340	17.6214	27.6886
f(η)	Re = 0.4, ϕ = 0.8, M = 5 , h = 20	0	-0.0437	0.5148	1.6539	3.2914	5.2561	7.24901	8.7811	9.0984	7.1770
f(η)	Re = 0.4, ϕ = 0.8, M = 5 , h = 2	0	-0.0408	0.1572	0.3113	0.7511	3.1767	12.3310	39.3433	107.8957	264.8520
f(η)	Re = 0.4, ϕ = 0.8, M = 5 , h = 3	0	-0.0468	0.3428	1.0968	2.4835	5.5616	13.2645	32.6461	79.0744	183.5005
f(η)	Re = 0.4, ϕ = 0.8, M = 5 , h = 4	0	-0.0477	0.4114	1.3620	3.0064	6.0719	12.5933	27.5608	62.0242	138.5052
f(η)	Re = 0.4, ϕ = 0.8, M = 5 , h = 5	0	-0.0476	0.4449	1.4804	3.2102	6.1598	11.8263	23.9538	50.9495	110.1257
f(η)	Re = 0.4, ϕ = 0.8, M = 5 , h = 10	0	-0.0459	0.4944	1.6253	3.3706	5.8445	9.4732	15.4893	26.8997	50.2566
f(η)	Re = 0.4, ϕ = 0.8, M = 5 , h = 20	0	-0.0444	0.5104	1.6518	3.3272	5.4413	7.8869	10.6319	13.9246	18.7676

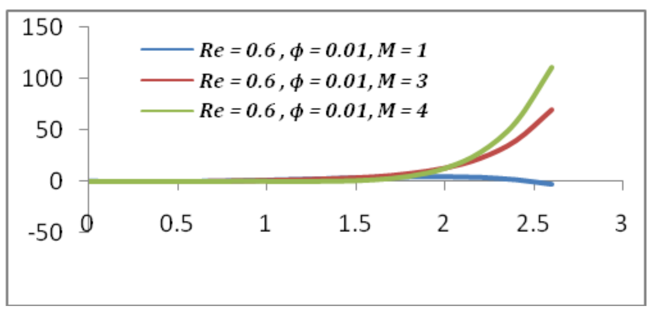




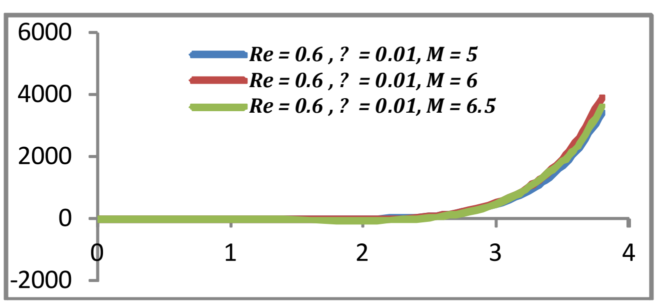
(a) (b)



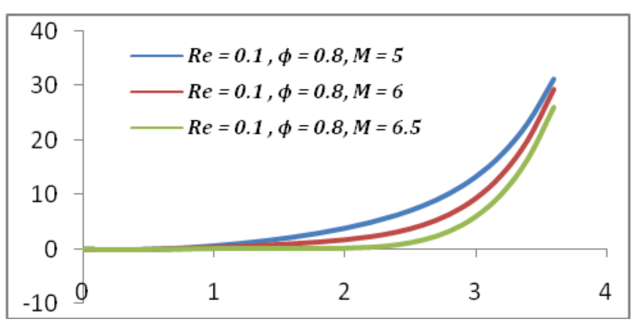
(c)



(d)

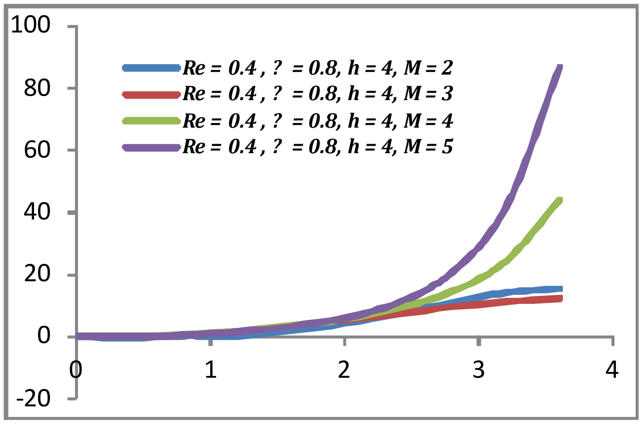


(e)

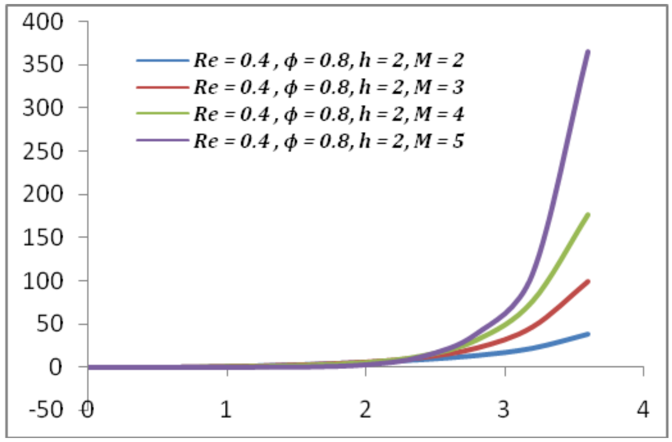


(f)

Fig. 2. Graph between dimensionless variable η and velocity of fluid flow $f(\eta)$ in different magnetic field at constant Reynolds number Re and slip coefficient ϕ.

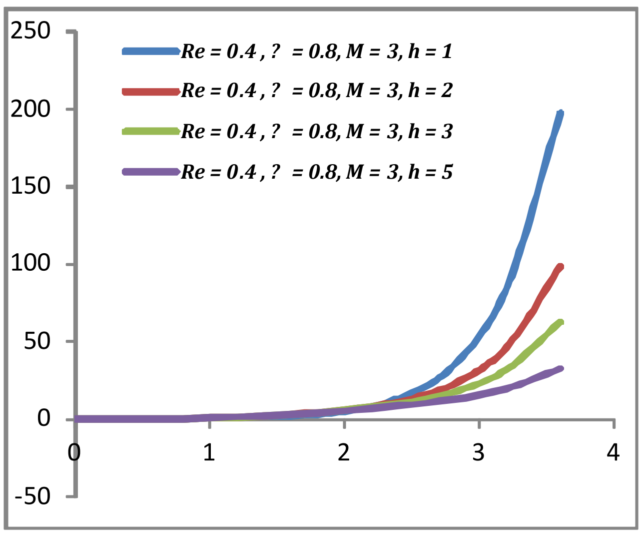


(a)

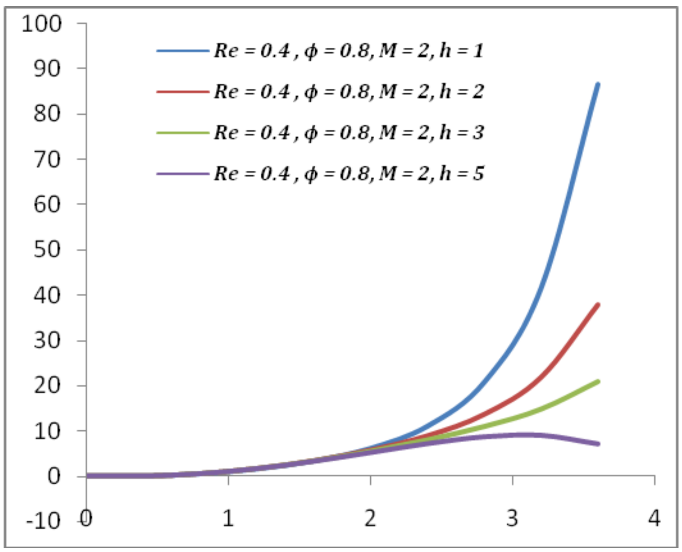


(b)

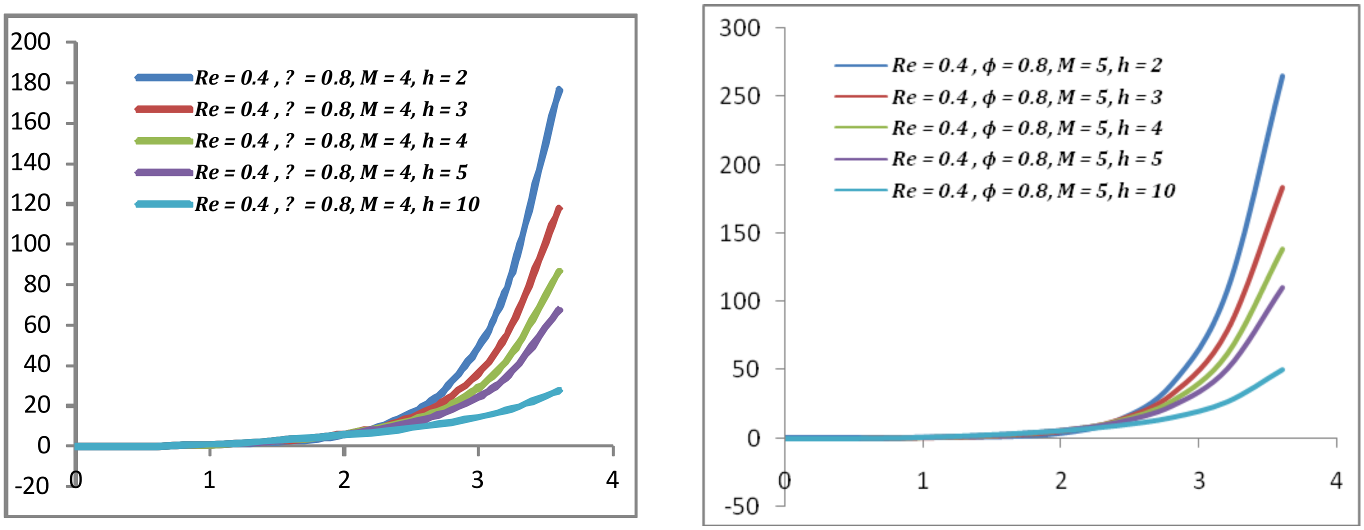
Fig. 3. Graph between dimensionless variable η and velocity of fluid flow $f(\eta)$ in different magnetic field at constant Reynolds number Re, slip coefficient ϕ and height of the channel



(a)



(b)



(c) (d)

Fig. 4. Graph between dimensionless variable η and velocity of fluid flow $f(\eta)$ in different height of channel at constant magnetic field M, Reynolds number Re and slip coefficient ϕ.

3. Conclusion

In this paper we investigated the combined effect of magnetic field and height of channel on the steady flow of fluid of conducting viscous incompressible fluid in a channel with porous bounding wall. Our results revealed that the velocity of fluid flow is reduced both increase of magnetic field and increase of height of channel. When the magnetic field M increases in the in this range 0<M<4, The velocity of fluid flow is increases with increase of magnetic field and when we increase the magnetic field more than 5 the velocity of fluid is start of to decrease. We also notice when we increase the slip coefficient $\emptyset$ as compared the Reynolds number, the velocity of fluid is increases, the pressure of fluid flow reversal near the wall due to wall slip. Generally, wall skin friction increases with suction and decreases with injection, however, wall slip, height of channel & magnetic field also have great influence of wall skin friction.

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