

Free Vibrational Analysis of Cracked and Un-cracked Cantilever Beam

Rupali Ramesh Rao Patil¹, Prof. Dheeraj Verma²

¹P.G.Student, Mechanical Engineering Department,

²Associate Prof. Mechanical Engineering Department,

Padm. Dr. V. B. Kolte College of Engineering, Malkapur. Maharashtra, India.

Abstract : Early detection of damage is of special concern for engineering structures. A comparatively recent development for the diagnosis of structural crack location and size by using the finite element method and Fuzzy logics techniques has improved. The traditional methods of damage detection include visual inspection or instrumental evaluation. A method based on measurement of natural frequencies is presented for detection of the location and size of a crack in a cantilever beam. Numerical and programming in MATLAB is used for solving the Euler equation for un-crack beam to obtain first three natural frequencies of different modes of vibration considering boundary conditions for the beam.

Here in this, ANSYS software package is used for finite element analysis of both crack and un-crack cantilever beam taking input file as a CAD design developed in CATIA. Experiments is done for total 10 models of crack beam having different cross section, crack location and crack depth and it generates natural frequency for 3 modes of vibration. A part of Matlab introduced is curve fitting toolbox and Fuzzy logic controller. The curve fitting toolbox introduced for crack identification by using simple curve. The Fuzzy controller here used comprises of three input variables (FNF, SNF and TNF) and two output variables (RCD and RCL) are generated with Triangular MF. Fuzzy analysis is done based on some set of fuzzy rules obtained from the information supplemented by Finite Element Analysis and Numerical Analysis. The proposed approach has been verified by comparing results obtained from fuzzy logic technique and finite element-analysis.

Key Words: Cantilever beam, Free vibration, frequency, Crack, ANSYS

1. INTRODUCTION

1.1 CONCEPT OF VIBRATION

With the discovery of musical instruments, people started taking interest in vibration. Since then, people have applied investigation and theories to study the vibration phenomenon. Sir Galileo is considered to be the founder of vibration phenomenon. He has performed his experiments on free vibrations of simple pendulum. After sir Galileo, a lot

of research is made by Mersenne (Vibrating string), Newton (Laws of motion), Taylor (Taylor series for vibration analysis), Alembert (Principles of equilibrium), Euler (Euler's beam theory), Fourier (sine wave analysis), Rayleigh (frequency measurement) and Lagrange's method of vibration analysis. Since then the vibration has covered a brief history from Sir Galileo's work to modern vibration researches.

Any motion which repeats itself after a certain interval of time is called vibration. The swing of pendulum is a typical example of vibration. The theory of vibration deals with study of oscillatory motions of bodies and the forces associated with them. A vibration can be caused due to external unbalanced force also. A vibratory system, in general, includes elastic member for storing potential energy, a mass or inertia member for storing kinetic energy and damper by which gradual loss of energy takes place. A simple pendulum as shown in fig 1.1 is an example of vibration system. Pendulum has a string for elastic nature, mass of bob acts as a means for kinetic energy. Like pendulum, spring-mass system, vehicle suspension system, simply supported and cantilever beam, lateral vibrating string, vibration due to unbalanced reciprocating or rotating force, etc. are the examples of vibrating system.

Vibrations with certain amplitude and frequency may be reliable but excessive vibrations causes

- Resonance and excessive noise.
- Structural failure of the machine components.
- Power transmission loss in case of gears.
- Increases bearing clearances.
- Reduces the working life of components.
- Effect on financial growth.

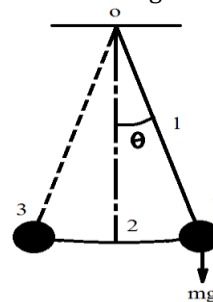


Fig. 1.1 Free Vibrations of Simple Pendulum

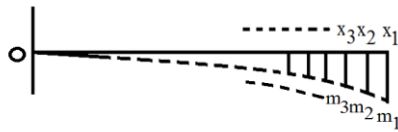


Fig. 1.2 Cantilever Beam as Multi Degree of Freedom System

1.1.1 Lateral vibrations of a cantilever beam

The vibrations in which particles of the system vibrates in the direction perpendicular to the axis of system is known as lateral vibration. A large number of practical systems can be described using finite number of degree of freedom; such a system is shown in fig.1.1. But some systems, especially those involving continuous elastic members, have an infinite number of degree of freedom. As an example, consider a cantilever beam shown in fig.1.2. Since the beam has infinite number of masses, we need infinite number of coordinates to specify the deflected configuration. Thus, cantilever beam is infinite degree of freedom system and it is necessary to study lateral vibrations of cantilever beam.

1.2 RECOGNITION OF NEED

Most of the structures and the mechanical systems are failed due to excessive vibrations in their working life. Since most human activities involve vibration in one form or other. The vibrations having certain amplitude and frequency may be reliable for human being. But excessive vibrations may lead to structural or mechanical failure of the system. Thus, study of vibration has more importance in order to avoid failures. The study of vibrations is necessary to understand the vibration behavior of the system. If the excessive vibrations from machine transmitted to the earth surface then the effect of excessive vibrations will be on human being as well as environment. Thus, study of vibration is important to avoid its effects.

The phenomenon of vibration can be applied to identify the crack size and location. Modern NDT techniques like dye penetrant testing, ultrasonic testing, acoustic emission techniques are hold the position of crack identification but such techniques require more time interval for processing. Thus it is necessary to develop new techniques for crack identification. The improved Fuzzy logic techniques and curve fitting in Mat lab are the alternatives for NDT techniques. These techniques give approximately solution for the problems of cracks.

This report contains certain knowledge about curve fitting and fuzzy logic techniques applied for different beam models

2. OBJECTIVES OF WORK

The objective of the thesis is to study

- Vibration phenomenon and root causes of cracks.

- Free Vibration of a cantilever beam and to measure the natural frequencies of various beam models by using –
 - (i) Euler’s Beam Model Theory
 - (ii) Finite Element Method (ANSYS)
 - (iii) Experimentation (FFT Analyzer)
- To compare the natural frequencies of models by above three methods. A comparison is made at the end of report to find the errors in the above methodologies.
- To identify the crack by using Curve fitting toolbox and Fuzzy logic toolbox in Mat lab.

3. PROBLEM IDENTIFICATION

Most of the structures and the mechanical systems are failed due to excessive vibrations in their working life. Since most human activities involve vibration in one form or other.

The phenomenon of vibration can be applied to identify the crack size and location.

To develop new techniques for crack identification using Fuzzy logic techniques and curve fitting in Mat lab are the alternatives for NDT techniques. These techniques give approximately solution for the problems of cracks. Crack location and depth are analyzed using ANSYS software. The results obtained from theoretical, ANSYS & Fuzzy logic are verified with experimentation.

4. THEORETICAL FORMULATIONS

4.1 CRACK THEORY

4.1.1 Physical parameters affecting Dynamic characteristics of cracked structures

Usually the physical dimensions, boundary conditions, the material properties of the structure play important role for the determination of its dynamic response. Their vibrations cause changes in dynamic characteristics of structures. In addition to this presence of cracking structures modifies its dynamic behaviour. The following aspects of the crack greatly influence the dynamic response of the structure.

- a) The position of crack
- b) The depth of crack
- c) The orientation of crack
- d) The number of cracks

4.1.2 Classification of cracks

Based on their geometries, cracks can be broadly classified as follows:

- Cracks perpendicular to the beam axis are known as “**transverse cracks**”. These are the most common and most serious as they reduce the cross-section and thereby weaken the beam. They introduce a local flexibility in the stiffness of the beam due to strain energy concentration in the vicinity of the crack tip.

- Cracks parallel to the beam axis are known as “longitudinal cracks”. They are not that common but they pose danger when the tensile load is applied is at right angles to the crack direction i.e. perpendicular to beam axis or the perpendicular to crack.

- “Slant cracks” (cracks at an angle to the beam axis) are also encountered, but are not very common. These influence the torsion behaviour of the beam. Their effect on lateral vibrations is less than that of transverse cracks of comparable severity.

- Cracks that open when the affected part of the material is subjected to tensile stresses and close when the stress is reversed are known as “breathing cracks”. The stiffness of the component is most influenced when under tension. The breathing of the crack results in non-linearity’s in the vibration behaviour of the beam. Cracks breathe when crack sizes are small, running speeds are low and radial forces are large. Most theoretical research efforts are concentrated on “transverse breathing” cracks due to their direct practical relevance.

- Cracks that always remain open are known as “gaping cracks”. They are more correctly called “notches”. Gaping cracks are easy to mimic in a laboratory environment and hence most experimental work is focused on this particular crack type.

- Cracks that open on the surface are called “surface cracks”. They can normally be detected by techniques such as dye-penetrates or visual inspection.

- Cracks that do not show on the surface are called “subsurface cracks”. Special techniques such as ultrasonic, magnetic particle, radiography or shaft voltage drop are needed to detect them. Surface cracks have a greater effect than subsurface cracks on the vibration behaviour of shafts.

4.2 GOVERNING EQUATION FOR FREE VIBRATION OF BEAM

The cantilever beam with a transverse edge crack is clamped at left end, free at right end and has same cross section and same length like model in Fig.4.1 and 4.2. The Euler- Bernoulli beam model is assumed for the theoretical formulation. The crack in this particular case is assumed to be an open surface crack and the damping is not being considered in this theory. Both single and double edged crack are considered for the formulation.

The free bending vibration of a beam of a constant rectangular cross section having length l , width b , and depth w is given by the Euler’s beam theory as follows:

If the cross sectional dimensions of beam are small compared to its length, the system is known as Euler-Bernoulli beam. Only thin beams are treated in it. The differential equation for transverse vibration of thin uniform beam is obtained with the help of strength of materials. The beam has cross section area A , flexural rigidity EI and density of material ρ .

Consider the small element dx of beam is subjected to shear force Q and bending moment M , as shown in Fig. 4.3.

While deriving mathematical expression for transverse vibration, it is assumed that there are no axial forces acting on the beam and effect of shear deflection is neglected. The deformation of beam is assumed due to moment and shear force.

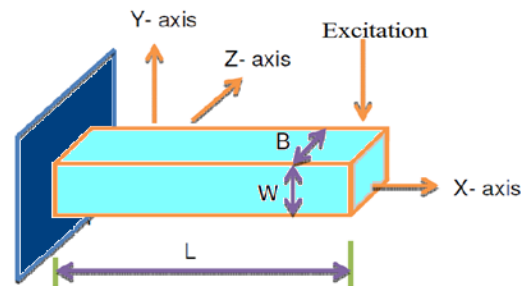


Fig. 4.1 Un-cracked cantilever beam model

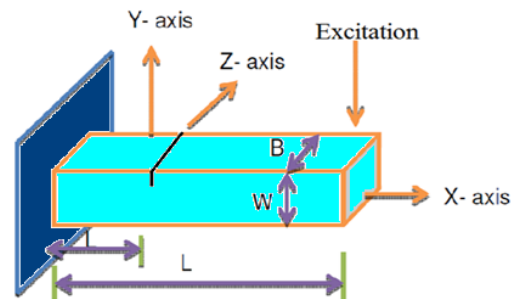


Fig. 4.2 Cracked cantilever beam model

The net force acting on the element,

$$Q - \left(Q + \frac{\partial Q}{\partial x} dx \right) = dm * acceleration$$

$$-\frac{\partial Q}{\partial x} dx = (\rho A dx) \frac{\partial^2 y}{\partial t^2}$$

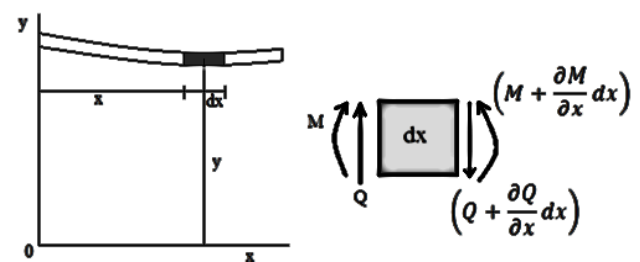


Fig. 4.3 Shear Force and Bending Moment acting on beam element

$$\frac{\partial Q}{\partial x} + \rho A \frac{\partial^2 y}{\partial t^2} = 0 \quad (\text{Equation 1})$$

Considering the moments about A, we get

$$M - \left(M + \frac{\partial M}{\partial x} dx \right) + \left(Q + \frac{\partial Q}{\partial x} dx \right) dx = 0$$

$$-\frac{\partial M}{\partial x} + Q + \frac{\partial Q}{\partial x} dx = 0$$

1	Structural Steel E= 210×10 ⁹ N/m ² , ρ =7850 1Kg/m ³ , Length=0.7m.	32×5	Un-cracked	0	0
2		32×5	Cracked	1	175
3		32×5	Cracked	2	175
4		32×5	Cracked	1	350
5		32×5	Cracked	2	350
6		10×10	Un-cracked	0	0
7		10×10	Cracked	1	175
8		10×10	Cracked	2	175
9		10×10	Cracked	1	350
10		10×10	Cracked	2	350

So $Q = \frac{\partial M}{\partial x}$ higher order derivatives are neglected here

$$\left(\frac{\partial Q}{\partial x} dx = 0\right)$$

$$\text{Or } \frac{\partial Q}{\partial x} = \frac{\partial^2 M}{\partial x^2} \text{ (Equation 2)}$$

From the above two equations 1 and 2, we get

$$\frac{\partial^2 M}{\partial x^2} = \rho A \frac{\partial^2 y}{\partial t^2} \text{ (Equation 3)}$$

We know from strength of materials that

$$M = -EI \frac{\partial^2 y}{\partial x^2}$$

$$\text{So } \frac{\partial^2 M}{\partial x^2} = -EI \frac{\partial^4 y}{\partial x^4} \text{ (Equation 4)}$$

Comparing equation 3 and 4 we get,

$$\frac{\partial^4 y}{\partial x^4} + \left(\frac{\rho A}{EI}\right) \frac{\partial^2 y}{\partial t^2} = 0 \text{ (Equation 5)}$$

This is the general equation for transverse vibration.

Thus the natural frequency can be found out by this theory as,

$$\omega_n = C * \sqrt{\frac{EI}{\rho A l^4}} \text{ (Equation 6)}$$

Where,

E= Young's modulus of the material,

I= Moment of inertia,

A= Area of cross section,

l=length of the beam,

C= Constant depending mode of vibration,

C₁=0.56 for first mode,

C₂= 3.52 for second mode,

C₃= 9.82 for third mode.

The moment of inertia can be found out by relation,

$$I = \frac{bd^3}{12} \text{ (Equation 7)}$$

Where,

b= width of the beam,

d= depth of the beam.

Due to presence of crack, moment of inertia of the beam changes and correspondingly the natural frequency also changes. For a constant beam material and cross section the reduced moment of inertia will be found by relation below.

$$I_1 = I - I_c \text{ (Equation 8)}$$

Where,

I₁=Moment of inertia of a cracked beam,

I=Moment of inertia of Un-cracked beam,

I_{ce}=Moment of inertia of cracked beam element.

Thus by the use of equation 6, 7 and 8 we can find out the different modes of natural frequencies for the cantilever beam.

Instead of using crack depth and location, the terms RCD and RCL are considered and calculated by the formula,

$$\text{RCD} = \frac{\text{depth of the crack}}{\text{depth of the beam}}$$

(Equation 9)

$$\text{RCL} = \frac{\text{distance of crack from fixed end}}{\text{length of the beam}}$$

(Equation 10)

4.3 THEORETICAL CALCULATIONS

The theoretical calculations are carried out by using above theory. Here total 10 models have been studied for natural frequency analysis having different crack depth and location. The table no. 4.1 shows various dimensions for the beam models. The beam models no. 1 to 5 are of rectangular cross section while beam model from 6 to 10 are of square cross section. It is assumed that the natural frequency changes due to the change in moment of inertia only. The beam made up of structural steel material by extrusion process. The Young's modulus for the beam material of length 700 mm is 210×10⁹ N/mm².

4.3.1 Theoretical calculation for Beam model 1

For Beam model 1,

Length= 700 mm, Cross-section= 32*5 mm, Un-cracked,

ρ= 7850Kg/mm³

$$A = 32 \times 10^{-3} \times 5 \times 10^{-3} \text{ m}^2 = 1.6 \times 10^{-4} \text{ m}^2$$

$$I = (0.032) (0.005)^3 / 12 \text{ mm}^4 = 3.33 \times 10^{-10} \text{ m}^4$$

Now from equation 6,

$$\omega = C \times \sqrt{\frac{210 \times 10^9 \times 3.33 \times 10^{-10}}{7850 \times 1.6 \times 10^{-4} \times 0.7^4}}$$

$$\omega = C \times 15.235 \text{ Rad/s.}$$

For first three modes we have the value of constant C,

Thus,

$$\omega_1 = 0.56 \times 15.235$$

$$\omega_1 = 8.5316 \text{ rad/s.}$$

Similarly ω₂ and ω₃ will be

$$\omega_2 = 3.52 \times 15.235 = 53.628 \text{ rad/s}$$

$$\omega_3 = 9.82 \times 15.235 = 149.61 \text{ rad/s.}$$

4.3.2 Theoretical calculation for Beam model 2

For Beam model 2,

Length= 700 mm, Cross-section= 32*5 mm, Cracked, RCD=0.2, RCL=0.25,

ρ= 7850Kg/mm³

$$A = 32 \times 10^{-3} \times 5 \times 10^{-3} \text{ m}^2 = 1.6 \times 10^{-4} \text{ m}^2$$

$$I = I - I_c$$

$$= 3.33 \times 10^{-10} - 2.6667 \times 10^{-12}$$

$$= 3.3033 \times 10^{-10} \text{ m}^4$$

Now from equation 6,

$$\omega = C \times \sqrt{\frac{210 \times 10^9 \times 3.3033 \times 10^{-10}}{7850 \times 1.6 \times 10^{-4} \times 0.7^4}}$$

$$\omega = C \times 15.166 \text{ rad/s.}$$

For first three modes we have the value of constant C,

Thus,

$$\omega_1 = 0.56 \times 15.166$$

$$\omega_1 = 8.4933 \text{ rad/s.}$$

Similarly ω_2 and ω_3 will be

$$\omega_2 = 53.40 \text{ rad/s}$$

$$\omega_3 = 148.98 \text{ rad/s.}$$

4.3.3 Theoretical calculation for Beam model 3

For Beam model 3,

Length= 700 mm, Cross-section= 32*5 mm, Cracked,

RCD=0.4, RCL=0.25,

$\rho = 7850 \text{ Kg/mm}^3$

$$A = 32 \times 10^{-3} \times 5 \times 10^{-3} \text{ m}^2$$

$$= 1.6 \times 10^{-4} \text{ m}^2$$

$$I = I - I_c$$

$$= 3.33 \times 10^{-10} - 2.133 \times 10^{-11}$$

$$= 3.12 \times 10^{-10} \text{ m}^4$$

Now from equation 6,

$$\omega = C \times \sqrt{\frac{210 \times 10^9 \times 3.12 \times 10^{-10}}{7850 \times 1.6 \times 10^{-4} \times 0.7^4}}$$

$$\omega = C \times 14.732 \text{ rad/s.}$$

For first three modes we have the value of constant C,

Thus,

$$\omega_1 = 0.56 \times 14.732$$

$$\omega_1 = 8.250 \text{ rad/s.}$$

Similarly ω_2 and ω_3 will be

$$\omega_2 = 51.88 \text{ rad/s}$$

$$\omega_3 = 144.24 \text{ rad/s.}$$

From the theory it is predicted that presence of crack in structure reduces the natural frequency. The natural frequency is the function of flexural rigidity and inversely proportional to the density of material and length of the beam.

The calculations show the natural frequency and mode shapes for first three beam models. Similarly, the natural frequency and different mode shapes for remaining beam models are calculated and tabulated in Table no. 4.2.

Table 4.2 Theoretical natural frequencies for the beam models

Beam model no.	RCD	RCL	1 st Natural Freq.	2 nd Natural Freq.	3 rd Natural Freq.
1	0	0	8.53	53.628	149.61
2	0.2	0.25	8.49	53.40	148.98
3	0.4	0.25	8.25	51.88	144.24
4	0.2	0.5	8.49	53.40	148.98
5	0.4	0.5	8.25	51.88	144.24
6	0	0	17.06	107.22	299.11
7	0.2	0.25	17.04	107.14	298.82
8	0.4	0.25	16.98	106.76	297.84
9	0.2	0.5	17.04	107.14	298.82
10	0.4	0.5	16.98	106.76	297.84

creations in one operation. Thus the process of subdividing all systems into their individual components or elements, whose behavior is all readily understood and then rebuilding the original system from such components to study its behavior is a natural way in which the engineer, the scientist or even the economist Proceeds .

Finite Element Analysis (FEA) was first developed in 1943 by R. Courant, who utilized the Ritz method of numerical analysis and minimization of variation calculus to obtain approximate solutions to vibration systems. Shortly thereafter, a paper published in 1956 by M. J. Turner, R.W. Clough, H.C. Martin, and L. J. Topp established a broader definition of numerical analysis.

The finite element method is a numerical method, which can be used for the solution of complex engineering problems with accuracy acceptable to engineers. In 1957 this method was first developed basically for the analysis of aircraft structures. There after the usefulness of this method for various engineering problems were recognized. Over the years, the finite element technique has been so well developed that, today it is considered to be one of the best method for solving a wide variety of practical problems efficiently. One of the main reasons for the popularity of the method in different fields of engineering is that once a general computer program is written, it can be used for the solution of any problem simply by changing the input data.

In FEM since the actual problem is replaced by a simpler one in finding the solution we will be able to find only an approximate solution rather than the exact solution. In most of the practical problems, the existing mathematical tools are not even able to find approximate solution of the problem. Thus, in the absence of any other convenient

method to find even the approximate solution of a given problem, we have to prefer the FEM.

The digital computer provided a rapid means of performing many calculations involved in FEA. Along with the development of high speed computers, the application of the FEM also progressed at a very impressive rate.

5.2 BRIEF DESCRIPTION OF FINITE ELEMENT METHOD

In the finite element method, the actual continuum or body of matter like solid, liquid or gas is represented as an assemblage of sub-divisions called finite elements. These elements are considered to be interconnected at specified joints which are called 'nodal points' or nodes. The nodes usually lie on the elements boundaries where adjacent elements are considered to be connected. Since the actual variation of the field variable (displacements, stress, temperature, pressure, velocity) inside the continuum is not known, we assume that, the variation of the field variable inside a finite element can be approximated by a simple function. These approximating functions (are called interpolating models) are defined in terms of field variables at the nodes. When field equations for the whole continuum are written, the new unknowns will be the nodal values of the field variable.

By solving the field equations, which are generally in the form of matrix equation, the nodal values of the field variables will be known. Once these are known, the approximating functions define the field variables throughout the assemblage of elements.

5.2.1 Steps for Finite Element Analysis

The basic steps adopted for analyzing an engineering problem by the FEM are,

i) Discretization of the Continuum: - The continuum is the body, structure or solid being analyzed. Discretization is the process of dividing the body into a finite number of elements, as shown in Fig.. These elements may have different sizes. The choice of the element type, its shape and refinement are required to be decided before discretization. The success of the discretization, which is also known as *meshing*, lies in how closely the discretized continuum represents the actual continuum.

The size and orientation of the elements are selected in such a manner that an element is exclusively composed of only one material.

ii) Formation of Element Stiffness Matrix: -After the continuum is discretized with the desired type of element, the element stiffness matrixes are formulated for all the elements. The elements stiffness matrix, which depends upon the material and geometry of an element, can be formulated by following methods,

- 1) Direct Stiffness Methods and
- 2) Energy Method



Fig. 5.1 Non-discretized beam section

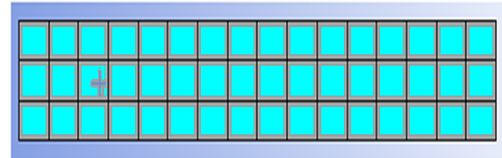


Fig. 5.2 Discretized beam section

In some problems, such as trusses, the local coordinate system of an individual element is different from the global coordinate system. In such cases, the element stiffness matrix in local coordinate system $[k]_e$ is converted to the element stiffness matrix in global coordinate system $[k]_e$ by using the transformation matrix $[L]$. The element stiffness matrix in global coordinate system is given by,

$$[k]_e = [L]^T [k']_e [L]$$

iii) Formation of Global Stiffness Matrix: -After element stiffness matrix in global coordinate system is obtained for all the elements, they are assembled to form the global stiffness matrix $[K]$ for the entire body.

$$[K] = \sum [k]_e$$

The global stiffness matrix is symmetric, banded and singular.

iv) Formation of Global Load Vector: -The element force vectors in the global coordinate system $\{f\}_e$ for all elements are assembled to form the global vector $\{F\}$ for the entire body.

$$\{F\} = \sum \{f\}_e$$

v) Formation of Global Node Displacement Vector: -The global nodal displacement vector $\{U_N\}$ is formed for the entire body.

vi) Assembly of Global stiffness vector -The relation between the global stiffness matrix $[K]$, global nodal displacement vector $\{U_N\}$ and the global load vector $\{F\}$ is expressed as a set of simultaneous algebraic equation.

$$[K] \{U_N\} = \{F\}$$

vii) Incorporation of Specified Boundary conditions: - The specified boundary conditions are incorporated in equilibrium equation by using one of the following two approaches:

- a) The Elimination approach and
- b) The Penalty approach

viii) Solution of Simultaneous Equations: -After including the specified boundary conditions in equilibrium equation, the modified equation are solved for the unknown nodal displacements by using methods like Gaussian elimination, Chomsky's Factorization, Gauss Seidel, Jacobi iterations, Frontal Technique, etc. In solving the equations, advantage is taken of the banded property of the global stiffness matrix.

ix) **Computation of element strains and stresses:** - Knowing the nodal displacements, the element strains and stresses are calculated. The components of strains at any point within the element are given by,

$$\{\epsilon\} = [B]\{U_N\}$$

Similarly, the components of stresses at any point within the element are given by

$$\{\sigma\} = [D]\{\epsilon\}$$

Where, [B] = Element strain-nodal displacement matrix

[D] = Element stress-strain matrix.

5.3 FEA SOFTWARE PACKAGES AND APPLICATIONS

The rapid advance made in computer hardware and software led to significant developments in FEA software. FE programming has emerged as a specialized discipline which requires knowledge and experience in the diverse areas such as FE technology including foundations of machines, and numerical analysis on the one hand and the computational skills in areas of software technology including programming techniques, data structure, data base management and computer graphics on the other hand. It requires several man years to develop general purpose finite element analysis software with a processing capability and facility for the user to have a wide choice of several types of elements, analysis for different types of problem- static, dynamic, material and geometric nonlinear, coupled situations, heat transfer, interaction problems etc. and pre and post-processing features.

In 1975 the FEA software package was used for the analysis of aero plane parts. From the analysis it was predicted that the Andy's packages gives better solution as compare to defining theory models. Thus the packages cover the industrial sector for analysis of various structural and mechanical parts. The main purpose of such packages is to obtain the solution without creating physical model.

The analysis software packages are applicable to static linear analysis, non-linear analysis, modal analysis, fluid flow analysis, thermal analysis, harmonic response analysis, steady state analysis, etc.

As it is not possible here to review the capabilities and compare different commercially available finite element analysis packages only the names of some of the popular packages are given below:

ABAQUS, ADINA, ANSYS, ALGOR, ASKA, COSMOS, GT-STRU DL, LISA, NISA, ASTRAN, PAFEC, PASTRAN, SAP.

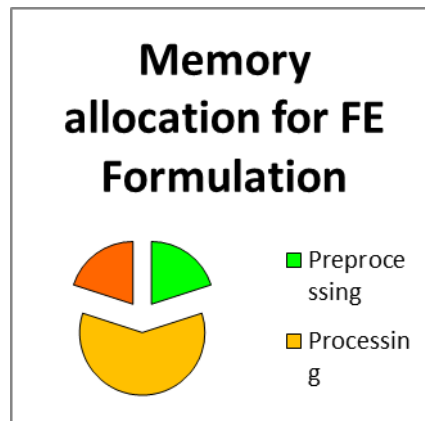


Fig. 5.3 Memory allocations for analysis software package.

5.3.1 General steps of Finite Element Software Package

Use of general-purpose FEA software involves the following several steps.

Preprocessing- This is the first step of FE formulation. It includes input data to provide for finite element formulation. Input data describes geometry, material properties, loads, and boundary conditions. Software can automatically prepare much of the FE mesh. But must be given direction as to the type of element and the mesh density describe. That is, the analyst must choose one or more element formulations that suit the mathematical model, and state how large or how small elements should be in selected portions of the FE model. All data should be reviewed for correctness before proceeding.

Processing- This step will performed by software package to analyze the input data and generate possible solution. Software automatically generates matrices that described the behavior of each element, combines these matrices into a large matrix equation that represents the FE structure, and solves this equation to determine values of field quantities at nodes. Substantial additional calculations are performed if behavior is nonlinear or time dependent.

Post processing-The FEA solution and quantities derived from it are listed or graphically displayed. This step also automatic, expect that the analyst must tell the software what lists or displays to prepare. In stress analysis, typical displays include the deformed shape, with deformations exaggerated and probably animated, and stresses of various types on various planes. The resulted data is shown in animated images and video. Animated images may include color coded form of a data. A short video of deformation can be animated in the package.

The Fig. 5.3 shows the memory allocation required for finite element formulation of analysis problems. The memory allocation depends on how the processing is required and what type of processing is covered in that memory.

5.4 MODAL ANALYSIS OF A BEAM MODELS

Finite element analysis has been carried out by ANSYS12 software. ANSYS is a general-purpose finite-element modeling package for numerically solving a wide

variety of mechanical problems. These problems include static/dynamic, structural analysis (both linear and nonlinear), heat transfer, and fluid problems, as well as acoustic and electromagnetic problems.

Here the, beam is discretized into approximately 17000 elements having 88000 nodes. The solution is obtained by using ANSYS Work Bench modeler. First 10 modes of natural frequencies are set down and from the resulted data; first three modes of transverse vibration are taken for comparison of 10 beam models. The results are tabulated in table no. 5.1.

Following steps show the guidelines for carrying out Modal analysis.

Define Materials

1. Set preferences. (Structural steel)
2. Define constant material properties. (Density, Young’s modulus, Poisson’s ratio)

Model the Geometry

3. Follow bottom up modelling and create/import the geometry.

Generate Mesh

4. Define element type. (Default mesh of element size 2 mm)
5. Mesh the area.

Apply Boundary Conditions

6. Apply constraints to the model. (Fixed support at the end of beam)

Obtain Solution

7. Specify analysis types and options. (Modal analysis up to 10 modes)
8. Solve

The ANSYS 12 finite element program was used for free vibration of the cracked beams. For this purpose, the total 10 models are created at various crack positions in CAD software (CATIA) and imported in ANSYS (.stop file). The beam model was discretised into no. of elements with N nodes. Cantilever boundary conditions can also be modelled by constraining all degrees of freedoms of the nodes located on the left end of the beam. The subspace mode extraction

method was used to calculate the natural frequencies of the beam.

The models are created with different dimensions which are taken from table no. 5.2. The results of modal analysis i.e. values of natural frequencies and mode shapes are shown in Fig. 5.4 to 5.33. The result is tabulated in table no. 5.1.

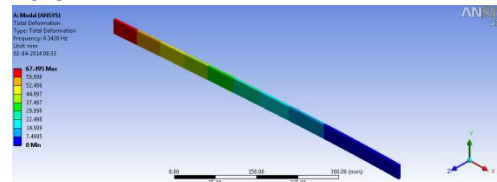


Fig. 5.4 First natural frequency of beam model 1

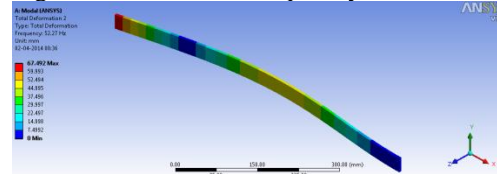


Fig. 5.5 Second natural frequency of beam model 1

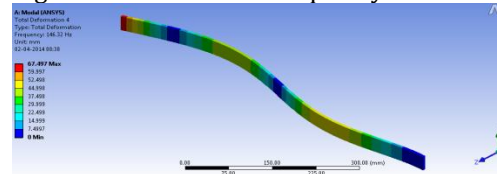


Fig. 5.6 Third natural frequency of beam model 1

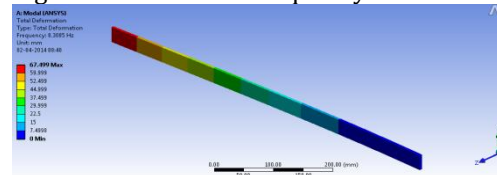


Fig. 5.7 First natural frequency of beam model 2

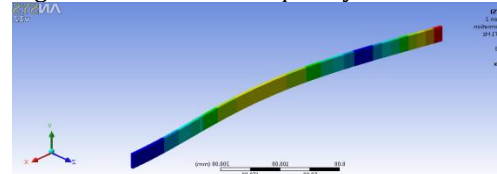


Fig. 5.8 Second Natural Frequency of Beam Model 2

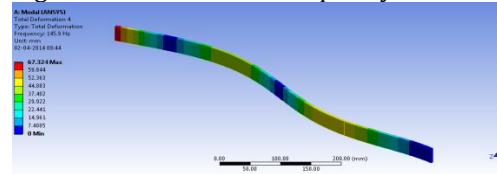


Fig. 5.9 Third natural frequency of beam model 2

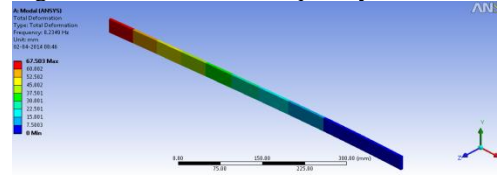


Fig. 5.10 First natural frequency of beam model 3

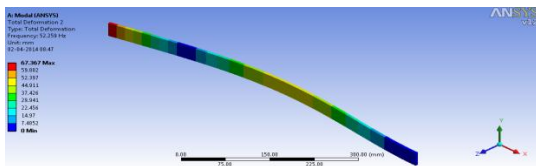


Fig. 5.11 Second natural frequency of beam model 3

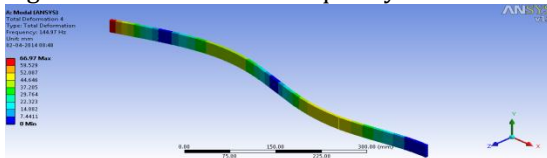


Fig. 5.12 Third natural frequency of beam model 3

6. EXPERIMENTAL FORMULATIONS

6.1 INTRODUCTION

In this report the vibrational, behavior of the beam models has been done by experimental approach with the help of FFT analyzer. The FFT analyzer is nothing but the Fast Fourier Transform analyzer contains different electronic components system for measuring natural frequencies of various beam models.

A spectrum analyser measures the magnitude of an input signal verses frequency within the full frequency range of the instrument. The primary use is to measure the power of the spectrum of known and unknown signals. The input signal a spectrum analyser measures is electrical, however, spectral compositions of other signals, such as acoustic pressure waves and optical light waves, can be considered through the use of an appropriate transducer. Optical spectrum analysers also exist, which use direct optical techniques such as a monochromatic to make measurements.

By analysing the spectra of electrical signals, dominant frequency, power, distortion, harmonics, bandwidth, and other spectral components of a signal can be observed that are not easily detectable in domain waveforms. These parameters are useful in the characterization of electronic devices, such as wireless transmitters.

The display of a spectrum analyser has frequency on the horizontal axis and the amplitude displayed on the vertical axis. To the casual observer, a spectrum analyser looks like an oscilloscope and, in fact, some lab instruments can function either as an oscilloscope or a spectrum analyser.

An FFT spectrum analyser works in an entirely different way. The input signal is digitized at a high sampling rate, similar to a digitizing oscilloscope. Nyquist's theorem says that as long as the sampling rate is greater than twice the highest frequency component of the signal, the sampled data will accurately represent the input signal. In the SR7xx (SR760, SR770, SR780 or SR785), sampling occurs at 256 kHz. To make sure that Nyquist's theorem is satisfied; the

input signal passes through an analogue filter which attenuates all frequency components above 156 kHz by 90 db. This is the anti-aliasing filter. The resulting digital time record is then mathematically transformed into a frequency spectrum using an algorithm known as the Fast Fourier Transform, or FFT. The FFT is simply a clever set of operations which implements Fourier's theorem. The resulting spectrum shows the frequency components of the input signal.

Now here's the interesting part. The original digital time record comes from discrete samples taken at the sampling rate. The corresponding FFT yields a spectrum with discrete frequency samples.

In fact, the spectrum has half as many frequency points as there are time points. (Remember Nyquist's theorem.) Suppose that you take 1024 samples at 256 kHz. It takes 4 Ms to take this time record. The FFT of this record yields, 512 frequency points but over what frequency range? The highest frequency will be determined by the period of two time samples or 128 kHz. The lowest frequency is just the period of the entire record or 1/ (4 Ms) or 250 Hz. everything below 250 Hz is considered to be DC. The output spectrum thus represents the frequency range from DC to 128 kHz, with points every 250 Hz.

6.1.1 Advantages of FFT Analysers

The advantage of this technique is its speed. Because FFT spectrum analysers measure all frequency components at the same time, the technique offers the possibility of being hundreds of times faster than traditional analogue spectrum analysers. In the case of a 100 kHz span and 400 resolvable frequency bins, the entire spectrum takes only 4 Ms to measure. To measure the signal with higher resolution, the time record is increased. But again, all frequencies are examined simultaneously providing an enormous speed advantage.

In order to realize the speed advantages of this technique we need to do high speed calculations. And, in order to avoid sacrificing dynamic range, we need high-resolution ADCs. SRS spectrum analysers have the processing power and frontend resolution needed to realize the theoretical benefits of spectrum analysers.

6.2 COMPONENTS OF FFT ANALYZER

A lot of components are evolved for sensing, data analysis and storing in electronic field. The main components of FFT analyser are FFT signal analysing unit, memory unit for processing and handling data of testing, impact hammer for impact testing, and microphone sensors for obtaining reading of experimental work. Here some components are discussed that are required for experimental work.

6.2.1 FFT Analyzer with data handling unit

This is the main part of FFT analyzer system. It consists of function generator, data processing unit, A/D converter, memory unit together called as FFT analyzer. The function generator generates the sine function required as per the input signal. Processing unit processes on input

signal and perform various processes like sampling, windowing, averaging and maintaining the signal proportional to input. A/D converter convert analogue signal to digital signal. Memory unit stores the information and suitable data in graphical user interfacing and software package use for converting that data for analysis purpose and hardware copies. The analyzer for experimentation is four-channel spectrum analyzer data collector and balance with software along with acceleration sensor sensitivity 100 m V/g ($g=9.81 \text{ m/s}^2$).

6.2.2 Impact Hammer

It is used to provide an impact on beam for excitation. The impact is positioned such that the mode cannot present at impact position. The used impact hammer is dynamic quartz sensor -9722A- for light to medium structure at med. to high frequencies. It has Measuring range up to 2000N with cable and other accessories and sensitivity at 100 Hz=2mV/N. the overload capacity is 500N with Resonance frequency = 27 KHz, hammer mass=100 gm., Rigidity=0.8 KN/Micron, temperature range = - 20 to 70 °c.

After impacting excitations are setup in beam and knowing the magnitude of impact and amplitude of excitation the natural frequency is calculated by FFT analyzer.

6.2.3 Microphone with cable arrangement

The microphone type sensor with cable arrangement is used to measure the input signal. The sensitivity of sensor is 46.17 mV/Pa. The Sensor senses the exciting signal i.e. acoustic sound and amplitude of input signal and transferred to FFT unit for further processing to obtain the appropriate results.

6.2.4 Software interfacing module

The processing and results of experiment are stored in the memory unit of FFT analyzer. Such results can obtain for hard copy by use of software interfacing module. The software interfacing module converts results in a graphical interface and use for windowing and hard copy from computers and hardware.

6.3 STEPS FOR EXPERIMENTATION

To perform the experimental working on beams, certain steps have to be followed. The following steps are used for experimentation on beams. Firstly, the beam is clamped to fixed arrangement and then excited by impact hammer to obtain natural frequencies. The Fig. 6.1 shows the experimental setup required to calculate the natural frequencies and Fig. 6.2 to 6.11 shows the graphical results of FFT analyzer system. The numerical values of average natural frequencies are tabulated in table no. 6.1.

The beam dimensions for experimentation are
 Beam length: 700 mm.
 Cross-section: 32*5 mm² for first 5 beams, 10*10 mm² for next five beams.

Crack depth: 0 mm, 1 mm, and 2 mm.

Crack location: 0 mm, 175 mm, and 350 mm from fixed end.

Following steps are adopted for experimentation work.

1. Clamp the beam on clamping arrangement as shown in Fig. 6.1.
2. Attach the microphone sensor at some position where modes cannot occur.
3. Start the FFT analyzer by powering on button on front panel.
4. Take new file in FFT analyzer and named as per date.
5. Take the impact hammer in hand and impact on beam at certain location.
6. Repeat step 5 for various positions of hammering.
7. Check the frequency spectrum obtained on FFT display.
8. Save the file in memory unit of FFT analyzer.
9. Now change the beam models and repeat procedure for all beam models.
10. At the end copy all files from FFT analyzer to hard disk drive by suitable interfacing.
11. Obtain reading and spectrum pattern from software installed in computer system.

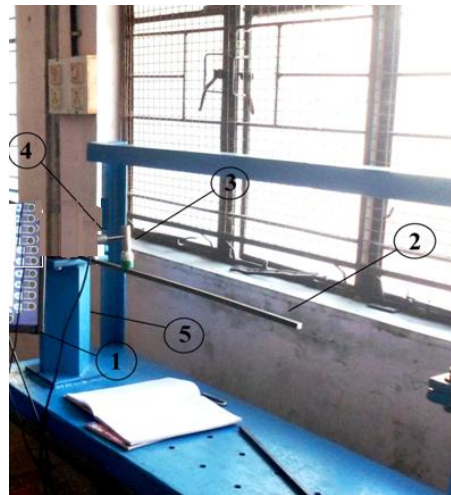


Fig. 6.1 Experimental setup for measuring Natural Frequency Details:

- 1- FFT analyzing machine
- 2- Cantilever Beam model
- 3- Impact hammer
- 4- Microphone sensor
- 5- Clamping arrangement

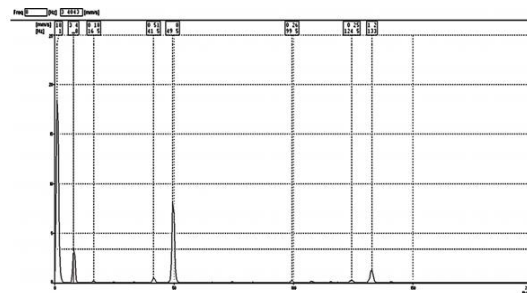


Fig. 6.2 FFT reading for beam 1

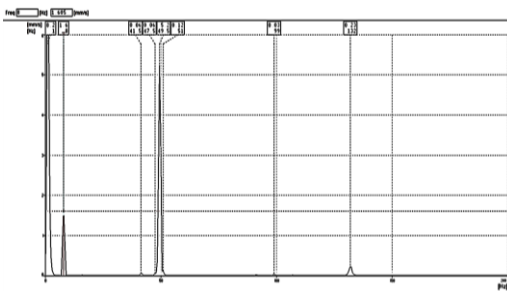


Fig. 6.3 FFT reading for beam 2

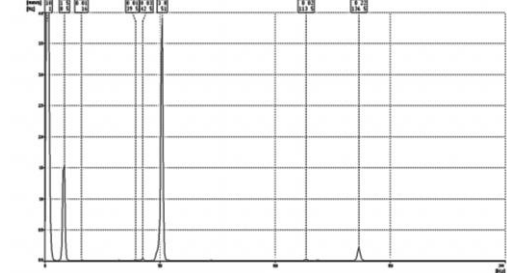


Fig. 6.4 FFT reading for beam 3

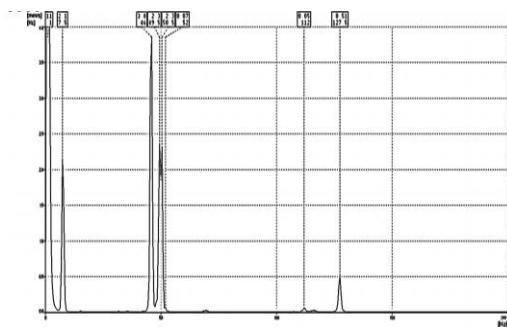


Fig. 6.5 FFT reading for beam 4

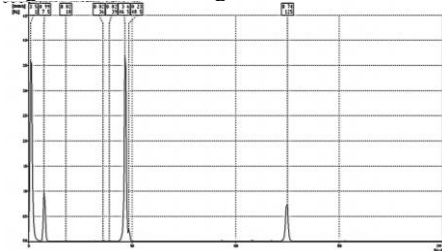


Fig. 6.6 FFT reading for beam 5

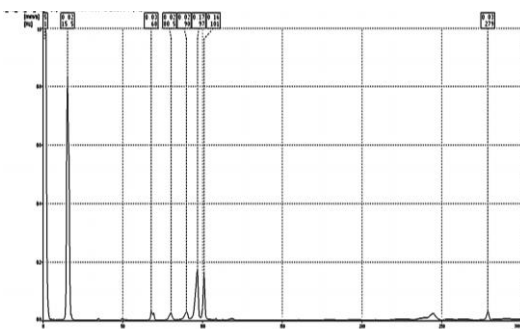


Fig. 6.7 FFT reading for beam 6

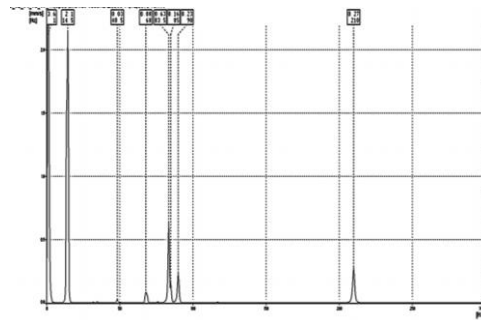


Fig. 6.8 FFT reading for beam 7

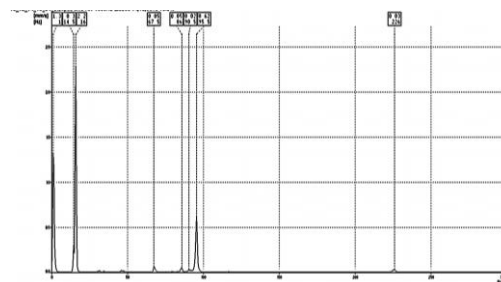


Fig. 6.9 FFT reading for beam 8

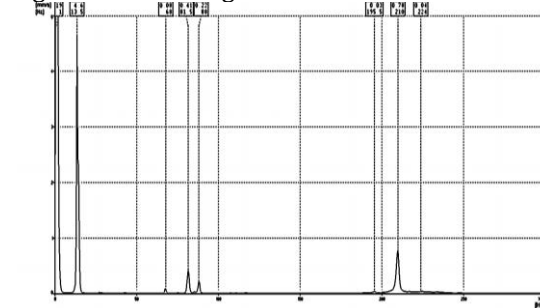


Fig. 6.10 FFT reading for beam 9

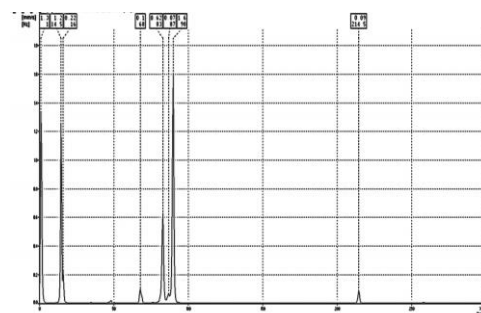


Fig. 6.11 FFT reading for beam 10

7. CRACK IDENTIFICATION USING MATLAB

7.1 MATLAB: A TOOL FOR ENGINEERING ANALYSIS

MATLAB is high performance language for technical computing. It integrates computation, visualization and programming in an easy to use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses includes following:

- Math and Computation

- Algorithm development
- Data Acquisition
- Modelling, Simulation and Prototyping
- Data Analysis, Exploration and Visualization
- Scientific and Engineering graphics
- Application development including graphical user interface.

MATLAB is interactive system whose basic data element is an array that does not require dimensioning. This allows formulating solutions to many technical computing problems, especially those involving matrix representations, in a fraction of time it would take to write program in a scalar non interactive language such as C or FORTRAN.

The name MATLAB stands for matrix laboratory. MATLAB was written originally to provide easy access to matrix software developed by the LINPACK (Linear System Package) and EIPACK(Eigen System Package) projects. Today, a MATLAB engine incorporates the LAPACK (Linear Algebra Package) and BLAS (Basic Linear Algebra Subprogram) libraries, constituting the state of the art in software computation. In university environment, MATLAB is the standard computational tool for introductory and advanced courses in mathematics, engineering, and science. In industry, it is the computational tool of choice for research and development, and analysis. It is complemented by a family of application specific solutions called toolboxes.

The image processing toolbox is a collection of MATLAB environment for the solution of digital image processing problems. Other toolboxes that sometimes complement IPT are Signal Processing, Neural Network, Fuzzy Logic and Wavelet Toolboxes.

In this report, two types of toolboxes namely CF toolbox and Fuzzy logic toolbox are studied. The CF toolbox commonly called as Curve Fitting toolbox used to fit a curve by analysing suitable data matrix. The curve fitting is required when we have available only one of the natural frequency instead of different modes. Here curve is fitted by knowing the first natural frequency. The variation of first natural frequency with crack depth and crack location is shown in terms of curve fitting in Fig. no. 7.1

7.2 CURVE FITTING TOOLBOX

Curve Fitting Toolbox software is a collection of graphical user interfaces (GUIs) and functions for curve and surface fitting that operate in the MATLAB® technical computing environment. The toolbox supplements MATLAB features with:

1. Data pre-processing capabilities, such as sectioning, excluding data, and smoothing
2. Data fitting using parametric and nonparametric models.
3. The toolbox includes a library of parametric models, with polynomials, exponentials, rationales, sums of Gaussians, Fourier polynomials, and many others.

4. You can also define custom models to precisely reflect the goals of your data analysis.
5. Nonparametric models are available through a variety of smoothers and interplants.
6. Fitting methods for linear least squares, nonlinear least squares, weighted least squares, constrained least squares, and robust fitting are available
7. Data and fit statistics to assist you in analysing your models
8. Post processing capabilities that allow you to interpolate, extrapolate, differentiate, and integrate the fit
9. The ability to save your work in various formats, including workspace variables, binary files, and automatically generated MATLAB code

Curve Fitting Toolbox provides an application and functions for fitting curves and surfaces to data. The toolbox lets we perform exploratory data analysis,pre-process and post-process data, compare candidate models, and remove outliers. We can conduct regression analysis using the library of linear and nonlinear models provided or specify your own custom equations. The library provides optimized solver parameters and starting conditions to improve the quality of our fits. The toolbox also supports nonparametric modelling techniques, such as splines, interpolation, and smoothing.

After creating a fit, we can apply a variety of post-processing methods for plotting, interpolation, and extrapolation; estimating confidence intervals and calculating integrals and derivatives.

7.2.1 Key Features

- Curve Fitting app for curve and surface fitting.
- Linear and nonlinear regression with custom equations.
- Library of regression models with optimized starting points and solver parameters.
- Interpolation methods, including B-splines, thin plate splines, and tensor-product splines.
- Smoothing techniques, including smoothing splines, localized regression,Savitzky-Golay filters, and moving averages.
- Pre-processing routines, including outlier removal and sectioning, scaling, and weighting data.
- Post-processing routines, including interpolation, extrapolation, confidence intervals, integrals and derivatives.

7.2.2 Steps for curve fitting

1.Load First frequency data matrix and crack depth data matrix in the MATLAB command line:

```
load First frequency data matrix
load crack depth data matrix
load crack location data matrix
```

2.Open the Curve Fitting app by entering:

Cytosol

Alternatively, click **Curve Fitting** on the **Apps** tab.

1 Select **X data** and **Y data**. The Curve Fitting app creates a default polynomial fit to the data.

2 Select **File > Generate Code**.

The Curve Fitting app creates a file in the Editor containing MATLAB code to recreate all fits and plots in your interactive session.

Here the data for curve fitting is taken from table no. 7.1. Since there are two cross sections of beam for analysis only first three beams are taken for curve fitting. The Fig. no. 7.1, 7.3, 7.5 shows curve fitting for natural frequency versus crack depth and Fig. no. 7.2, 7.4, 7.6 shows natural frequency versus crack location.

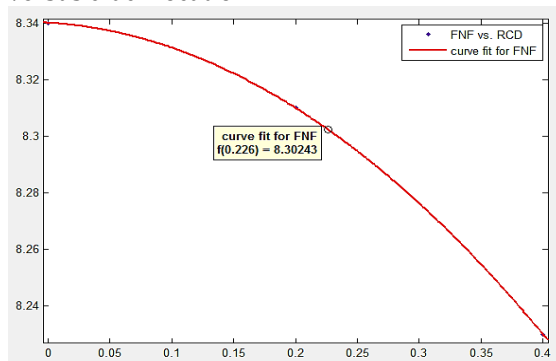


Fig. 7.1 Curve fitting for First Natural Frequency with RCD

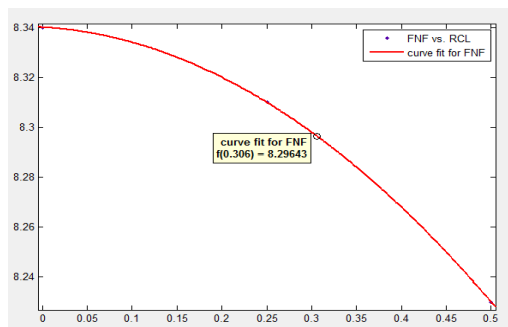


Fig. 7.2 Curve fitting for First Natural Frequency with RCL

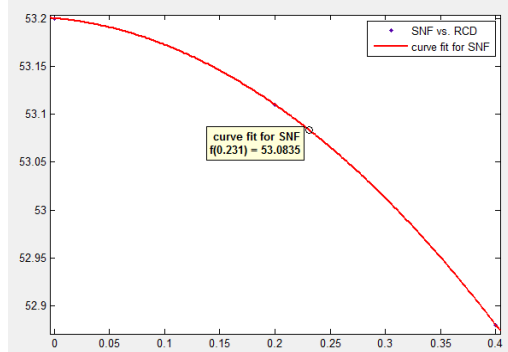


Fig. 7.3 Curve fitting for Second Natural Frequency with RCD

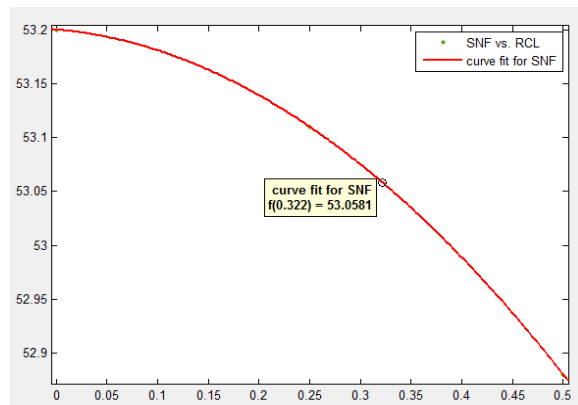


Fig. 7.4 Curve fitting for Second Natural Frequency with RCL

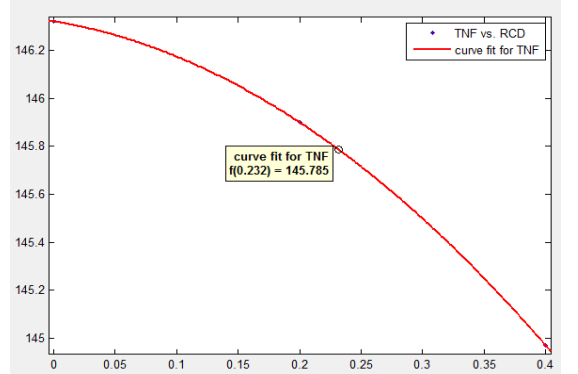


Fig. 7.5 Curve fitting for Third Natural Frequency with RCD

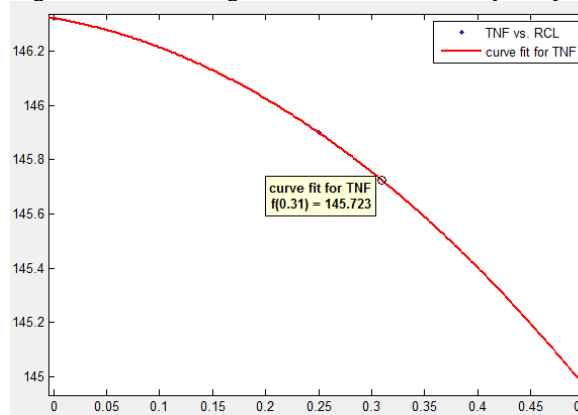


Fig. 7.6 Curve fitting for Third Natural Frequency with RCL

7.3 FUZZY LOGIC TOOLBOX

It is observed that the human beings do not need precise, numerical information input to make a decision, but they are able to perform highly adaptive control. Human have a remarkable capability to perform a wide variety of physical and mental tasks without any explicit measurements or computations. Examples of everyday tasks are parking a car, driving in city traffic, playing golf, and summarizing a story. In performing such familiar tasks, human use perceptions of time, distance, speed, shape, and other attributes of physical and mental objects. Fuzzy logic is a problem-solving control system methodology that lends itself for implementation in systems ranging for simple, small, embedded micro-controller storage, networked, workstation-based acquisition and control systems. The

story of fuzzy logic systems is inspired by the remarkable human capability to operate on and reason with perception-based information. The rule-based fuzzy logic provides a scientific formalism for reasoning and decision making with uncertain and imprecise information. This methodology can be implemented in hardware, software, or a combination of both. Fuzzy logic approach to control problems mimic show a person would make decisions. Fuzzy systems allow for easier understanding as they are expressed in terms of linguistic variables. Damage detection is one of the key aspects in structural engineering both for safety reasons and because of economic benefits that can result. Many non-destructive testing methods for health monitoring have been proposed and investigated. These methods include modal analysis, strain analysis, photo-elastic techniques, and ultra sound and acoustic emissions. A fuzzy logic methodology can be presented for structural fault detection based on Eigen value, and dynamic responses of vibrating structure.

This proposes online crack detection methodology embedded with a new intelligent Fuzzy Interface System. In this approach a fuzzy logic controller is designed and implemented for identifying relative crack depth and location. The designed fuzzy controller has three inputs and two outputs. The inputs for fuzzy controller are three modes of natural frequencies and two outputs are relative crack depth and relative crack location.

7.3.1 Fuzzy Interface System

A fuzzy inference system (FIS) essentially defines a nonlinear mapping of the input data vector into a scalar output, using fuzzy rules. The mapping process involves input/output membership functions, FL operators, fuzzy if-then rules, aggregation of output sets, and defuzzification. An FIS with multiple outputs can be considered as a collection of independent multi input ,single-output systems. A general model of a fuzzy inference system (FIS) is shown in Fig. 6.7. The FLS maps crisp inputs into crisp outputs. It can be seen from the Fig. that the FIS contains four components: the fuzzifier, inference engine, rule base, and defuzzifier. The rule base contains linguistic rules that are provided by experts. It is also possible to extract rules from numeric data. Once the rules have been established, the FIS can be viewed as a system that maps an input vector to an output vector. The fuzzifier maps input numbers into corresponding fuzzy memberships. This is required in order to activate rules that are in terms of linguistic variables. The fuzzifier takes input values and determines the degree to which they belong to each of the fuzzy sets via membership functions. The inference engine defines mapping from input fuzzy sets into output fuzzy sets. It determines the degree to which the antecedent is satisfied for each rule. If the antecedent of a given rule has more than one clause, fuzzy operators are applied to obtain one number that represents the result of the antecedent for that rule. It is possible that one or more rules may fire at the same time. Outputs for all rules are then aggregated. During aggregation, fuzzy sets that represent the output of each rule are combined into a single fuzzy set.

Fuzzy rules are fired in parallel, which is one of the important aspects of an FIS. In an FIS, the order in which rules are fired does not affect the output. The defuzzifier maps output fuzzy sets into a crisp number. Given a fuzzy set that encompasses a range of output values, the defuzzifier returns one number, thereby moving from a fuzzy set to a crisp number. Several methods for defuzzification are used in practice, including the centroid, maximum, mean of maxima, height, and modified height defuzzifier. The most popular defuzzification method is the centroid, which calculates and returns the centre of gravity of the aggregated fuzzy set. FISs employ rules. However, unlike rules in conventional expert systems, a fuzzy rule localizes a region of space along the function surface instead of isolating a point on the surface.

For a given input, more than one rule may fire. Also, in an FIS, multiple regions are combined in the output space to produce composite region. A general schematic of an FIS is shown in Fig. 7.8.

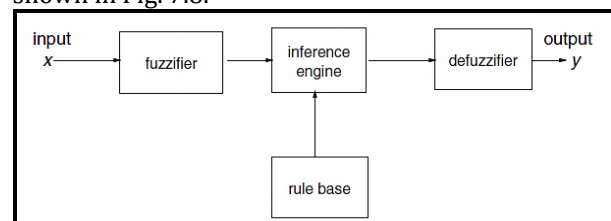


Fig. 7.7 Block diagram of Fuzzy Interface System

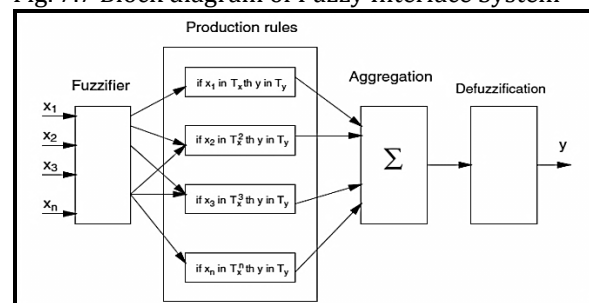


Fig. 7.8 Schematic diagram of a Fuzzy Interface System

7.3.2 Importance of Fuzzy Logic

Here is a list of general observations about fuzzy logic:

- **Fuzzy logic is conceptually easy to understand:** The mathematical concepts behind fuzzy reasoning are very simple. What makes fuzzy nice is the "naturalness" of its approach and not its far-reaching complexity.
- **Fuzzy logic is flexible:** With any given system, it's easy to massage it or layer more functionality on top of it without starting again from scratch.
- **Fuzzy logic is tolerant of imprecise data:** Everything is imprecise if you look closely enough, but more than that, most things are imprecise even on careful inspection. Fuzzy reasoning builds this understanding into the process rather than tacking it onto the end.
- **Fuzzy logic can model nonlinear functions of arbitrary complexity:** You can create a fuzzy system to match any set of input-output data. This process is made particularly easy by adaptive techniques like Adaptive

Neuron-Fuzzy Inference Systems (ANFIS), which are available in the Fuzzy Logic Toolbox.

- **Fuzzy logic can be built on top of the experience of experts:** In direct contrast to neural networks, which take training data and generate opaque, impenetrable models, fuzzy logic lets you rely on the experience of people who already understand your system.
- **Fuzzy logic can be blended with conventional control techniques:** Fuzzy systems don't necessarily replace conventional control methods. In many cases fuzzy systems augment them and simplify their implementation.
- **Fuzzy logic is based on natural language:** The basis for fuzzy logic is the basis for human communication. This observation underpins many of the other statements about fuzzy logic.

7.3.3 Foundations of Fuzzy Logic

- 1) Fuzzy Set
- 2) Membership Function
- 3) Logical Operations
- 4) If-Then Rules

1) Fuzzy Set: Fuzzy logic starts with the concept of a fuzzy set. A fuzzy set is a set without a crisp, clearly defined boundary. It can contain elements with only a partial degree of membership. To understand what a fuzzy set is, first consider what is meant by what we might call a classical set. A classical set is a container that wholly includes or wholly excludes any given element. For example, consider the set of days comprising a weekend. The Fig.7.9 below is one attempt at classifying the weekend days.

Most would agree that Saturday and Sunday belong, but what about Friday? It feels like a part of the weekend, but somehow it seems like it should be technically excluded. So in the Fig.7.9 above Friday tries its best to sit on the fence. Classical or normal sets wouldn't tolerate this kind of thing. Either you're in or you're out. Human experience suggests something different, though: fence sitting is a part of life.

"In fuzzy logic, the truth of any statement becomes a matter of degree."

The Fig. no.7.10, left is a plot that shows the truth values for weekend-ness if we are forced to respond with an absolute yes or no response. On the right is a plot that shows the truth value for weekend-ness if we are allowed to respond with fuzzy in-between values.

2) Membership Function: A membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The input space is sometimes referred to as the universe of discourse, a fancy name for a simple concept. A classical set might be expressed as

$$A = \{x \mid x > 6\}$$

A fuzzy set is an extension of a classical set. If X is the universe of discourse and its elements are denoted by x, then a fuzzy set A in X is defined as a set of ordered pairs.

$$A = \{x, \mu_A(x) \mid x \in X\}$$

$\mu_A(x)$ is called the membership function (or MF) of x in A. The membership function maps each element of X to a

membership value between 0 and 1. The Fuzzy Logic Toolbox includes 11 built-in membership function types. These 11 functions are, in turn, built from several basic functions: piecewise linear functions, the Gaussian distribution function, the sigmoid curve, and quadratic and cubic polynomial curves. For detailed information on any of the membership functions mentioned Fig. 7.11, turn to Functions — Alphabetical List. By convention, all membership functions have the letters mf at the end of their names.

3) Logical Operations: The most important thing to realize about fuzzy logical reasoning is the fact that it is a superset of standard Boolean logic. In other words, if we keep the fuzzy values at their extremes of 1 (completely true), and 0 (completely false), standard logical operations will hold.

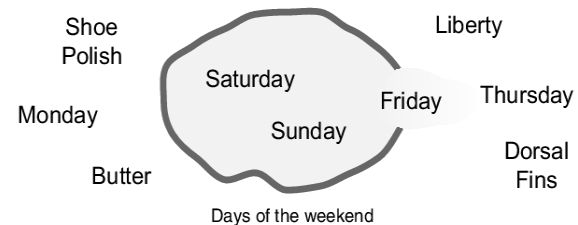


Fig. 7.9 Days of the Weekend

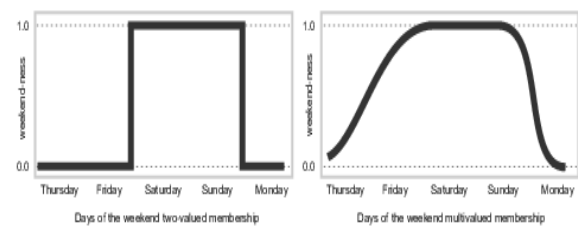
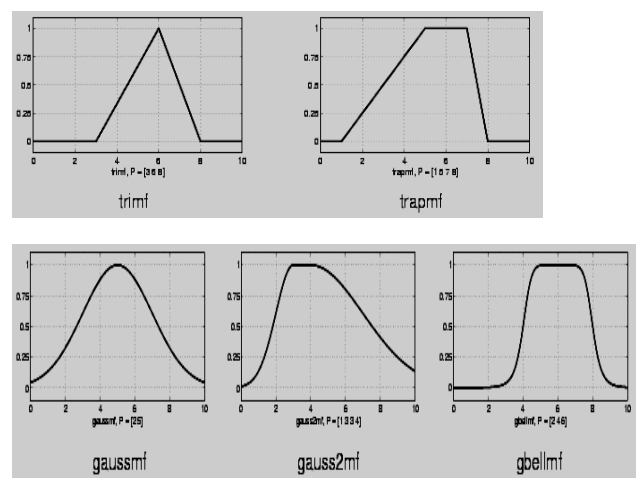


Fig. 7.10 Days of Weekend with Two Valued and Multi-valued Membership



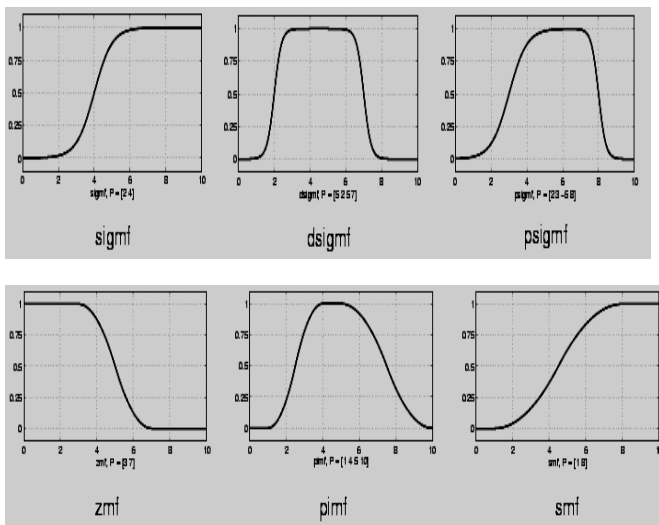


Fig. 7.11 Different Membership Functions

4) If-Then Rules: Fuzzy sets and fuzzy operators are the subjects and verbs of fuzzy logic. These if-then rule statements are used to formulate the conditional statements that comprise fuzzy logic. A single fuzzy if-then rule assumes the form if x is A then y is B where A and B are linguistic values defined by fuzzy sets on the ranges (universes of discourse) X and Y, respectively. If part of the rule "x is A" is called the antecedent or premise, while the then-part of the rule "y is B" is called the consequent or conclusion.

7.4 STEPS FOR CREATING FUZZY MODEL

Step 1: Fuzzy Inputs

The first step is to take inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions.

Step 2: Apply Fuzzy Operators

Once the inputs have been fuzzified, we know the degree to which each part of the antecedent has been satisfied for each rule. If a given rule has more than one part, the fuzzy logical operators are applied to evaluate the composite firing strength of the rule.

Step 3: Apply the Implication Method

The implication method is defined as the shaping of the output membership functions on the basis of the firing strength of the rule. The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set. Two commonly used methods of implication are the minimum and the product.

Step 4: Aggregate all Outputs

Aggregation is a process whereby the outputs of each rule are unified. Aggregation occurs only once for each output variable. The input to the aggregation process is the truncated output fuzzy sets returned by the implication process for each rule. The output of the aggregation processes the combined output fuzzy set.

Step 5: Defuzzify

The input for the defuzzification process is a fuzzy set (the aggregated output fuzzy set), and the output of the defuzzification process is a crisp value obtained by using some defuzzification method such as the centroid, height, or maximum.

This is the approach to obtain the required approximate values from the fuzzy theory. Here in this report firstly the values of RCD and RCL have given and the natural frequencies are found out. In fuzzy logic approach, the natural frequencies have been used as input for fuzzy model creation and by using these values of natural frequencies the RCD and RCL have been found out. This is an approach to find out the RCD and RCL values by fuzzy logic. It is possible that the fuzzy logic may not give accurate solution since it deals with fuzzy rules and logical parameters.

The membership functions are common for all beam models except that their display range is different. The input and output variables are shown in Fig. 7.12 and the rules viewer window for first five beam model is shown in Fig. no. 7.13 and for next five beam models shown in Fig. no. 7.14. the result of fuzzy logic or outputs from fuzzy logic has been tabulated in table no. 7.1.

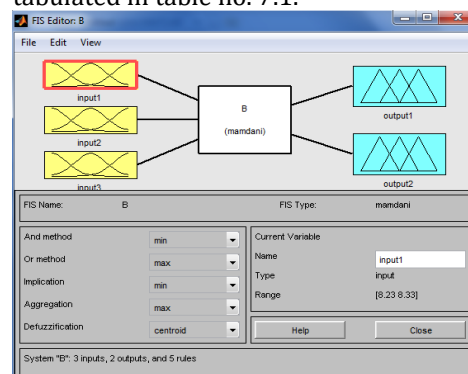


Fig. 7.12 Input and Output variables

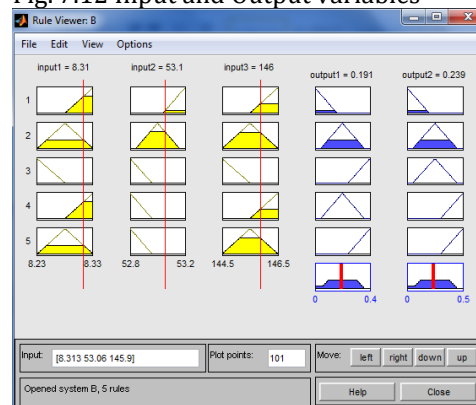


Fig. 7.13 Rules viewer for first five beam models

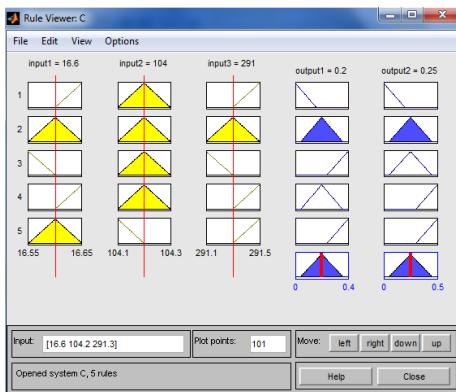


Fig. 7.14 Rules viewer last five beam models

If we compare the results of fuzzy logic parameters and theoretical parameters, we came to know that the results from fuzzy logic method gives better results.

8.RESULTS AND DISCUSSION

Discussion based on the output generated by Theoretical analysis and the information supplemented by FEA analysis in ANSYS and experimental analysis is as follows:

It is already known that the natural frequency decreases as the crack depth increases in a structural part. Firstly determination of natural frequency of different modes of vibration is done for un-cracked beam theoretically (solving Euler’s Equation for Beam in vibration analysis), then FEA analysis in ANSYS and by using FFT analyser in experimental work. Here total 10 models have been used taking different combinations of relative crack location and relative crack depth. Several steps have been shown to develop a natural frequency modal based on FEA which is explained through an example and all the frequency values are tabulated in chapter 5 along with the knowledge of finite element analysis. Certain steps are followed to carry on analysis by FFT analyser for experimentation. Several decisions are made to carry out experimental work in chapter 6. The brief information has been obtained due to experimentation. It is clear from analysis that the natural frequency of different modes of vibration can be precisely obtained from these methods and tabulated in table 8.1. The variation of natural frequency with crack depth and location is shown in Fig. 8.1 and 8.2

Now considering the second approach in which fuzzy inference system is used. The present fuzzy controller used Triangular MF for input variables and Triangular MF for output variables shown in chapter 7. Linguistic terms used in this fuzzy controller for variables are given in this chapter. The Fuzzy logic has been successfully applied to beam models and gives satisfactory results as tabulated in table 7.2.

A comparison is made in between theoretical values of natural frequencies with teensy values of natural frequencies and experimental values of natural frequencies. The result shows that all the values obtained by three methods are closed to the agreement. It is clear that crack

position can be successfully identified by Fuzzy Logic approach and can be verified by theoretical method.

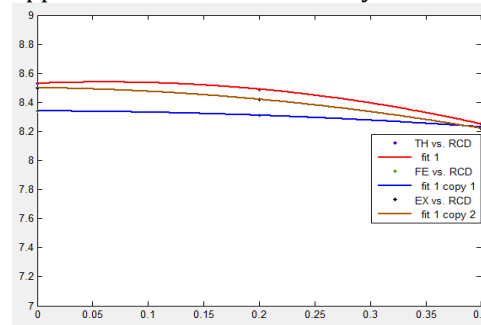


Fig. 8.1 Variation of First Natural Frequency with Crack Depth

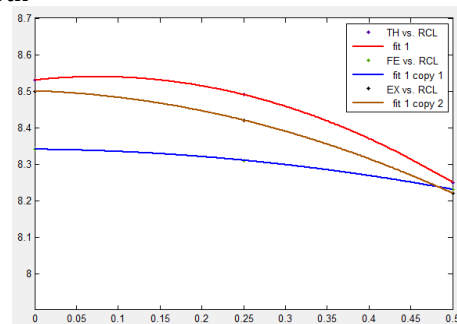


Fig. 8.2 Variation of First Natural Frequency with Crack Location

Table 8.1 Comparison of Natural frequencies by three methods

The comparison of natural frequencies shows that the results from the method of Finite Element Analysis (ANSYS) are much accurate than the other methods of calculation.

9.CONCLUSIONS AND SCOPE FOR FUTURE WORK

9.1 CONCLUSIONS

The present investigation based on the theoretical Analysis, FEA Analysis and experimental analysis draws the following conclusion.

1. As clear cut deviation, the mode shapes, natural frequencies can be detected for the cracked and Un-cracked beams at the vicinity of crack location.
2. The theoretical results, ANSYS results obtained by the methods proposed in this work and the FFT experiment result vibrations are in fair matching in terms of natural frequency.
3. The results based on ANSYS are much accurate as it is based on finite elements.
4. A new approach can be suggested which is based on the combination of both ANSYS and Fuzzy in which natural frequency obtained in ANSYS can be used as input for Fuzzy Controller for determination of accurate value of crack depth and crack location.

9.2 SCOPE FOR THE FUTURE WORK

The investigation presented here is based on theoretical, FEA and FFT analysis. It draws certain scope for future work.

- The variable cross-section beam can be analysing under excitation.
- The cracked beam can be analysing under external force.
- The dynamic response of the cracked beams can be analysed for different crack orientations.
- Stability study of the cracked beams can be done.
- Use hybrid neuron genetic technique like artificial neural network for crack detection.

"Smart crack detection of a cracked cantilever beam using fuzzy logic technology with hybrid membership functions", Journal of Engineering And Technology Research, Vol. 3(8), Aug. 2011, PP270-300

[9]MeysamSiyahMansoori, "Automatic crack detection in eggshell based on SUSAN Edge Detector using Fuzzy Thresholding", Journal of Modern Applied Science, Vol.5 No.6, Dec 2011, pp.1-9

10. REFERENCES

[1]DK.Agarwalla, D.R. Parham, "Effect of Crack on Modal Parameters of a Cantilever Beam Subjected to Vibration", journal of Procedia Engineering, vol. 51, 2013, pp. 665 - 669

[2]Z.A.Jassim,N.N.Ali,F.Mustapha,N.A.AbdulJalil."A review on the vibration analysis for a damage occurrence of a cantilever beam", Journal of Engineering Failure Analysis,vol.31,2013,pp. 442-461

[3]Kasha H. Bared, D. S. Sharma, Vishal Visa, "Crack detection in cantilever beam by frequency based method", journal of Procedia Engineering, vol. 51, 2013, pp. 770-775

[4]PankajCharan Jena, Deal R. Parham, GoutamPohit, "Faults detection of a single cracked beam by theoretical and experimental analysis using vibration signatures", IOSR Journal of Mechanical and Civil Engineering, Volume 4, Issue 3 (Nov-Dec. 2012), PP 01-18

[5]P. K. Jenna,, D. N. Tahoe, J. Nanda, D. R. K. Parham, "Effect of damage parameters on vibration signatures of a cantilever beam" journal of Procedia Engineering, vol. 38, 2012, pp.3318 - 3330

[6]N.V.NarsimhaRao,"Fault diagnosis of dynamic cracked structure using Fuzzy interface system", International Journal of Emerging Trends and Engineering Development,Vol.5,Jul 2012,pp.1-10

[7]Mohammad R. Jahanshahi, "An innovative methodology for detection and quantification of cracks through incorporation of depth perception", Journal of Machine Vision and Applications, Feb 2011, pp. 3-11

[8]Deal R. Parham And SasankaChoudhary,