Split and Non Split Neighborhood Connected Domination in Fuzzy Graphs

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Abstract - Let G be a connected fuzzy graph without isolated vertices. A dominating set D of G is said to be neighborhood connected dominating set if the induced subgraph $\langle N(D)\rangle$ is connected. A neighborhood connected dominating set D of G is a split neighborhood connected dominating set D if the induced subgraph $\langle V-D\rangle$ is disconnected. The split neighborhood connected domination number $\gamma_{snc}(G)$ of G is the minimum cardinality of a split neighborhood connected dominating set. A neighborhood connected dominating set D of G is a nonsplit neighborhood connected dominating set D if the induced subgraph $\langle V-D\rangle$ is connected. The nonsplit neighborhood connected domination number $\gamma_{nsnc}(G)$ of G is the minimum cardinality of a non split neighborhood connected dominating set. In this paper we study these parameters. Also we present some bounds on these parameters.

***Key Words:* :** neighborhood connected dominating set ,split neighborhood connected dominating set, nonsplit neighborhood connected dominating set.

1.INTRODUCTION: A fuzzy graph $G=(\sigma,\mu)$ is a set with a pair of relations

$$\sigma:V\to[0,1]\ and\ \mu:V\times V\to[0,1]\ \ such\ that\ \mu(u,v)\le\sigma(u),$$

for all $u,v\in V$.

A non empty set $D\subseteq V$ of a fuzzy graph $G=(\sigma,\mu)$ is a dominating set of G if every vertex in V-D is adjacent to some vertex in D .The domination number $\gamma(G)$ is the minimum cardinality taken over all the minimal dominating sets of G.

The order of a fuzzy graph G is O (G)= $\sum_{u\in V}\sigma(u)$

The size of a fuzzy graph G is S (G)= $\sum_{uv\in E}\mu(uv)$

A dominating set of a fuzzy graph G is a split (non split) dominating set if the induced subgraph $\langle V-D\rangle$ is disconnected (connected).

The split (non split) domination number $\gamma_s(G)[\gamma_{ns}(G)]$ is the minimum cardinality of a split(non split) dominating set.

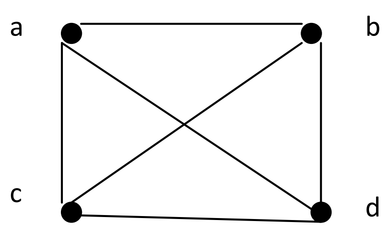
Two nodes that are joined by a path are said to be connected.

A fuzzy graph G is said to be connected if any two nodes are connected.
Let G be a connected fuzzy graph without isolated vertices. A dominating set D of G is said to be neighborhood connected dominating set if the induced subgraph $\langle N(D)\rangle$ is connected

A neighborhood connected dominating set D of G is a split neighborhood connected dominating set D if the induced subgraph $\langle V-D\rangle$ is disconnected.

A neighborhood connected dominating set D of G is a nonsplit neighborhood connected dominating set D if the induced subgraph $\langle V-D\rangle$ is connected.

Example:



$\sigma(a)=0.2,\ \sigma(b)=0.1\quad \sigma(c)=0.1\quad \sigma(d)=0.4$

Neighborhood connected dominating set D ={b,d}

$\langle N(D)\rangle$ is connected.

2.Split and nonsplit neighborhood connected domination number:

The split neighborhood connected domination number $\gamma_{snc}(G)$ of G is the minimum cardinality of a split neighborhood connected dominating set.

The nonsplit neighborhood connected domination number $\gamma_{nsnc}(G)$ of G is the minimum cardinality of a non split neighborhood connected dominating set.

2.1Propositions:

.Suppose a fuzzy graph G has γ_{snc} set, then $\gamma_{nc} \le \gamma_{snc}$

Proof:

Every split neighborhood connected dominating set of G is a neighborhood connected dominating set.

Hence the result.

.Suppose a fuzzy graph G has γ_{snc} set then $\gamma(G) \le \gamma_{snc}(G)$

Proof:

Clearly $\gamma(G) \le \gamma_{nc}(G)$

By proposition (1), $\gamma_{nc} \le \gamma_{snc}$

Hence the result

.If P_p is a path with $p \ge 4$ vertices then $\gamma_{snc}(P_p) = \left\lceil \frac{p}{2} \right\rceil$

2.2.Main Results

Theorem:

A neighborhood connected dominating set D of G is a split neighborhood connected dominating set iff there exists two vertices u, v in V-D such that every u-v path contains a vertex of D.

Proof:

Let u,v $\in V-D$ such that every u-v path contains a vertex of D. Then u and v are in different components of the induced subgraph $\langle V-D \rangle$. Thus $\langle V-D \rangle$ is disconnected and hence D is a split neighborhood connected dominating set. Conversely, suppose D is a split neighborhood connected dominating set of G. Suppose on the contrary assume that for every pair of vertices u, v in D, there exists u-v path which does not contain a vertex of D. Then $\langle V-D \rangle$ is connected which is a contradiction.Hence the proof.

Theorem:

If a connected fuzzy graph G contains an end vertex , then $\gamma_{nc}(G) = \gamma_{snc}(G)$

Let D be a γ_{nc} set of a connected graph G. Let u be an end vertex of G. Then there exist a cut vertex adjacent to u. Suppose v $\in D$. We consider the following two cases.

Case (i): Suppose $u \notin D$. Then $\langle V-D \rangle$ is disconnected. Hence the result.

Case (ii) Suppose u $\in D$ and $\langle V-D \rangle$ is connected.Then V-D has atleast two vertices. Then there exists a vertex w in V-D is adjacent to v. Then (D-u) $\cup \{w\}$ is γ_{snc} set of G.

Theorem:

If G has γ_{snc} set, then $\gamma_{snc}(G) \le p-2$

Proof:

Let D be a γ_{nc} set of a connected graph G. For some neighborhood connected dominating set D of G, $\langle V-D \rangle$ is not complete. Then there exists two non adjacent vertices u,v in V-D. Let $D' = (V-D) - \{u,v\}$

Then D $\cup$ D' is a split neighborhood connected dominating set of G.

Theorem:

If γ_{snc} set exists in a connected graph , then

$$\frac{p}{\Delta(G)+1} \le \gamma_{snc}(G) \le 2q-p$$

Proof :

Clearly $\frac{p}{\Delta(G)+1} \le \gamma(G)$ *and* $\gamma(G) \le \gamma_{snc}(G)$

Hence $\frac{p}{\Delta(G)+1} \le \gamma_{snc}(G)$

We know that if G has γ_{snc} set, then $\gamma_{snc}(G) \le p-2$

=2(p-1)-p

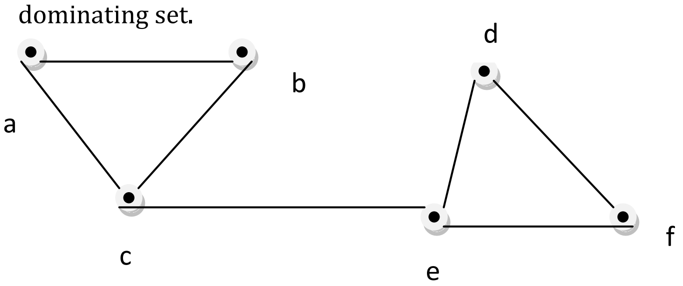
$\leq 2q - p$

3.Non split neighborhood connected domination Number

Definition:

A neighborhood connected dominating set D of G is a nonsplit neighborhood connected dominating set D if the induced subgraph $\langle V - D \rangle$ is connected. The nonsplit neighborhood connected domination number $\gamma_{nsnc}(G)$ of G is the minimum cardinality of a non split neighborhood connected dominating set. A γ_{nsnc}

set is a minimum nonsplit neighborhood connected dominating set.



$\sigma(a) = 0.2,\ \sigma(b) = 0.1,\ \sigma(c) = 0.3,\ \sigma(d) = 0.4,$
$\sigma(e) = 0.4,\ \sigma(f) = 0.1$

D={b ,f} is a nonsplit neighborhood connected dominating set.

3.1 Propositions:

.If G is a nontrivial connected fuzzy graph, then

$\gamma_{nc}(G) \leq \gamma_{nsnc}(G)$

Proof:

Every nonsplit neighborhood connected dominating set of G is a neighborhood connected dominating set.

Hence the result

For any nontrivial connected fuzzy graph G,
$\gamma_{nc}(G) = \min\{\gamma_{snc}(G), \gamma_{nsnc}(G)\}$

Theorem:

If H is a spanning subgraph of a connected graph G, then
$\gamma_{ncnc}(G) \leq \gamma_{nsnc}(H)$

Proof:

Let H be a spanning subgraph of a connected fuzzy graph G. Since every nonsplit neighborhood connected dominating set of H is a neighborhood connected dominating set of G.

4. CONCLUSIONS

In this paper we introduce the concept of split and non split neighborhood connected dominating sets in fuzzy graphs .We have obtained some bounds on these parameters.

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