Dufour effect on Unsteady MHD Flow Past an Impulsively Started Inclined Plate with Variable Temperature and Mass Diffusion

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Abstract - *Dufour effect on unsteady MHD flow past an impulsively started inclined plate with variable temperature and mass diffusion is studied here. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. The governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity profile and Skin friction have been studied for different parameters like thermal Grashof number, mass Grashof Number, Prandtl number, Dufour number, magnetic field parameter, Schmidt number and time. The effects of parameters are shown graphically and the value of the skin-friction for different parameters has been tabulated.*

Key Words: *MHD, Dufour* effect, inclined plate, variable temperature, mass diffusion.

1. INTRODUCTION

The application of Hydromagnetic incompressible viscous flow in science and engineering involving heat and mass transfer under the influence of Dufour effect is of great importance to many areas of science and engineering, like chemical engineering, mechanical engineering, biological science, petroleum engineering, biomechanics, irrigation engineering and aerospace technology. Free convective flow of visco-elastic fluid in a vertical channel with Dufour effect was studied by Acharya et al [10]. Kesavaiah and Satyanarayana [11] have considered radiation absorption and dufour effects to mhd flow in vertical surface. Alam et al[1] have analyzed Dufour and Soret effects on steady MHD combined free-forced convective and mass transfer flow past a semi-Infinite vertical plate. Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects was presented by Postelnicu [2]. Ibrahim and abdallah[3] have investigated analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall effect. Further Ibrahim[4] has worked on analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour–Soret effect and Hall effect. Dufour and Soret effects of a transient free convective flow with radiative heat transfer past a flat plate moving through a binary mixture was studied by Olanrewaju[5]. Srinivasacharya and Reddy[6] have investigated Soret and Dufour effects on mixed convection from an exponentially stretching surface. Effects of Soret, Dufour, chemical reaction and thermal radiation on MHD non Darcy unsteady mixed convective heat and mass transfer over a stretching was presented by Pal and Mondal[7]. Rajput and Kumar[8] have worked on MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Subhakar and Gangadhar [9] have analyzed Soret and Dufour effects on MHD free convection heat and mass transfer flow over a stretching vertical plate with suction and heat source/sink. We are extending their [8] work to study the effect of Dufour in the flow model. The effect of Dufour on the velocity is observed with the help of graphs, and the skin friction has been tabulated.

2. MATHEMATICAL ANALYSIS

An unsteady viscous incompressible MHD flow between two parallel electrically non conducting plates inclined at an angle α from vertical is considered. x axis is taken along the plate and y normal to it. A transverse magnetic field B_0 of uniform strength is applied on the flow. Initially it has been considered that the plate as well as the fluid is at the same temperature T_∞. The species concentration in the fluid is taken as C_∞. At time t > 0 the plate starts moving with a velocity $u = u_0$ in its own plane and temperature of the plate is raised to T_w. The concentration C near the plate is raised linearly with respect to time. So, under the above assumptions, the flow modal is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta Cos\alpha (T - T_\infty) + g\beta^* Cos\alpha (C - C_\infty) - \frac{\sigma B_0^2 u}{\rho}, \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2}, \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

with the corresponding initial and boundary conditions:

$$\left.\begin{aligned} & t \le 0: u = 0, T = T_\infty, C = C_\infty, \text{ for all y,} \\ & t > 0: u = u_0, \\ & T = T_\infty + (T_w - T_\infty)\frac{u_0^2 t}{\upsilon}, \quad \text{at y=0} \\ & C = C_\infty + (C_w - C_\infty)\frac{u_0^2 t}{\upsilon}, \\ & u \to 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty \end{aligned}\right\} \tag{4}$$

here u is the velocity of the fluid, g - the acceleration due to gravity, β-volumetric coefficient of thermal expansion, t-time, T-the temperature of the fluid, β^*-volumetric coefficient of concentration expansion, C-species concentration in the fluid, ν - the kinematic viscosity, y-the coordinate axis normal to the plates, ρ-the density, the specific heat at constant pressure, C_s-Concentration susceptibility, k-thermal conductivity of the fluid, D- the mass diffusion coefficient, D_m-is the effective mass diffusivity rate, T_w-temperature of the plate at y= 0, C_w-species concentration at the plate y= 0, B_0- the uniform magnetic field, σ- electrically conductivity.

The following non-dimensional quantities are introduced to transform equations (1), (2) and (3) into dimensionless form:

$$\left.\begin{aligned} &\bar{y}=\frac{yu_0}{\upsilon},\bar{u}=\frac{u}{u_0},\theta=\frac{(T-T_\infty)}{(T_w-T_\infty)},S_c=\frac{\upsilon}{D},\\ &\mu=\rho\upsilon,P_r=\frac{\mu C_p}{k},D_f=\frac{D_mK_TT_\infty(C_w-C_\infty)}{\upsilon C_sC_p(T_w-T_\infty)},\\ &G_m=\frac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^3},G_r=\frac{g\beta\upsilon(T_w-T_\infty)}{u_0^3},\\ &\bar{C}=\frac{(C-C_\infty)}{(C_w-C_\infty)},\bar{t}=\frac{tu_0^2}{\upsilon},M=\frac{\sigma B_0^2\upsilon}{\rho u_0^2}.\end{aligned}\right\} \tag{5}$$

Here the symbols used are:

$\bar{u}$ -is the dimensionless velocity, $\bar{t}$ - dimensionless time, θ - the dimensionless temperature, $\bar{C}$ -the dimensionless concentration, G_r - thermal Grashof number, G_r-mass Grashof number, μ-the coefficient of viscosity, P_r-the Prandtl number, S_c - the Schmidt number, D_f - Dofour number, K_T -Thermal diffusion ratio and M - the magnetic parameter,

Thus the model becomes

$$\frac{\partial\bar{u}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+G_rCos\alpha\theta+G_mCos\alpha\bar{C}-M\bar{u} \tag{6}$$

$$\frac{\partial\theta}{\partial\bar{t}}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial\bar{y}^2}+D_f\frac{\partial^2C}{\partial\bar{y}^2}, \tag{7}$$

$$\frac{\partial\bar{C}}{\partial\bar{t}}=\frac{1}{S_c}\frac{\partial^2\bar{C}}{\partial\bar{y}^2}, \tag{8}$$

with the following boundary conditions:

$$\left.\begin{aligned} &\bar{t}\le 0:\bar{u}=0,\theta=0,\bar{C}=0, \quad \text{for all } \bar{y},\\ &\bar{t}>0:\bar{u}=1,\theta=\bar{t},\bar{C}=\bar{t}, \text{at} \quad \bar{y}=0,\\ &\bar{u}\to 0,\theta\to 0,\bar{C}\to 0, \text{ as } \quad \bar{y}\to\infty.\end{aligned}\right\} \tag{9}$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t}=\frac{\partial^2u}{\partial y^2}+G_rCos\alpha\theta+G_mCos\alpha C-Mu, \tag{10}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}+D_f\frac{\partial^2C}{\partial y^2}, \tag{11}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2C}{\partial y^2}, \tag{12}$$

with the following boundary conditions:

$$\left.\begin{aligned} &t\le 0:u=0,\theta=0,C=0, \quad \text{for all y}\\ &t>0:u=1,\theta=t,C=t \quad \text{at} \quad \text{y=0}\\ &u\to 0,\theta\to 0,C\to 0 \quad \text{as} \quad y\to\infty\end{aligned}\right\} \tag{13}$$

The dimensionless governing equations (10) to (12), subject to the boundary conditions (13), are solved by the usual Laplace - transform technique with respect to time t.

$$u=\frac{e^{-\sqrt{A}y}A_{13}}{2}+(e^{-\sqrt{M}y}(2A_1+2MA_2t+A_3\sqrt{M}y+$$
$$2A_2P_r)+2A_{16}A_4(1-P_r))(\frac{G_rCos\alpha}{4M^2}-\frac{G_rD_fP_rS_cCos\alpha}{4M^2(S_c-P_r)})$$
$$-\frac{G_rD_fP_rS_cCos\alpha}{4M^2(S_c-P_r)}(e^{-\sqrt{A}y}(2A_1+2MA_2t+A_3\sqrt{M}y+$$
$$2A_2S_c)+2A_{17}A_5(1-S_c))+\frac{e^{-\sqrt{A}y}G_rCos\alpha}{4\sqrt{M}}(2\sqrt{M}t+$$
$$2A_{30}\sqrt{M}t-A_{31}y+e^{2\sqrt{A}y}A_{31}(2\sqrt{M}+y))(yMA_{11}\sqrt{P_r}$$
$$+A_6A_{16}\sqrt{\pi}-2A_{14}\sqrt{\pi}+2MA_{14}t\sqrt{\pi}+A_7A_{16}\sqrt{\pi}P_r+$$
$$2A_{14}P_r\sqrt{\pi})(\frac{G_rD_fP_rS_cCos\alpha}{4M^2\sqrt{\pi}(S_c-P_r)}-\frac{G_rCos\alpha}{4M^2\sqrt{\pi}})+$$
$$\frac{G_rD_fP_rS_cCos\alpha}{4M^2\sqrt{\pi}(S_c-P_r)}(yMA_{12}\sqrt{S_c}+A_8A_{17}\sqrt{\pi}-2A_{15}\sqrt{\pi}+$$
$$2MA_{15}t\sqrt{\pi}+A_8A_{17}\sqrt{\pi}S_c+2A_{15}S_c\sqrt{\pi})-G_mA_{10}Cos\alpha$$

$$\theta=A_{14}\left(t-\frac{y^2}{2}\right)-\frac{A_{29}y\sqrt{tP_r}}{\sqrt{\pi}}-\frac{D_fP_rS_c}{S_c-P_r}(t(A_{14}+A_{15})-$$
$$\frac{y^2(A_{14}P_r+A_{15}S_c)}{2}-\frac{y\sqrt{t}}{\sqrt{\pi}}(A_{29}\sqrt{P_r}+A_{28}\sqrt{S_c}))$$

$$c=t[(1+2\eta^2S_c)erfc(\eta\sqrt{S_c})-\frac{2\eta\sqrt{S_c}}{\sqrt{\pi}}e^{-\eta^2S_c}]$$

The expressions for the constants involved in the above equations are given in the appendix.

3. SKIN FRICTION

The dimensionless skin friction at the plate y=0 is obtained by

$$\tau=\left(\frac{du}{dy}\right)_{z=0}.$$

The numerical values of τ are given in table-1 for different parameters.

4. RESULT AND DISCUSSION

The velocity and temperature profile for different parameters like, mass Grashof number Gm, thermal Grashof number Gr, magnetic field parameter M, Dofour number, Prandtl number Pr and time t is shown in figures 1 to 12. It is observed from figure 1, that velocity of fluid decrease when the angle of inclination (α) is increased. It is observed from figure 2, when the mass Grashof number is increased then the velocity is increased. From figure 3, it is deduced that when thermal Grashof number Gr is increased then the velocity is decreased. It is observed from figure 4, that the effect of increasing values of the parameter M results in decreasing u. If Dofour number is increased then the velocity is decreased (figure 5). Further, it is observed that velocity increase when Prandtl number is increased (figure 6). When the Schmidt number increased then the velocity get increased (figure 7). Further, from figure 8, it is observed that velocities increase with time. It is observed that the temperature profile is increase when Dofour number, Prandtl number, Schmidt number and time is increased(figures 9,10,11,12).

Skin friction is given in table1. The value of τ increases with the increase in mass Grashof Number, Schmidt number, and decreases with the increase in the angle of inclination of the plate, thermal Grashof number, Prandtl number, the magnetic field ,Dufour number and time.

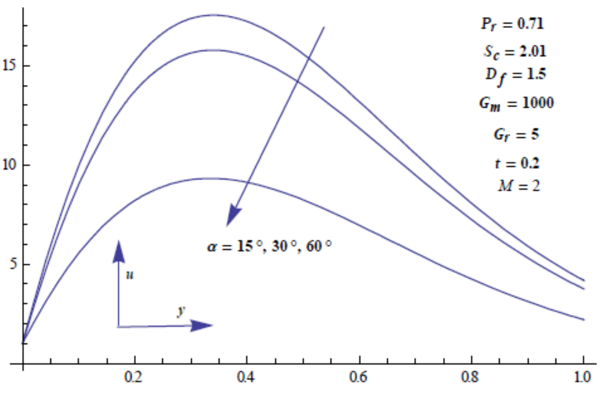

Figure 1: Velocity Profile for different value for α

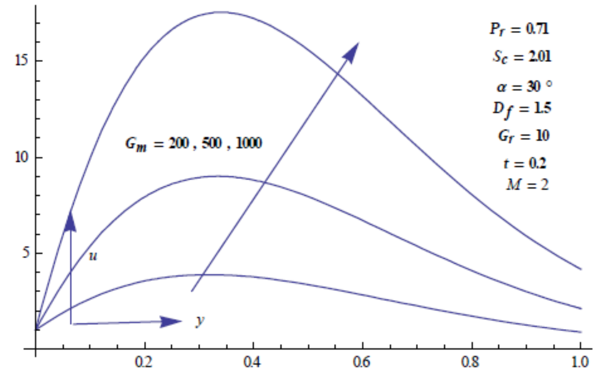

Figure 2: Velocity Profile for different value for *Gm*

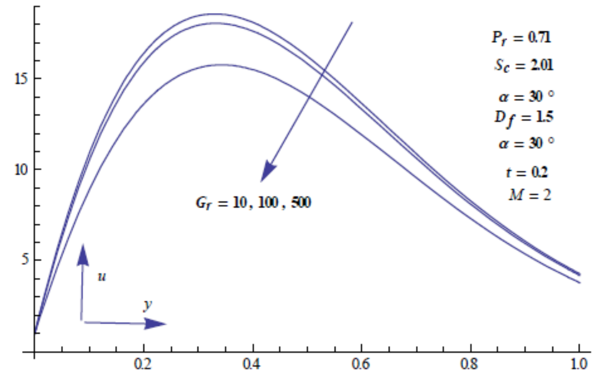

Figure 3: Velocity Profile for different value for *Gr*

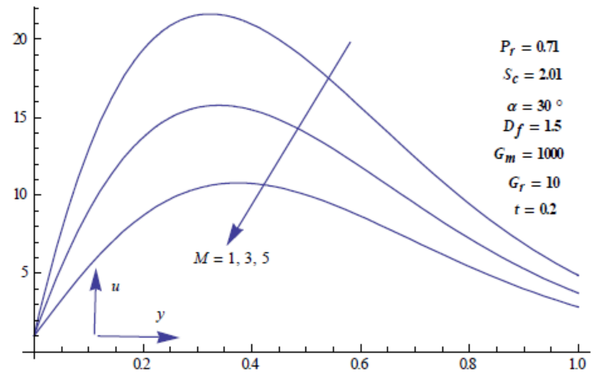

Figure 4: Velocity Profile for different value for *M*

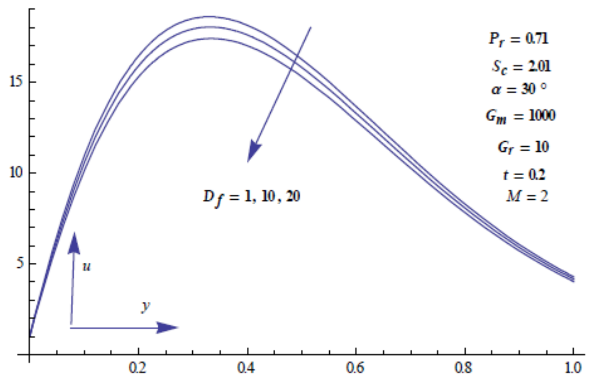

Figure 5: Velocity Profile for different value for *Df*

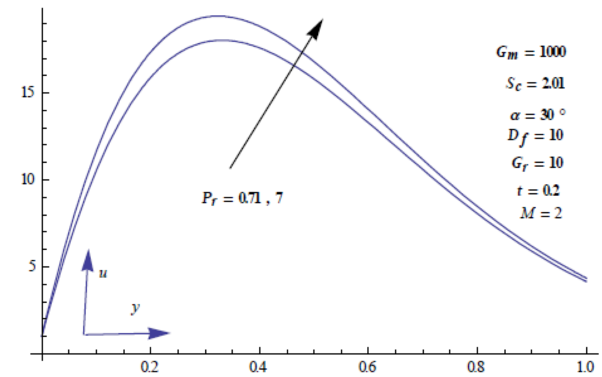

Figure 6: Velocity Profile for different value for *Pr*

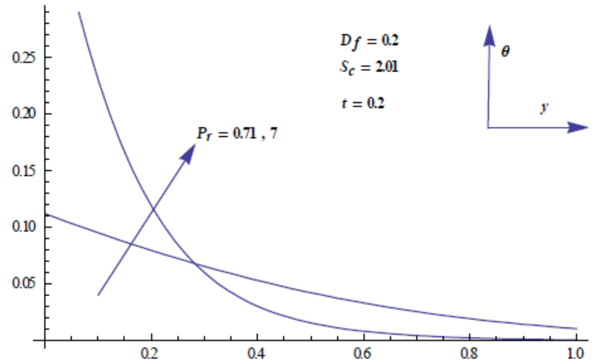

Figure 10: Temperature profile for different value for *Pr*

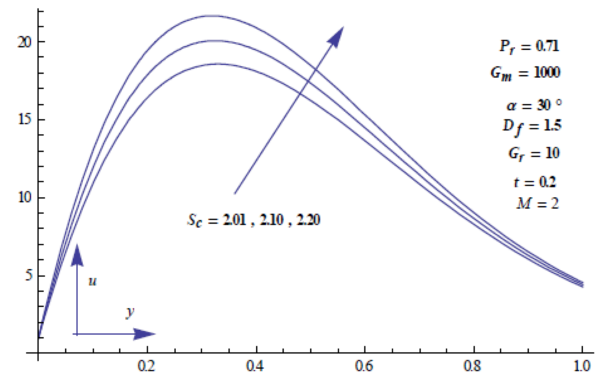

Figure 7: Velocity Profile for different value for *Sc*

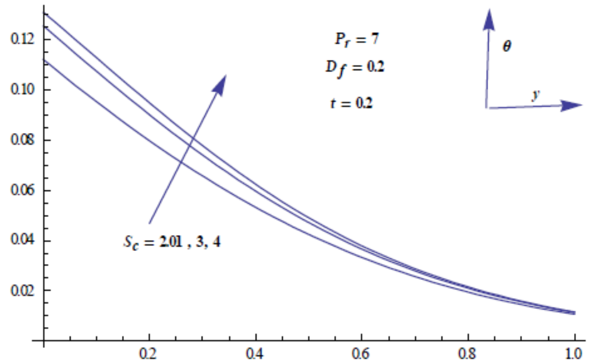

Figure 11: Temperature profile for different value for *Sc*

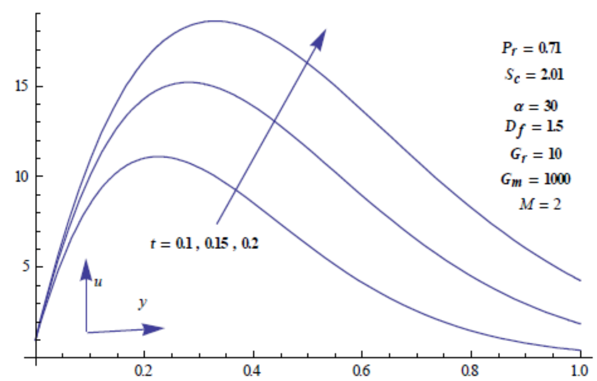

Figure 8: Velocity Profile for different value for *t*

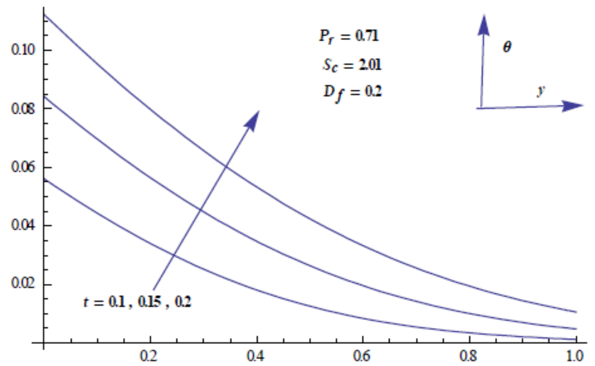

Figure 12: Temperature profile for different value for *t*

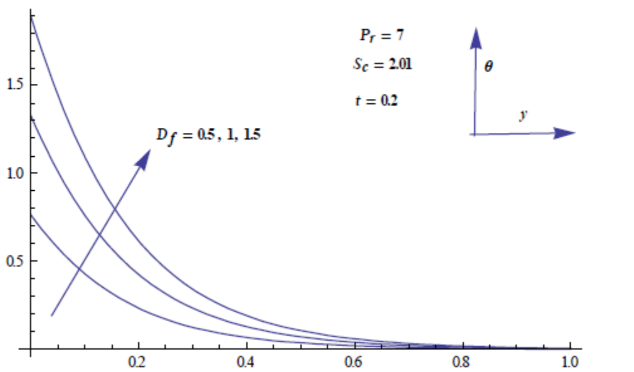

Figure 9: Temperature profile for different value for *Df*

TABLE1. Skin friction for different Parameters.

α (in degrees)	*M*	*Pr*	*Sc*	*Gm*	*Gr*	D_f	*t*	τ
15	2	7	2.01	10^3	5.0	1.5	0.2	12.3133
30	2	7	2.01	10^3	5.0	1.5	0.2	10.8604
45	2	7	2.01	10^3	5.0	1.5	0.2	8.54905
30	3	7	2.01	10^3	5.0	1.5	0.2	33.8919

30	5	7	2.01	10^3	5.0	1.5	0.2	16.0122
30	2	5	2.01	10^3	5.0	1.5	0.2	13.8856
30	2	7	3.00	10^2	5.0	1.5	0.2	-102.988
30	2	7	2.01	10^3	10	1.5	0.2	-52.3886
30	2	7	3.00	10^3	5.0	1.0	0.2	-103.042
30	2	7	4.00	10^3	5.0	1.5	0.3	3.02459

5. CONCLUSIONS

In this paper a theoretical analysis has been done to study of Dufour effect on unsteady MHD flow past an impulsively started inclined plate with variable temperature and mass diffusion. Solutions for the model have been derived by using Laplace - transform technique. Some conclusions of the study are as below:

- Velocity increases with the increase in mass Grashof Number, Prandtl number, Schmidt number and time.
- Velocity decreases with the increase in the angle of inclination of the plate, thermal Grashof number, the magnetic field and Dufour number.
- Concentration near the plate increases with the increase in Prandtl number, Dufour number. Schmidt number and time.
- Skin friction increases with the increase in mass Grashof Number, Schmidt number.
- Skin friction decreases with the increase in the angle of inclination of the plate, thermal Grashof number, Prandtl number, the magnetic field , Dufour number and time.

APPENDEX:

$$A_1 = 12 + A_{18} + e^{2\sqrt{M}y}(1 - A_{19}),$$

$$A_2 = -A_1,\ A_3 + A_1 = 2(1 + A_{18}),$$

$$A_4 = -1 + A_{20} - A_{26}(1 - A_{21}),$$

$$A_5 = A_4 - (1 + A_{21})(A_{27} - A_{26}),$$

$$A_6 = -1 + A_{22} - A_{26}(1 - A_{23}),$$

$$A_7 = -A_6,\ A_9 = -A_8,$$

$$A_8 = -1 + A_{24} - A_{27}(1 - A_{25}),$$

$$A_{10} = -tA_{14} - \frac{y^2 A_{15} S_c}{2} - \frac{A_{28} y\sqrt{tS_c}}{\sqrt{\pi}},$$

$$A_{11} = 2A_{29}\sqrt{t} + yA_{14}\sqrt{P_r \pi},$$

$$A_{12} = 2A_{28}\sqrt{t} + yA_{15}\sqrt{S_c \pi},$$

$$A_{13} = 1 + A_{30} + e^{2\sqrt{A}y} A_{31},$$

$$A_{14} = -1 + erf\left[\frac{y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{15} = -1 + erf\left[\frac{y\sqrt{S_c}}{2\sqrt{t}}\right],$$

$$A_{16} = e^{\frac{Mt}{-1+P_r} - y\sqrt{\frac{MP_r}{-1+P_r}}},$$

$$A_{17} = e^{\frac{Mt}{-1+S_c} - y\sqrt{\frac{MS_c}{-1+S_c}}},$$

$$A_{18} = erf\left[\frac{2\sqrt{M}t - y}{2\sqrt{t}}\right],$$

$$A_{19} = erf\left[\frac{2\sqrt{M}t + y}{2\sqrt{t}}\right],$$

$$A_{20} = erf\left[\frac{y - 2t\sqrt{\frac{MP_r}{-1+P_r}}}{2\sqrt{t}}\right],$$

$$A_{21} = erf\left[\frac{y + 2t\sqrt{\frac{MP_r}{-1+P_r}}}{2\sqrt{t}}\right],$$

$$A_{22} = erf\left[\frac{2t\sqrt{\frac{M}{-1+P_r}} - y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{23} = erf\left[\frac{2t\sqrt{\frac{M}{-1+P_r}} + y\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{24} = erf\left[\frac{2t\sqrt{\frac{M}{-1+S_c}} - y\sqrt{S_c}}{2\sqrt{t}}\right],$$

$$A_{25} = erf\left[\frac{2t\sqrt{\frac{M}{-1+S_c}} + y\sqrt{S_c}}{2\sqrt{t}}\right],$$

$$A_{26} =, e^{2y\sqrt{\frac{MP_r}{-1+P_r}}},$$

$$A_{27} =, e^{2y\sqrt{\frac{MS_c}{-1+S_c}}},$$

$$A_{28} =, e^{\frac{-y^2 S_c}{4t}},$$

$$A_{29} =, e^{\frac{-y^2 P_r}{4t}},$$

$$A_{30} = erf\left[\frac{2t\sqrt{M} - y}{2\sqrt{t}}\right],$$

$$A_{31} = erfc\left[\frac{2t\sqrt{M} + y}{2\sqrt{t}}\right],$$

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