Special Pairs of Rectangles and Narcissistic Numbers of Order 3 and 4

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Abstract - *We search for infinitely many pairs of rectangles such that in each pair the sum of their areas represents the Narcissistic numbers of order 3 and 4. Also we present the number of pairs of primitive and non-primitive rectangles.*

***Key Words*: Rectangle, Narcissistic numbers of order 3 and 4, primitive,non-primitive.**

1.INTRODUCTION

The ability to count dates back to prehistoric times is evident from archaeological artifacts, such as a 10,000-year-old bone from the Congo region of Africa with tally marks scratched upon it—signs of an unknown ancestor counting something. Very near the dawn of civilization, people had grasped the idea of "multiplicity" and thereby had taken the first step towards the study of numbers. Number theory**,** branch of mathematics concerned with properties of the positive integers (1, 2, 3, …). Sometimes called "higher arithmetic," it is among the oldest and most natural of mathematical pursuits.

For more ideas and interesting facts on numbers and special numbers like dhuruva number, nasty number and jarasandha number one can refer [1-5].For Ideas on rectangles and special numbers [6-9] has been studied. Recently in [10-12], Special Pythagorean triangles and rectangles in connections with narcissistic numbers of order 3 and 5 are obtained.

In this communication we search for infinitely many pairs of rectangles such that in each pair the sum of their areas represents the Narcissistic numbers of order 3 and 4. Also we present the number of pairs of primitive and non-primitive rectangles.

2.Basic Definitions

Definition 1-Narcissistic Numbers:

An n-digit number which is the sum of n^{th} power of its digits is called an n-narcissistic number. It is also known as Armstrong number.

Definition 2:

A rectangle is said to be primitive if u, v are of opposite parity and gcd(u,v)=1, where $x = u + v;\ y = u - v$ and $u > v > 0$.

3.METHOD OF ANALYSIS

Let R_1, R_2 be two distinct rectangles with dimensions p,q and x,y such that $p = u + v, q = u - v$ $(u > v > 0)$ and $x = w + v, y = w - v$ $(w > v > 0)$ respectively provided u,w,v are generators.

Let A_1, A_2 represent the areas of R_1, R_2 such that $A_1 + A_2$ equals the narcissistic numbers of order 3 and 4.

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The above relation which leads to the equation

$$u^2 + w^2 - 2v^2 = \text{narcissistic numbers of order 3 and 4} \quad (1)$$

case(i)

$$u^2 + w^2 - 2v^2 = 153 (\text{narcissistic number of order 3}) \quad (2)$$

After performing numerical computations, it is noted that there are 2 distinct values for u, v, w satisfying (2). For simplicity and clear understanding, we have presented the values of u, v, w, A_1 and A_2 in Table 1.

Table 1

S.No	u	v	w	A_1	A_2	$A_1 + A_2$
1	12	6	9	108	45	153
2	17	14	16	93	60	153

It is clear that out of 2 pairs of rectangles, one pair is non-primitive and in the other, one rectangle is primitive and the other is non-primitive.

case(ii)

$$u^2 + w^2 - 2v^2 = 370 (\text{narcissistic number of order 3}) \quad (3)$$

Proceeding as in case(i) we see that there are 6 distinct values for u,v,w satisfying (3)

Table 2

S.No	u	v	w	A_1	A_2	$A_1 + A_2$
1	18	3	8	315	55	370
2	18	7	12	275	95	370
3	19	6	9	325	45	370
4	27	20	21	329	41	370
5	26	21	24	235	135	370
6	48	45	46	279	91	370

It is clear that out of 6 pairs of rectangles, 2 pairs are primitive and in the remaining pairs, one rectangle is primitive and the other is non-primitive.

case(iii)

$$u^2 + w^2 - 2v^2 = 371 (\text{narcissistic number of order 3}) \quad (4)$$

Proceeding as in case(i) we see that there are 6 distinct values for u,v,w satisfying (4)

Table 3

S.No	u	v	w	A_1	A_2	A_1+A_2
1	18	1	7	323	48	371
2	17	3	10	280	91	371
3	15	5	14	200	171	371
4	18	11	17	203	168	371
5	22	13	15	315	56	371
6	25	17	18	336	35	371

It is clear that out of 6 pairs of rectangles, in each pair, one rectangle is primitive and the other is non-primitive.

case(iv)

$$u^2+w^2-2v^2=407 \text{(narcissistic number of order 3)} \quad (5)$$

Proceeding as in case(i) we see that there are 8 distinct values for u,v,w satisfying (5)

Table 4

S.No	u	v	w	A_1	A_2	A_1+A_2
1	20	1	3	399	8	407
2	20	3	5	391	16	407
3	19	3	8	352	55	407
4	16	3	13	247	160	407
5	21	7	8	392	15	407
6	19	7	12	312	95	407
7	20	9	13	319	88	407
8	27	19	20	368	39	407

It is clear that out of 8 pairs of rectangles, in each pair, one rectangle is primitive and the other is non-primitive.

case(v)

$$u^2+w^2-2v^2=1634 \quad \text{(narcissistic number of order 4)} \quad (6)$$

Proceeding as in case (i) we see that there are 16 distinct values for u,v,w satisfying (6)

Table 5

S.No	u	v	w	A_1	A_2	A_1+A_2
1	40	1	6	1599	35	1634
2	39	2	11	1517	117	1634
3	35	4	21	1209	425	1634
4	30	5	28	875	759	1634
5	34	7	24	1107	527	1634
6	41	8	9	1617	17	1634
7	40	9	14	1519	115	1634
8	26	13	36	507	1127	1634
9	40	15	22	1375	259	1634
10	46	23	24	1587	47	1634
11	45	26	31	1349	285	1634
12	44	27	34	1207	427	1634
13	41	28	39	897	737	1634
14	53	40	45	1209	425	1634
15	59	44	45	1545	89	1634
16	56	47	54	927	707	1634

It is clear that out of 16 pairs of rectangles, 11 pairs are primitive and in the remaining pair, one rectangle is primitive and the other is non-primitive.

case(vi)

$$u^2+w^2-2v^2=8208 \text{(narcissistic number of order 4)} \quad (7)$$

Proceeding as in case(i) we see that there are 32 distinct values for u,v,w satisfying (7)

Table 6

S.No	u	v	w	A_1	A_2	A_1+A_2
1	67	1	61	4488	3720	8208
2	89	1	17	7920	288	8208
3	75	2	51	5621	2597	8208
4	83	5	37	6864	1344	8208
5	91	7	5	8232	-24	8208
6	80	8	44	6336	1872	8208
7	91	11	13	8160	48	8208
8	89	11	23	7800	408	8208
9	85	11	35	7104	1104	8208
10	47	11	79	2088	6120	8208
11	89	13	25	7752	456	8208
12	87	15	33	7344	864	8208
13	68	16	64	4368	3840	8208
14	87	21	39	7128	1080	8208
15	79	23	55	5712	2496	8208
16	71	23	65	4512	3696	8208
17	84	24	48	6480	1728	8208
18	91	31	43	7320	888	8208
19	89	31	47	6960	1248	8208
20	75	33	69	4536	3672	8208
21	89	37	55	6552	1656	8208
22	85	37	61	5856	2352	8208
23	93	39	51	7128	1080	8208
24	79	41	73	4560	3648	8208
25	96	49	60	6815	1199	8208
26	101	49	53	7800	408	8208
27	93	51	69	6048	2160	8208
28	107	59	61	7968	240	8208
29	91	59	83	4800	3408	8208
30	97	61	79	5688	2520	8208
31	103	61	71	6888	1320	8208
32	100	64	80	5904	2304	8208

It is clear that out of 32 pairs of rectangles, 30 pairs are non- primitive and the remaining 2 pairs are primitive.

case(vii)

$$u^2 + w^2 - 2v^2 = 9474 \text{ (narcissistic number of order 4)} \qquad (8)$$

Proceeding as in case(i) we see that there are 13 distinct values for u,v,w satisfying (8)

Table 7

S.No	u	v	w	A_1	A_2	A_1+A_2
1	91	4	35	8265	1209	9474
2	70	5	68	4875	4599	9474
3	74	7	64	5427	4047	9474
4	89	8	41	7857	1617	9474
5	95	14	29	8829	645	9474
6	95	16	31	8769	705	9474
7	86	23	56	6867	2607	9474
8	101	28	29	9417	57	9474
9	100	29	34	9159	315	9474
10	106	43	44	9387	87	9474
11	86	43	76	5547	3927	9474
12	107	50	55	8949	525	9474
13	91	58	89	4917	4557	9474

It is clear that out of 13 pairs of rectangles, 10 pairs are primitive and in the remaining pairs, one rectangle is primitive and the other is non-primitive.

4. CONCLUSION

To conclude, one may search for the connections between the pairs of rectangles and other Narcissistic numbers of higher order and other number patterns.

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