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Vibration analysis of rotating tapered composite beam

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Abstract

The present work deals with the vibration analysis of rotating tapered composite beam. The characteristics of rotating beams differ significantly from non-rotating beams. Centrifugal inertia force due to rotational motion causes variation in bending stiffness, which naturally results in variations in natural frequencies and mode shapes. Moreover, the stiffness property of composite structures can be easily modulated through changing their fiber orientation angles and number of layers. The taper of the composite beam changes not only its geometric properties but also the stiffness of the oblique plies. This causes the mechanical behavior of the tapered composite beam to be different from that of the uniform beam. Hence the objective of the present work is to conduct vibration analysis of Rotating Composite Taper beam. Analysis is carried out using finite element analysis based software ANSYS. In the present work the effect of rotating speed, fiber angle, hub radius ratio, taper ratio on natural frequency of rotating composite taper beam to be investigated.

Keywords: *vibration analysis, tapered beam, composite, natural frequency, hub radius, fiber angle*

I. INTRODUCTION

Composite structures are increasingly being used in aerospace, mechanical and automotive industries due to their high strength-to-weight and stiffness-to-weight ratios. The laminated tapered beams are used in engineering applications, such as turbine blades, helicopter blades. In modern engineering, the heavy metallic beams are gradually being substituted by composite beams because of their high stiffness and high modulus to weight ratios. Rotating composite beam structures generally found in engineering applications, including robotics, wind turbine blades, and helicopter rotors.

The vibration analysis of rotating composite beams has been performed by a number of researchers. H. Kargarnovin *et al*. [1] Discussed about an analytical approach for the free vibration analysis of generally laminated composite beams with shear effect and rotary inertia based on Timoshenko beam theory. Lagrange multipliers method is employed for the free vibration analysis of generally laminated composite beam. Metin Aydogdu *et al.* [2] Investigated Flapwise vibration of rotating composite beams based on different beam theories such as Euler–Bernoulli, Timoshenko and Reddy beam theories. The Ritz method with the algebraic polynomials is used in the formulation of the problem. The effect of the displacement field has been investigated using different theories.

Hong HeeYoo *et al.*[3] to[4] used Timoshenko beam theory in order to examine behavior of flapwise vibration of rotating multilayer composite beam. Investigated vibration analysis of rotating tapered cantilever beams.The derived equations (governing stretching and bending motions), which are coupled through gyroscopic coupling terms, are all linear, so they can be directly used for the vibration analysis including the coupling effect, which could not be considered in the conventional modeling method. With the coupling effect ignored, the analysis results are consistent with the results obtained by the conventional modeling method. With the coupling effect considered, eigenvalue loci veerings and mode shape variations could be observed through numerical study.

F. Bakhtiari-Nejad *et al.*[5] discussed about nonlinear free vibration analysis of rotating composite Timoshenko beams. The Galerkin discretization approach was applied on the linear updated equations of motion for

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study on the linear free vibration, finding eigenpairs, of the rotating anti-symmetric cross-ply laminated Timoshenko beam. Faruk Firat Calim [6] Discussed about Free and forced vibrations of non-uniform composite beams in the Laplace domain. The Timoshenko beam theory is adopted in the derivation of the governing equation. The material of the beam is assumed to be homogeneous, linear elastic and anisotropic and the effects of shear deformation, rotary inertia, non-uniformity of the cross-section are considered in the formulation.

Ramazan-Ali Jafari-Talookolaei *et al*.[7] Analytical solution for the free vibration characteristics of the rotating composite beams with a delamination. The Hamilton principle is used to derive the coupled governing differential equations and boundary conditions for a rotating delaminated beam taking into account the effects of shear deformation, rotary inertia, material couplings (bending–tension, bending–twist, and tension–twist couplings), and Poisson's effect. B. P. Pate1et.al.[8] Investigated Free Vibrations Analysis of Laminated Composite Rotating Beam using C' Shear Flexible Element .The governing equations for the free vibration of rotating beam are derived using Lagrange's equation of motion. The element employed is based on shear flexible theory. It also includes in plane and rotary inertia terms.

Jun Li *et al*. [9] Presented Comparison of various shear deformation theories for free vibration of laminated composite beams with general lay-ups. A unified type of assumed displacement field is considered and the influences of shear deformation, rotary inertia and Poisson effect are included in the formulation. Free vibration of rotating tapered beams using the dynamic stiffness method. The governing differential equation of motion of the rotating tapered beam in free flap bending vibration is derived for the most general case using Hamilton's principle, allowing for the effects of centrifugal stiffening, an arbitrary outboard force and the hub radius term. The expressions for bending rotation, shear force and bending moment at any cross-section of the beam are also obtained in explicit analytical form. The differential equation of motion and the corresponding variation finite element formulation for the tapered composite beam have been developed. [10-15].

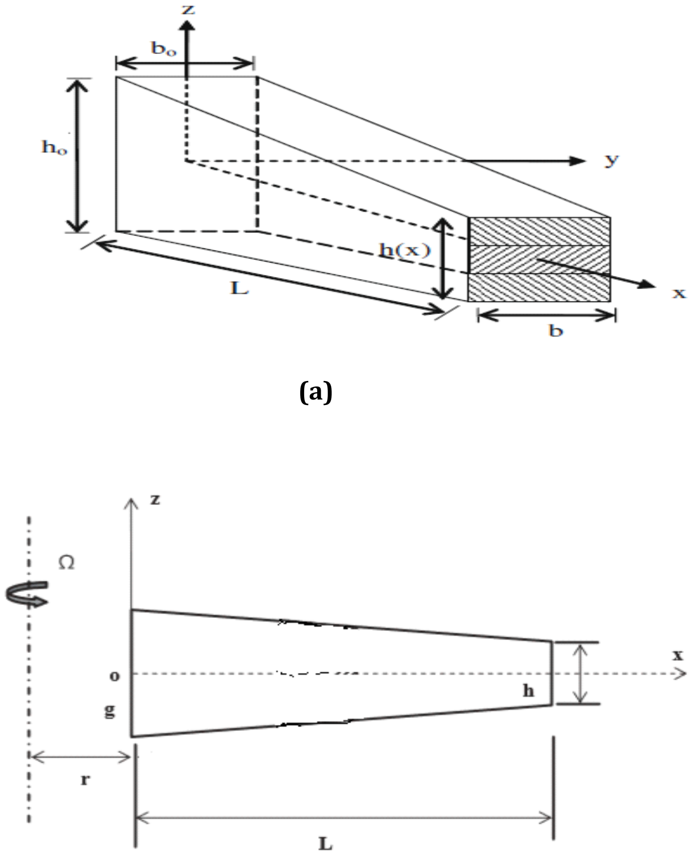
The vibration analysis for a tapered beam generally approximates the shape function because this approach can produce more accurate results and can simplify the computation. O.Ozdemir *et al*.[16] Presented flapwise bending vibration analysis of a rotating tapered cantilever Bernoulli-Euler beam by using differential transform method. The effects of taper ratio, nondimensional angular velocities and nondimensional hub radius are discussed. Jung woo lee *et al.* [17] Discussed about free vibrational analysis of tapered beam using transfer matrix method. This study examines the effect of various taper ratios on the Eigen pairs of these beams, in which the height of the cross section along the length is linearly reduced. Jagadish Babu Gunda *et al.* [18] A numerically efficient superelement is proposed as a low degree of freedom model for dynamic analysis of rotating tapered beams. The element developed in this work can be used to model rotating beam substructures as a part of complete finite element model of helicopters and wind turbines.

V.Ravindra *et al*.[19] Investigated vibration analysis of tapered beam. The equation of motion of a double tapered cantilever Euler beam is derived to find out the natural frequencies of the structure. Indrajit V. Tate et al.[20] presents modal strain energy based structural damage detection methodology for damage localization of a composite cantilever beam. The method is based on decomposing the modal strain energy of each element into two parts which are associated with element's translational and rotational coordinates respectively.

free vibration analysis of rotating laminated composite beams are investigated by authors [1] to [5].Authors [6] to[9] Presented Comparison of various shear deformation theories for free vibration of laminated composite beams with general lay-ups and the effects of non-uniformity parameters and angle of fiber orientation on dynamic behavior are investigated. Vibration analysis of tapered composite beams using a higher-order finite element and parametric study on tapered composite beams is conducted, considering the effects of taper angle, types of taper, boundary conditions, laminate configuration and geometric parameters on the undamped natural frequencies are investigated by authors [10] to [12]. Most of the authors concentrated on vibration analysis on rotating uniform beams. Less work is commenced on rotating tapered composite beams so,there is a scope for vibration analysis of rotating tapered composite beams.

II. THEORY

The vibration analysis for a tapered beam generally approximates the shape function because this approach can produce more accurate results and can simplify the computation. For tapered Euler-Bernoulli beam which is examined in this study, the total beam length is assumed to be significantly larger than its cross- sectional dimensions. Therefore, the shear deformation and rotary inertia can both be ignored and only the bending vibration needs to be investigated.



(a)

(b)

Fig 1 Notation and coordinate system of a rotating tapered beam with constant width and a linearly varying depth

The range of problems considered includes beams with linearly varying taper in depth and of the cross-section along the length. In terms of cross-sectional properties this essentially means that the area and the second moment of area of the beam can vary in two different ways. If '*L*' is the length of the beam, bo and ho are the width and depth of fixed end, which tapers to a depth *h* and which is fixed at o to a rigid hub with radius *r* is shown in fig 1(b).the beam is assumed to be rotating at a constant rotational speed Ω.The taper ratio C is such that $0<C<1$, Clearly, when $C=0$ the beam is uniform. However, when $C=1$, the thin end of the beam converges to a sharp point which makes the elastic critical buckling load of the beam zero [13].

III. NUMERICAL RESULTS

The present work deals with vibration analysis of rotating composite tapered beams. the analysis performed on two models which are composite uniform beam and composite tapered beam.

a. Vibration analysis of uniform composite beam:

A composite uniform beam was developed for vibration analysis. To validate the uniform beam model it is compared with previous literature. The natural frequencies are obtained by taking the dimensions and the material properties for a uniform fixed free composite beam studied in [20]. Beam has 8 layer of glass fiber, thickness of each layer is 0.33mm and therefore total thickness of the beam (T) = 2.64mm.

The mechanical properties of beam are as follows
E_{11}=40.5GPa; E_{22}=12.5GPa; G_{12}=3.96GPa; μ_{12}=0.29
Density of material is 1900 kg/m3
The cross section of the quasi-isotropic ($[0/45/-45/90]_s$) beam. Table 1 shows the comparison of frequencies and it shows a good agreement with numerical results presented by Indrajit. V[20].

Table 1: comparison of frequencies to reference paper with present work results

Mode	Reference [20] (Hz)	Present work (Hz)
1	18.06	18.420
2	113.13	115.40
3	316.48	322.92

The analysis is performed to determine the effect of Rotating Speed, hub radius ratio, fibre angle, and number of layers of the composite on Natural frequency of the rotating Composite beam. the first three natural

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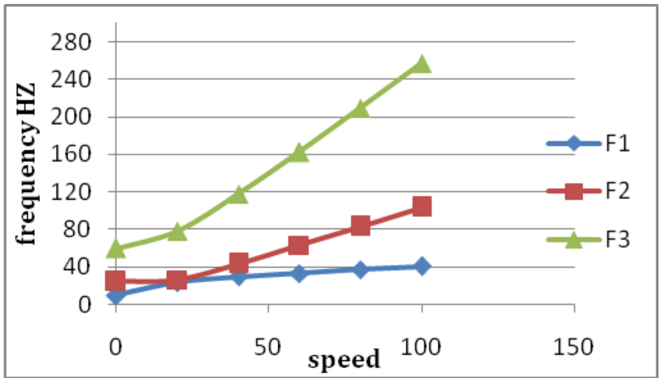
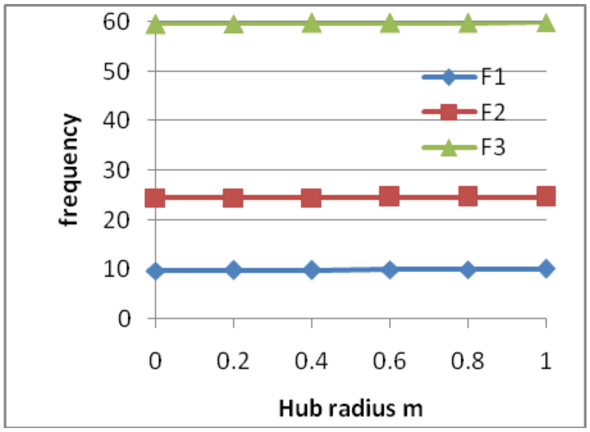
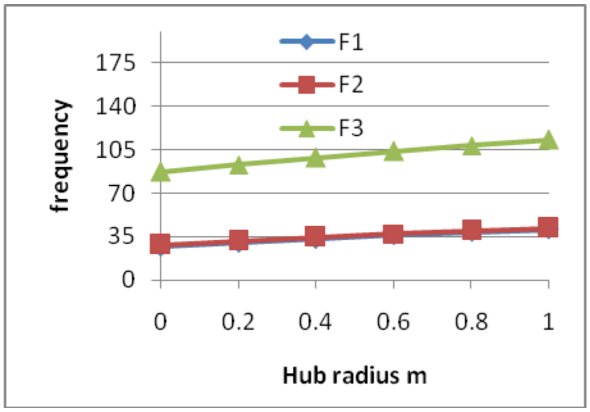
frequencies are obtained and plotted in chart-1 which represents the variation of frequency with respect to speed. It is observed that the natural frequencies increase with speed due to increase in centrifugal force leads to increase in stiffness of the beam

Chart -1: Variation of first three natural Frequencies with respect to speed



To study the effect of hub radius on natural frequency of the rotating composite beam. The natural frequencies are obtained for various hub radius ratio from 0 to 1m at 2 rps, 25 rps and 50 rps speeds. The variation of natural frequencies with respect to hub radius ratio is plotted in chart-2, chart-3 and chart-4 for speeds 2 rps, 25 rps and 50 rps respectively. From these figures it is observed that the frequency increases with increasing hub radius ratio. This increase is more pronounced especially for high rotation speed i.e. the hub radius affects the frequency in medium and higher speeds. This is due to increase centrifugal force is very less in lower speeds irrespective of hub radius ratio. At higher speeds increase in hub radius ratio leads to increase in centrifugal force causes increment in frequency.





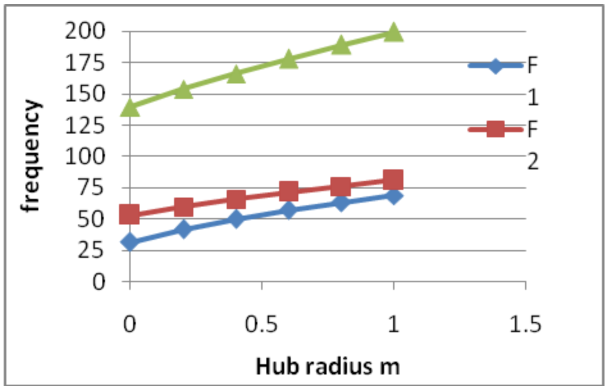
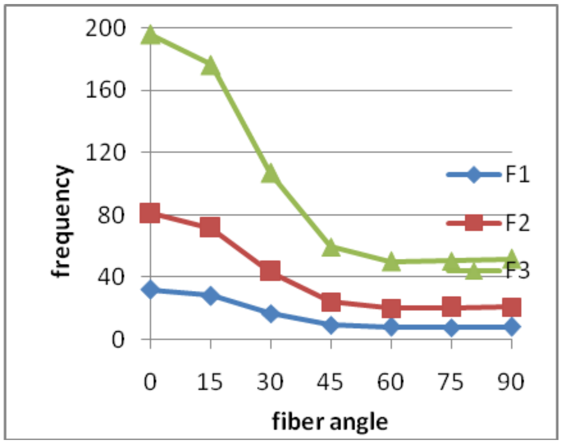


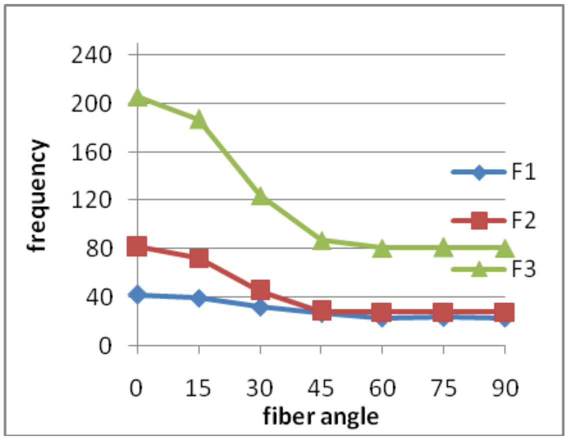
Chart-2: Variation of first three natural frequencies with respect to hub radius ratio at 5 rps

Chart-3: Variation of first three natural frequencies with respect to hub radius ratio at 25 rps

Chart-4: Variation of first three natural frequencies with respect to hub radius ratio at 50 rps

To study the effect of fibre orientation angle on natural frequency of rotating composite beam. First three natural frequencies obtained at 2Rps, 25Rps and 50Rps speed for various fiber angles. The variation of frequency with respect to fibre angle at 2rps, 25rps and 50rps speed is plotted in chart-5, chart-6 and chart-7 respectively. From the Figures it is observed that the natural frequencies of composite beam decreases as the angle of fibre orientation increases. The natural frequencies decrease monotonically as the fibre Orientation angle increases. It is observed that the natural frequency variation occurs rapidly in the range of 30°≤θ≤60°However, the variation occurs slowly in the range of 60°≤θ≤90°.





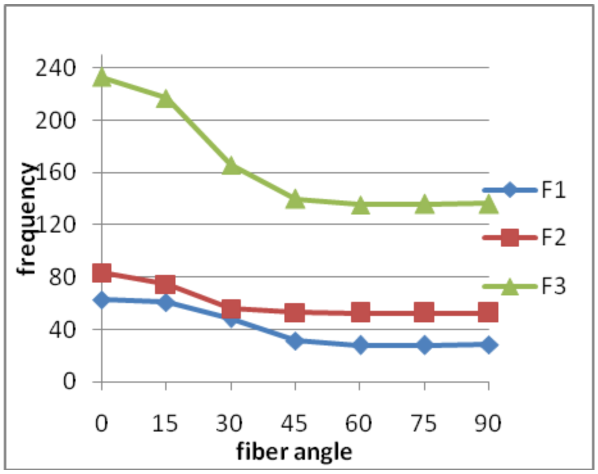


Chart-5: variation of first three natural frequencies with respect to fiber angle at 2rps

Chart-6: variation of first three natural frequencies with respect to fiber angle at 25rps

Chart-7: variation of first three natural frequencies with respect to fiber angle at 50rps

b. Vibration analysis of a rotating tapered composite beam:

The vibration analysis of rotating tapered composite beam is performed by creating a tapered beam model. In order to validate the tapered beam model results are compared with the previous literature based on dynamic stiffness method [13] and differential transformation method [14]. Variation of the first three non-dimensional natural frequencies ω, with respect to the non-dimensional rotational speed η, and the non-dimensional hub radius r, for taper ratio c=0.5 and n=1 by present model and previous methods represented in table 4. The comparison shows good agreement with numerical results.

Table 2: comparison of tapered beam frequencies to reference paper with present work

η	Dynamic stiffness method Ref [13]ω_1	Differential transform method Ref [14]	Based on ANSYS present	Dynamic stiffness method Ref [13] ω_2	Differential transform method Ref [14]	Based on ANSYS present
0	3.826	------	3.837	18.317	---------	17.972
1	3.986	3.986	3.897	18.47	18.47	18.194
2	4.43	4.43	4.42	18.937	18.937	18.65

3	5.092	5.092	5.138	19.683	19.683	19.369
4	5.878	5.878	5.7053	20.685	20.685	20.118
5	6.743	---------	6.599	21.905	-------	21.109

After good agreement with previous results vibration analysis is carried out on present model with following specifications. Total length (L) =0.45m, thickness of each layer is 0.00575m, A Composite beam with four layers and the stacking Sequence is [45/-45/-45/45] is considered. And taper ratio is c=0.5. The material properties are represented in table3.

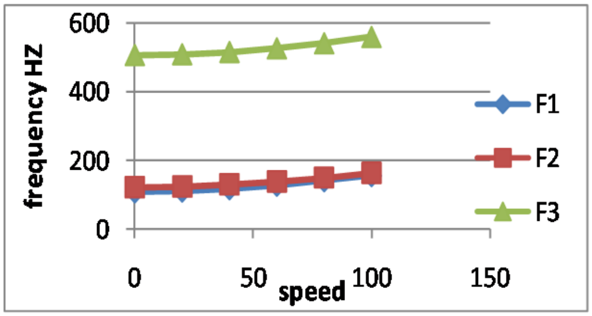
Table 3: Mechanical properties of the rotating tapered composite beam

E3 (GPa)	E1 = E2 (GPa)	G12 (GPa)	G23 = G23 (GPa)	μ21	μ32 = μ 31	ρ (kg/m3)
145	9.6	3.4	4.1	0.5	0.3	1389

IV. Results and Discussion

The first three natural frequencies are obtained using composite tapered beam model to determine the effect of Rotating Speed, hub radius ratio, fiber angle, taper ratio and number of layers of the composite on Natural frequency of the rotating tapered Composite beam. Chart -8 shows the variation of the first three natural frequencies with respect to rotating speed. It is clearly observed that all the natural frequencies increase with increase in rotating speed. This is due to increase in centrifugal force which causes increment of stiffness of the beam.

Chart-8: Variation of first three natural frequencies of with respect speed (tapered beam)



The first three natural frequencies are obtained for different hub radius ratios at three speeds are obtained and tabulated in table 4. Table represents increment of frequencies for higher speeds, there is no considerable variation of speed with respect to hub radius ratio at low speed.

Table 4: The effect of hub radius ratio (r_H) and rotational speed parameter (η) on the natural frequencies (ωi) for a rotating tapered composite beam for the case of C = 0.5

r_H	η = 2			η = 25			η = 50		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0	117.05	546.70	1104.0	120.37	549.93	1104.6	129.83	559.56	1106.2
0.2	117.07	546.72	1104.0	122.29	551.83	1104.8	136.78	566.99	1107.2
0.4	117.08	546.73	1104.0	124.17	553.72	1105.1	143.38	574.32	1107.2
0.6	117.09	546.74	1105.2	126.03	555.61	1105.3	149.69	581.56	1109.2
0.8	117.10	546.75	1104.0	127.87	557.49	1105.6	155.73	588.70	1110.1

First three natural frequencies of tapered composite beam at moderate speed 25 rps and 50 rps are plotted in chart-9 and chart-10 respectively. From these figures it is observed that the hub radius affects the frequency in moderate and higher speeds. Natural frequencies increase as the rotational speed increases and the rate of increase becomes larger with the increase in hub radius. This is due to the effect of centrifugal tension force which increases as the angular velocity and the hub radius are increased.

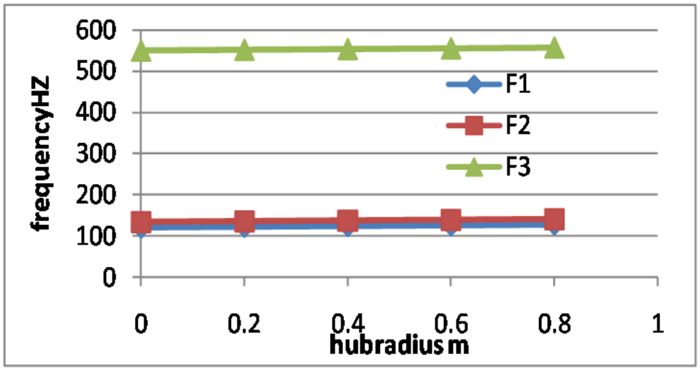


Chart-9: Variation of first three natural frequencies with respect to hub radius at 25rps

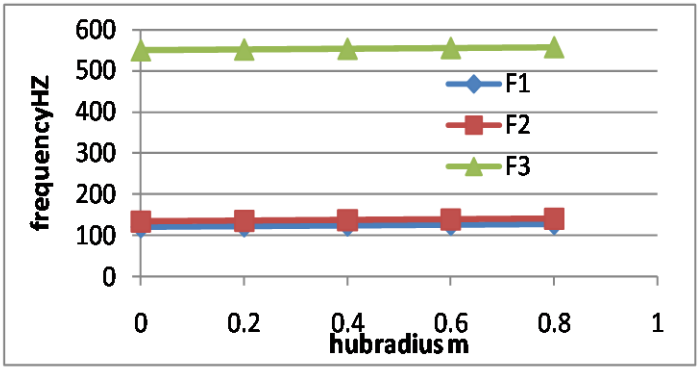
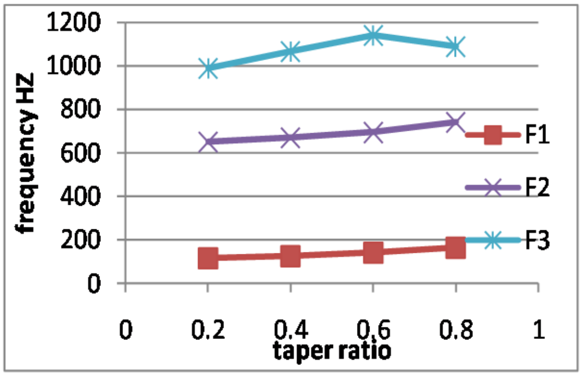


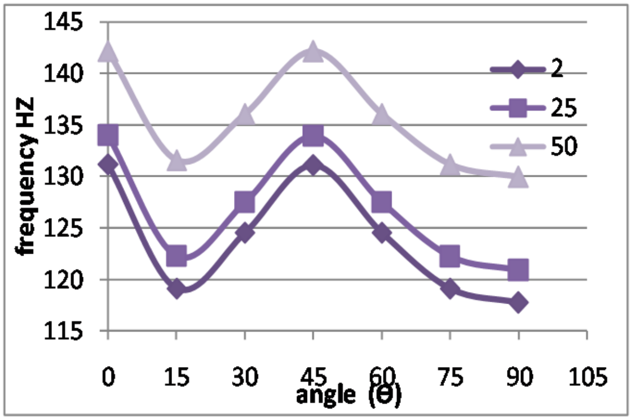
Chart-10: Variation of first three natural frequencies with respect to hub radius at 50rps

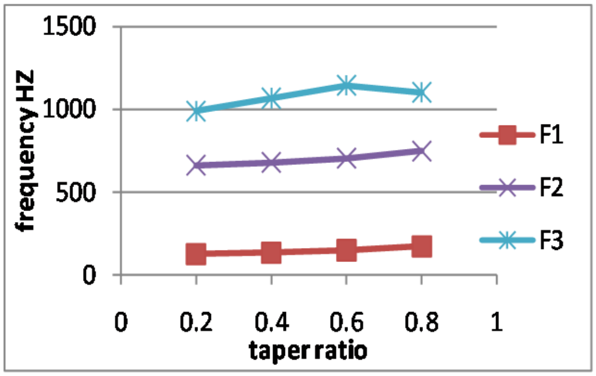
Table 5: The effect of taper ratio (c) and rotational speed parameter (η) on the natural frequencies (ω_i) for a rotating tapered composite beam for the case of rH/L=0

C	η = 2			η = 25			η = 50		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0.2	110.37	607.96	988.58	113.73	611.08	989.12	123.28	620.39	990.74
0.4	114.37	568.21	1065.6	117.70	571.39	1066.2	127.20	580.91	1067.8
0.5	117.05	546.70	1104.0	120.37	549.93	1104.6	129.83	559.56	1106.2
0.6	120.47	523.98	1141.4	123.76	527.25	1141.9	133.15	537.02	1143.5
0.8	131.49	475.27	1088.2	134.68	478.66	1091.6	143.84	488.78	1101.7

To study the effect of taper ratio (C) the natural frequencies are obtained for different taper ratios at 5 rps, 25 rps and 50 rps speeds. The taper ratio C is such that 0<C<1, clearly, when C=0 the beam is uniform. However, when C=1, the thin end of the beam converges to a sharp point which makes the elastic critical buckling load of the beam zero [13]. The taper ratios considered are 0.2 to 0.8 with step increment of 0.2. The variations natural frequencies with respect to taper ratio are plotted for speed 5 rps, 25 rps and 50 rps in chart-11, chart-12 and chart-13 respectively. The figures represents the first natural frequency is increases with increasing taper ratio while the second and third natural frequencies are decreased by increasing taper ratio. The decrease in taper ratio takes place due to reduction in cross-sectional area by taper which causes decrease in stiffness of the beam.







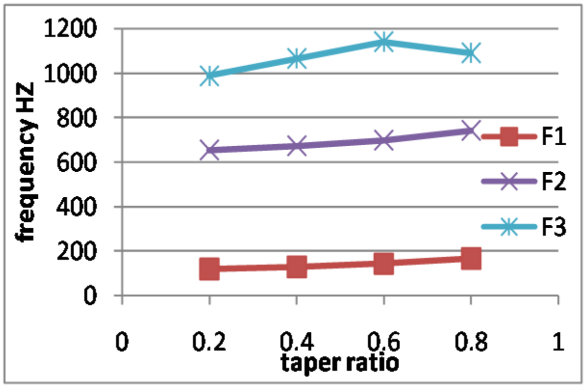


Chart-11: variation of first three natural frequencies with respect to taper ratio at 2 rps

Chart-12: variation of first three natural frequencies with respect to taper ratio at 25 rps

Chart -13: variation of first three natural frequencies with respect to taper ratio at 50 rps

The effect of fibre angle on natural frequency of the beam is studied by obtaining the frequencies for three speeds. The variation of frequencies with respect to fibre angle are plotted for first, second and third frequencies at three speeds in chart-14, chart-15 and chart-16 respectively. It shows the effect of natural frequency with respect to the fibre angle orientation. Natural frequency decreases monotonically as the fibre Orientation angle increases. The natural frequencies of composite tapered beam decreases as the angle of fibre orientation increases, But the natural frequency variation reaches peaks at fibre angle 45 degree the change in all three frequencies are decreased.

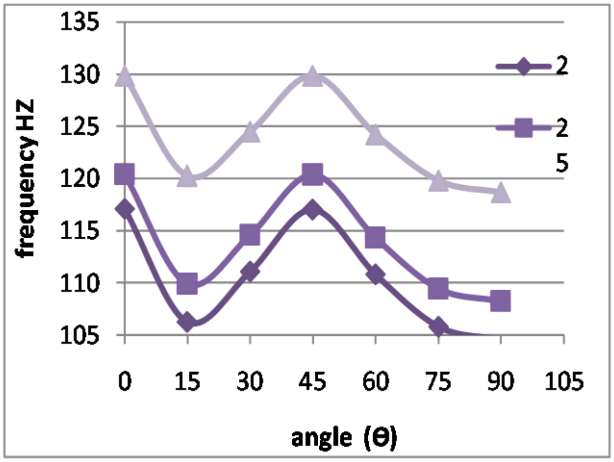
Chart-14: variation of first natural frequency with respect to fiber angle at three speeds

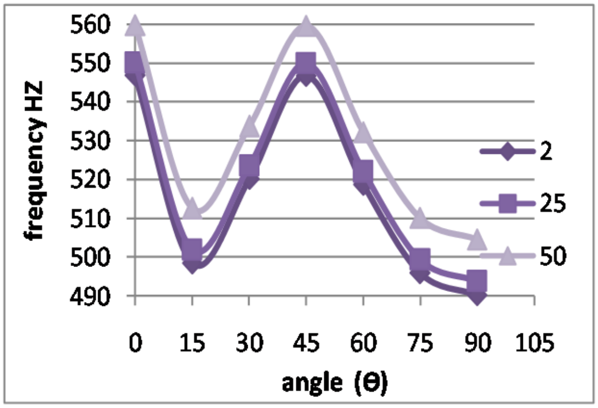
Chart-15: variation of second natural frequency with respect to fiber angle at three speed

Chart-16: variation of third natural frequency with respect to fiber angle at three speeds

V. Conclusion

In the present work, vibration analysis of rotating tapered composite beam is performed. A composite uniform beam model and composite tapered beam model are developed and validated with available literature. A numerical result shows good agreement. Further analysis is carried out to determine the effect of rotating speed, fiber angle, hub radius ratio, taper ratio on natural frequency of rotating composite taper beam. It is





concluded that the rotating speed affects the frequency due to variation of centrifugal force which leads variation in stiffness of the beam. The hub radius ratio affects the frequency at higher speeds only. The natural frequencies of composite tapered beam decreases as the angle of fibre orientation increases, But the natural frequency variation reaches peaks at fibre angle 45 degree the change in all three frequencies are decreased. The taper angle of the composite beam changes not only its geometric properties but also the stiffness of the oblique plies. This causes the mechanical behavior of the tapered composite beam to be different from that of the uniform beam. Therefore, it is necessary to consider the effect of the laminate stiffness of the composite beam caused by the taper angle.

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