Validity of Principle of Exchange of Stabilities of Rivilin- Ericksen Fluid Permeated with Suspended Particles in Porous Medium Under The effect of Rotation with Variable Gravity Field By Using Operator Method.

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Abstract - *The Thermosolutal Convection in Rivilin-Ericksen elastico-viscous fluid in porous medium is considered to include the effect of suspended particles and rotation under variable gravity. In the present, to establish the Principle Of Exchange of Stabilities (PES) by using a method of a Positive Operator, a generalization of a positive matrix Wherein, the resolvent of the linearized stability operator is analyzed which is in the form of a composition of certain integral operators. Motivated by the analysis of Weinberger and the works of Herron , our objective here is to extend this analysis of positive operator to establish the PES. It is established by the method of positive operator of Weinberger that PES is valid for this problem under sufficient conditions and* g *(z) is nonnegative throughout the fluid layer .*

Keywords : **Rivilin-Ericksen, Positive Operator, Principle of Exchange of Stabilities, linearized Stability Operator, Suspended Particles.**

1. INTRODUCTION

Convection in porous medium has been studied with great interest for more than a century and has found many applications in underground coal gasification, solar energy conversion, oil reservoir simulation, ground water contaminant transport, geothermal energy extraction and in many other areas. With the growing importance of non–Newtonian fluids in modern technology and industries, the investigations of such fluids are desirable. Rivlin–Ericksen [1955] proposed a non–linear theory of a class of isotropic incompressible elastico–viscous fluids with the constitutive relations

$$T_{ij} = -p\delta_{ij} + \tau_{ij},$$

$$\tau_{ij} = 2\left(\mu + \mu' \frac{\partial}{\partial t}\right)e_{ij},$$

Here μ' is the coefficient of viscoelasticity. Such elastico–viscous fluids have relevance and importance in agriculture, communication appliances, chemical technology and in biomedical applications.

Keeping in mind the importance of non–Newtonian fluids in modern technology, industries, chemical engineering and owing to the importance of variable gravity field in astrophysics etc.

Our objective here is to extend the analysis of Weinberger & Rabinowitz's [1969] based on the method of positive operator to establish the PES for a more general convective problems from the domain of non-Newtonian fluid, namely,Thermal convection of a Rivlin - Ericksen fluid in porous medium heated from below with variable gravity. Lata [2010,2012,2013,2015,2016] has exclusively worked for the validity of principle of exchange of stabilities by using Positive Operator Method.

The present work is partly inspired by the above discussions, and the works of Herron [2000,2001] and the striking features of convection in non-Newtonian fluids in porous medium and motivated by the desire to study the above discussed problems. Our objective here is to extend the analysis of Weinberger & Rabinowitz's [1969] based on the method of positive operator to establish the PES to these more general convective problems from the domain of non-Newtonian fluid. In the present paper, the problem of Thermal convection of a Rivilin- Ericken fluid layer heated from below in porous medium under the effect of suspended particles with variable gravity g(z) is positive throughout the fluid layer in porous medium heated from below with variable gravity is analyzed and using the positive operator method, when g (z)(the gravity field) It is established from the present analysis that PES is valid .

2. Mathematical Formulation of the Physical Problem

Consider an infinite horizontal Rivlin -Ericksen fluid layer of thickness d bounded by the horizontal plane z=0 and z=d in porous medium permeated with suspended particles. This layer is heated from below so that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained across the layer. This layer is acted upon by a vertical variable gravity field $\vec{g}(0,0-g(z))$.

3. Basic hydrodynamical equations governing the physical configuration

The basic hydrodynamic equations that govern the physical configurations (c.f. Rivlin and Ericksen [1955], Spiegel and Veronis [1960], Stokes [1966] and Scanlon and Segal [1973)]) under Boussinesq approximation[1903] are given by;

3.1. Equation of Continuity

$$\nabla.\vec{v} = 0 \tag{1}$$

3.2. Equations of Motion

$$\frac{1}{\varepsilon}\left[\frac{\partial \vec{v}}{\partial t} + \frac{1}{\varepsilon}(\vec{v}.\nabla)\vec{v}\right] = -\frac{\nabla p}{\rho_0} + \vec{X}\left(1+\frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\left(\upsilon + \upsilon'\frac{\partial}{\partial t}\right)\vec{v} + 2(q\times\Omega) + \frac{SN}{\rho_0\varepsilon}(\vec{u}-\vec{v}) \tag{2}$$

3.3. The equations of motion and continuity for the particles

The force exerted by the fluids on the particle is equal and opposite to the force exerted by the particles on fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The buoyancy force on the particles are neglected. Inter –particle reactions are ignored for we assume that the distances between particles are quite large as compared with their diameter . If mN is the mass of particles per unit volume, then the equation of motion and continuity for the particles ,under the above assumptions are :

$$m\,N\left[\frac{\partial \vec{u}}{\partial t} + \frac{1}{\varepsilon}(\vec{u}.\nabla)\vec{u}\right] = SN(\vec{v}-\vec{u}) \tag{3}$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla(N.\vec{u}) = 0 \tag{4}$$

3.4. The equation of heat conduction

Since the volume fraction of the particles is assumed small, the effective properties of the suspension are taken to be those of the clean fluid. Assuming that the particles and fluid are in thermal equilibrium, the equation of heat conduction is given as;

$$\left[\rho_0 c_v \varepsilon + \rho_s c_s (1-\varepsilon)\right]\frac{\partial T}{\partial t} + \rho_0 c_v (\vec{v}.\nabla)T + mNc_{pt}\left(\varepsilon\frac{\partial}{\partial t} + \vec{u}.\nabla\right)T = q\nabla^2 T \tag{5}$$

3.5. The equation of state

$$\rho = \rho_0 \left[1 - \alpha (T - T_0)\right] \quad (6)$$

In the above equations, p, ρ, υ, υ', ε, k_1, α, $\vec{v}(u, v, w)$,Tand $\vec{X}$ denote respectively the pressure, density, temperature, viscosity, viscoelasticity, medium porosity, medium permeability, thermal coefficient of expansion, the external force field, gradient operator; and velocity of the fluid; S=6 $\pi\mu\eta'$, (η' being particle radius), is the Stokes' drag coefficient, $\overline{x} = (x,y,z)$, ρ_s, c_s, ρ, c_v denote the density and heat capacity of solid (porous) matrix and fluid respectively, c_{pt} the heat capacity of the particles and q the "effective" thermal conductivity of the fluid . $\vec{u}(x, t)$ and N(x,t) denote the filter velocity and number density of the suspended particles, respectively.

Following the usual steps of the linearized stability theory, it is easily seen that the nondimensional linearized perturbation equations governing the physical problem described by equations (1)-(4) can be put into the following forms, upon ascribing the dependence of the perturbations of the form $exp\left[i\left(k_x x + k_y y\right) + \sigma t\right]$, $(\sigma = \sigma_r + i\sigma_i)$ (*c.f.* Chandrasekhar [1961] and Siddheshwar and Krishna [2001]);

$$\left[\frac{\sigma}{\varepsilon}\left(\frac{\Gamma\sigma + B}{\Gamma\sigma + 1}\right) + \frac{1}{P_l}\{1 + F\sigma\}\right](D^2 - k^2)w = -R_T k^2 g(z)\theta \quad (7)$$

$$(\Gamma\sigma + 1)\left[D^2 - k^2 - (E + h\varepsilon)\Pr\sigma\right]\theta = -R_T\ (H + \Gamma\sigma)w \quad (8)$$

together with following dynamically free and thermally and electrically perfectly conducting boundary conditions

$$w = 0 = \theta = D^2 w \quad \text{at } z = 0 \text{ and } z = 1 \quad (9)$$

In the forgoing equations, z is the real independent variable, $D \equiv d/dz$ is the differentiation with respect to z, k^2 is the square of the wave number, $\Pr = \frac{\upsilon}{\kappa}$ is the thermal Prandtl number, $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability, Where $\Gamma = \frac{m\upsilon}{Sd^2}$ and H=h+1 and B=b+1 where b= $\Gamma S\sigma$, $R_T = R^2 = \frac{g_0 \alpha\beta d^4}{\kappa\upsilon}$ is the thermal Rayleigh number, $P_l = \frac{k_1}{d^2}$ is the dimensionless medium permeability and $\Pr = \frac{\upsilon}{\kappa}$ is the Prandtl number and $F = \frac{\upsilon'}{d^2}$ is the dimensionless Rivilin-Ericksen parameter, $R^2 = \frac{g_0\alpha\beta d^4}{\kappa\upsilon}$ is the thermal Rayleigh number, $\sigma(= \sigma_r + i\sigma_i)$ is the complex growth rate associated with the perturabations and w, θ are the perturbations in the vertical velocity, temperature, respectively.

Hence, the system of equations (7) and (8) together with boundary conditions (9) constitutes an eigen value problem for σ for given values of the parameters k^2, R, F Pr, H ,B and Γ for the present problem.

The system of equations (7)-(8) together with the boundary conditions (9) constitutes an eigenvalue problem for σ for the given values of the parameters of the fluid and a given state of the system is stable, neutral or unstable according to whether σ_r is negative, zero or positive.

It is remarkable to note here that equations (7)-(8) contain a variable coefficient and an implicit function of σ , hence as discussed earlier the usual method of Pellew and Southwell is not useful here to establish PES for this general problem. Thus, we shall use the method of positive operator to establish PES.

3. METHOD OF POSITIVE OPERATOR

We seek conditions under which solutions of equations (7)-(8) together with the boundary conditions (9) grow. The idea of the method of the solution is based on the notion of a 'positive operator', a generalization of a positive matrix, that is, one with all its entries positive. Such matrices have the property that they possess a single greatest positive eigenvalue, identical to the spectral radius. The natural generalization of a matrix operator is an integral operator with non-negative kernel. To apply the method, the resolvent of the linearized stability operator is analyzed. This resolvent is in the form of certain integral operators. When the Green's function Kernels for these operators are all nonnegative, the resulting operator is termed positive. The abstract theory is based on the Krein – Rutman theorem [1962], which states that;

"If a linear, compact operator A, leaving invariant a cone $\hbar$, has a point of the spectrum different from zero, then it has a positive eigen value λ , not less in modulus than every other eigen value, and this number corresponds at least one eigen vector $\phi \in \hbar$ of the operator A, and at least one eigen vector $\varphi \in \hbar^*$ of the operator A^* ". For the present problem the cone consists of the set of nonnegative functions.

To apply the method of positive operator, formulate the above equations (7) and (8) together with boundary conditions (9) in terms of certain operators as;

4. MATHEMATICAL ANALYSIS BY USING THE METHOD OF POSITIVE OPERATORS

In the following analysis, we shall first of all construct an equivalent eigen -value problem to the eigen -value problem described by equations (7) and (8) together with boundary conditions (9) in terms of certain operators.

Let $(-D^2+k^2)\,w= mw$

and define

$$\tilde{M}w = mw, \qquad w \in \mathrm{dom}\tilde{M}$$
$$\tilde{M}^2 w = m^2 w, \qquad w \in \mathrm{dom}\left(\tilde{M}\tilde{M}\right)$$
$$\tilde{M}\theta = m\theta, \qquad w \in \mathrm{dom}\tilde{M}$$

We have the following forms of equations (2A.38) and (2A.39)

$$\left[\frac{\sigma}{\varepsilon}\left(\frac{\Gamma\sigma+H}{\Gamma\sigma+1}\right)+\frac{1}{p_1}\{1+F\sigma\}\right]M\,w = Rk^2 g(z)\theta \qquad (10)$$

$$(\Gamma\sigma+1)\left[D^2-a^2-(E+h\varepsilon)p_1\sigma\right]\theta = -R\,(H+\Gamma\sigma)\,w \qquad (11)$$

The above define domains are contained in cone $\hbar$, where

$\hbar = L^2(0,1) = \left\{\phi \;\middle|\; \int_0^1 |\phi|^2\, dz < \infty\right\}$ is a Hilbert space with a finite magnitude, by definition [],

with scalar product

$$\langle \phi, \varphi\rangle = \int_0^1 \phi(z)\,\overline{\varphi(z)}\,dz, \quad \phi,\varphi \in \hbar$$

and norm

$$\|\phi\| = \langle \phi, \phi \rangle^{1/2}$$

So, the domain of $\tilde{M}$ is

dom $\tilde{M}$ = $\{\phi \in \tilde{\lambda} : D\phi, m\phi \in \tilde{\lambda}, \quad \phi(0) = \phi(1) = 0\}$.

In the present a case, we have $\phi = \{w, \theta\}$.

Substituting the value of θ from equation (11) in equation (12), we get

$$w = \left[\frac{\sigma}{\varepsilon}\left(\frac{\Gamma\sigma + H}{\Gamma\sigma + 1}\right) + \frac{1}{p_1}\{1 + F\sigma\}\right]^{-1} M^{-1}\left(\frac{\Gamma\sigma + B}{\Gamma\sigma + 1}\right) R^2 k^2 g(z)[M + \sigma \Pr(E + h\varepsilon)]^{-1} \tag{12}$$

$$\text{or} \quad w = K(\sigma) w \tag{13}$$

We know that $L^2(0,1)$ is a Hilbert space, so, the domain of M is

dom M = $\{\phi \in B /\ D\phi, m\phi \in B, \quad \phi(0) = \phi(1) = 0\}$.

In (12), we have

$T(E + \varepsilon h)\Pr\sigma) = (M + \{E + \varepsilon h\}\Pr\sigma)^{-1}$ and exists for

$\sigma \in T_{\frac{k}{\sqrt{\Pr(E+\varepsilon h)}}} = \left\{\sigma \in C \ \middle|\ \mathrm{Re}(\sigma) > \frac{-k^2}{(E + \varepsilon h)\Pr}, \mathrm{Im}(\sigma) = 0\right\}$ and $\|T\sigma P_r(E + \varepsilon h)\Pr\|^{-1} > \left|\sigma + \frac{k^2}{(E + \varepsilon h)\Pr}\right|$ for $\mathrm{Re}(\sigma) > -\dfrac{k^2}{\sigma(E + \varepsilon h)\Pr}$.

Now, $T(E\Pr\sigma)$ is an integral operator such that for $f \in B$,

$$T(\sigma\{E + \varepsilon h\}\Pr) f = \int_0^1 g(z, \xi; \sigma\{E + \varepsilon h\}\Pr) f(\xi) d\xi,$$

where, $g(z, \xi, (E + \varepsilon h)\Pr\sigma)$ is Green's function kernel for the operator $(M + \sigma\{E + \varepsilon h\}\Pr)$, and is given as

$$g(z, \xi, \Pr\sigma\{E + \varepsilon h\}P_r) = \frac{\cosh[r(1 - |z - \xi|)] - \cosh[r(-1 + z + \xi)]}{2r\sinh r}$$

where, $r = \sqrt{k^2 + \sigma(E + \varepsilon h)\Pr}$.

In particular, taking $\sigma = 0$, we have $M^{-1} = T(0)$ is also an integral operator.

$K(\sigma)$ defined in (12), which is a composition of certain integral operators, is termed as *linearized stability operator*. $K(\sigma)$ depends analytically on σ in a certain right half of the complex plane. It is clear from the composition of $K(\sigma)$ that it contains an implicit function of σ.

$$[I-K(\sigma)]^{-1} = \left\{I-[I-K(\sigma_0)]^{-1}[K(\sigma)-K(\sigma_0)]\right\}^{-1}[I-K(\sigma_0)]^{-1} \quad (14)$$

If for all σ_0 greater than some a,

(1) $[I-K(\sigma_0)]^{-1}$ is positive,

(2) $K(\sigma)$ has a power series about σ_0 in $(\sigma_0-\sigma)$ with positive coefficients; i.e., is positive for all n, then the h

right side of (13) has an expansion in $(\sigma_0-\sigma)$ with positive coefficients. Hence, we may apply the methods of Weinberger [1969] and Rabinowitz [1969], to show that there exists a real eigenvalue σ_1 such that the spectrum of $K(\sigma)$ lies in the set $\{\sigma : \mathrm{Re}(\sigma) \le \sigma_1\}$. This is result is equivalent to PES, which was stated earlier as "the first unstable eigenvalue of the linearized system has imaginary part equal to zero."

5. THE PRINCIPLE OF EXCHANGE OF STABILITIES (PES)

It is clear that $K(\sigma)$is a product of certain operators. Condition (1) can be easily verified by following the analysis of Herron [2000, 2001] for the present operator $K(\sigma)$.The operator $M^{-1} = T(0)$ is an integral operator whose Green's function $g(z,\xi;0)$ is nonnegative so M^{-1} = T(0) is a positive operator. It is mentioned above that $T(\{E+\varepsilon h\}\Pr\sigma)$is an integral operator its Green's function kernel g $(z,\xi,\{E+\varepsilon h\}\Pr\sigma)$ is the Laplace transform of the Green's function $\frac{1}{(E+\varepsilon h)\Pr} G\left(z,\xi;\frac{t}{(E+\varepsilon h)\Pr}\right)$ for the boundary value problem

$$\left(-\frac{\partial^2}{\partial z^2}+k^2+(E+\varepsilon h)\Pr\frac{\partial}{\partial t}\right)G = \delta(z-\xi,t),$$

where, $\delta(z-\xi,t)$ is Dirac –delta function in two-dimension,

with boundary conditions $G(0,\xi;t) = G(1,\xi;t) = G(z,\xi;0) = 0$

Following Herron [2000], by direct calculation of the inverse Laplace transform, we can have Green's function kernel g $(z,\xi;\sigma(E+\varepsilon h)\Pr)$ is the Laplace transform of the Green's function $\frac{1}{(E+\varepsilon h)\Pr} G\left(z,\xi;\frac{t}{(E+\varepsilon h)\Pr}\right)$, thus by definition [] of Laplace transform,

$$g(z,\xi;\sigma(E+\varepsilon h)\Pr) = \int_0^\infty e^{-(E+\varepsilon h)\Pr t}\,\frac{1}{(E+\varepsilon h)\Pr} G\left(z,\xi;\frac{t}{(E+\varepsilon h)\Pr}\right)dt$$

$$\left(-\frac{d}{d\sigma}\right)^n g(z,\xi;\sigma(E+\varepsilon h)\Pr) = \int_0^\infty t^n e^{-(E+\varepsilon h)\Pr t}\,\frac{1}{(E+\varepsilon h)\Pr} G\left(z,\xi;\frac{t}{(E+\varepsilon h)\Pr}\right)dt \;\ge 0$$

for al l n and for all real $\sigma_0 > -\frac{k^2}{(E+\varepsilon h)\Pr}$. So, $T((E+\varepsilon h)\Pr\sigma)$ has a power series about σ_0 in $((\sigma_0-\sigma)$ with positive coefficients forl all real $\sigma_0 > -\frac{k^2}{(E+\varepsilon h)\Pr}$

Theorem. The PES holds for (7)- (8) when g(z) is nonnegative throughout the layer

$$\sigma > \max\{-\sqrt{\{\frac{P_1H+\varepsilon F+\varepsilon\Gamma}{2(\Gamma P_1+\Gamma\varepsilon F)}\}^2-\frac{\varepsilon}{(\Gamma P_1+\Gamma\varepsilon F)}}-\frac{(P_1H+\varepsilon F+\varepsilon\Gamma)}{2(\Gamma P_1+\Gamma\varepsilon F)}),-\frac{k^2}{(E+\varepsilon h)\Pr},-\frac{B}{\Gamma}\}$$,for

$(P_1H+\varepsilon\Gamma+\varepsilon F)^2 > 4\Gamma\varepsilon(P_1+\varepsilon F)$.

Proof: As $[I-K(\sigma)]$ is a nonnegative compact integral operator for

$$\sigma_0 > \max\{-\sqrt{(\frac{P_1H+\varepsilon F+\varepsilon\Gamma}{2\Gamma P_1+\Gamma\varepsilon F})^2-\frac{\varepsilon}{(\Gamma P_1+\Gamma\varepsilon F)}}-\frac{(P_1H+\varepsilon F+\varepsilon\Gamma)}{2(\Gamma P_1+\Gamma\varepsilon F)}),-\frac{k^2}{(E+\varepsilon h)\Pr},-\frac{B}{\Gamma}\}$$, for

$(P_1H+\varepsilon\Gamma+\varepsilon F)^2 > 4\Gamma\varepsilon(P_1+\varepsilon F)$ .Thus all the conditions of the Krein-Rutman theorem are satisfied ,therefore $[I-K(\sigma)]$ has a positive eigen value σ_1, which is an upper bound for the absolute values of all the eigenvalues, and the corresponding eigen function $\phi(\sigma)$ is nonnegative. We observe that

$$[I-K(\sigma)][\phi(\sigma)] = (1-\sigma_1)\phi \ge 0,$$

Thus, if $[I-K(\sigma)]$ is nonnegative, then $\sigma_1 \le 1$,so the methods of Weinberger[] and Rabinowitz []apply and showing that "there exits a real eigenvalue $\sigma_1 \le a$ such that the spectrum of $K(\sigma)$ lies in the set "$\{\sigma \mid \mathrm{Re}(\sigma) \le \sigma_1\}$". This is equivalent to the PES.

6.Conclusions:

It is established from the present analysis that PES is valid for Rivilin- Ericken fluid layer heated from below in porous medium under the effect of suspended particles with variable gravity g(z) is positive throughout the fluid layer and $(P_1H+\varepsilon\Gamma+\varepsilon F)^2 > \Gamma\varepsilon(P_1+\varepsilon F)$. The following conclusions are deduced from the above result in the light of Remark 1.

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