

# SPARSE CHANNEL ESTIMATION BY PILOT ALLOCATION IN MIMO-OFDM SYSTEMS

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**Abstract-** The channel estimation problem in multi-input-multi-output orthogonal frequency division multiplexing (MIMO-OFDM) systems is due to multipath delay spread and high sampling rate is investigated from the compressed sensing (CS) theory. In CS theory two algorithms are proposed such as SP and CoSaMP. These two algorithms are greedy algorithms and minimize the mutual coherence of the measurement matrix. Simulation results show that estimated channel patterns designed by CS theory algorithms gives much better performance than using normal channel estimation technique patterns in term of mean square error and bit error rate of the systems.

**Keywords:** Channel estimation, Compressed sensing (CS), MIMO, OFDM, MIMO-OFDM, Pilot allocation

## 1. INTRODUCTION

Radio spectrum is a rare resource for all nations in the world. As the number of wireless subscriber grows there will be increase in demand for high rate services. The modulated carriers must be put as close as possible allowing more number of bits for transmission without any Inter Carrier Interference (ICI) and Inter Symbol Interference (ISI). This results in efficient utilization of the spectrum.

ISI can be mitigated by using special multiplexing and modulation technique called Orthogonal Frequency Division Multiplexing (OFDM). In OFDM, the high rate data information is converted into groups of parallel lower rate information and therefore extends the symbol duration, in this way making a difference to suppress the ISI. Another advantage is that bandwidth of each subcarrier can be overlapped

and mitigate the effect of ICI as these subcarriers are orthogonal to each other.

Using Multiple Antenna (MA), one can achieve high-data-rate over wireless multipath channel. The Multiple Input Multiple Output (MIMO) technology enhances the data rate without any additional transmission power or extra bandwidth. In multi-input-multi-output orthogonal frequency division multiplexing (MIMO-OFDM) systems, channel estimation is of crucial importance to the performance of coherent demodulation. It also helps obtain channel state information (CSI) to support precoding and scheduling. In broadband wireless communication, in which the delay spread could be very large but the number of significant paths is normally very small, By taking into account the inherent sparsity of the channel, sparse channel estimation [4]–[6] can give a better estimation performance than conventional channel-estimation methods such as least squares (LS) and minimal mean square error (MMSE).

## 2. MIMO-OFDM

### 2.1. OFDM

In both wireless and a wired environment OFDM technology can be employed. The basic principle of OFDM is to get a number of parallel lower data rate streams from single high data rate information. These lower data rate streams are sent simultaneously over some narrower sub-channels. Henceforth, it is called multiplexing (Frequency Division Multiplexing) technique. In OFDM the letter 'O' indicates orthogonal which is major

difference between OFDM and FDM. Due to orthogonality it is possible to get major advantages from OFDM compared to FDM. The figure shows the graphical illustration of OFDM receiver and transmitter structure and it also shows the how OFDM is different from FDM. In the OFDM system, at the transmitter, the output of the modulator is in frequency-domain and it is converted into time-domain signal by performing IFFT, and transmitted through a wireless channel. Due to noise in the channel, the received signal is distorted, hence it becomes essential to compensate and estimate the CIR (Channel Impulse Response) at the receiver. CIR is estimated at each subcarrier and the transmitted signal can be recovered.

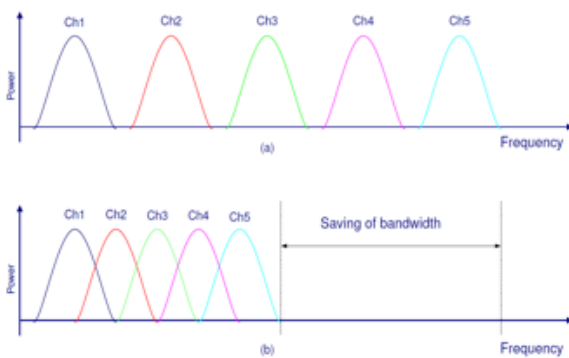


Figure 1: Difference between FDM and OFDM

From Figure 1 (a), we can observe that there is inefficient utilization of spectrum. In conventional FDM guard band must be introduced between different sub-carriers to eliminate the ICI, leading to an inefficient use of the spectrum. Hence multicarrier modulation with overlapping spectra in FDM scheme was needed. And perfect orthogonality is also needed to eliminate the effect of ICI between the modulated subcarriers in FDM systems.

### 2.2. MIMO Systems:

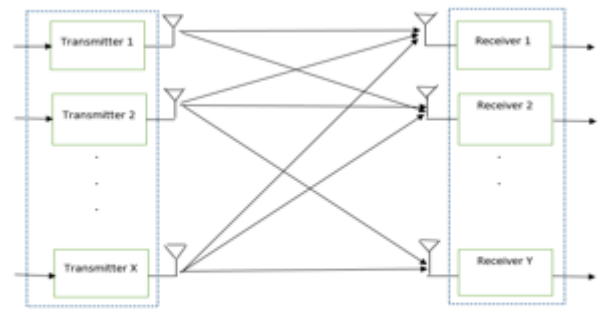


Figure 2: MIMO System

MIMO system consists of multiple antennas that can be used for multiplexing gain in order to increase the data rate. There is one more technique known as beam forming that can be applied to get a robust channel with increase in diversity gain and coherently combine the channel gains. It is not the compulsion that the antennas are specifically used for multiplexing or diversity. But grouping of some antenna can be done for diversity, in which each group can be used to transmit independent data streams.

### 2.3. MIMO-OFDM

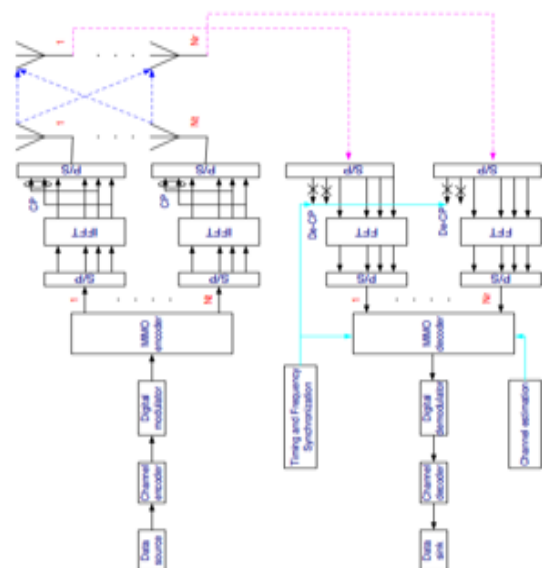


Figure 3: Block diagram of MIMO-OFDM system

MIMO-OFDM system is similar to the OFDM system except that multiple antennas are present instead of single antenna. There are  $M_t$  branches similar to single antenna OFDM system. Every branch serial to parallel conversion, pilot insertion, IFFT and adding CP

before up-converting to RF and transmitted through wireless channel. After collecting the signal at receiver, by using sub carriers receiver must estimate the channel. The received signal is sum of all signals transmitted from different branches. In the Receiver first removed the Cyclic prefix (CP) and performs the N-point DFT to estimate the pilot subcarriers in frequency domain. The estimated channel coefficients are used to detect the rest of the subcarriers. MIMO detection is done per subcarrier. Demodulation and decoding is done after the detection of the bits. The received signal per branch is combined with all the transmitted signals from  $M_t$  transmit antennas.

### 3. CHANNEL ESTIMATION IN MIMO-OFDM SYSTEMS

In MIMO-OFDM system, frequency domain channel estimation is considered because number of parameters ( $M_t M_r L$  coefficient) required to handle is less compared to time domain response. More number of pilots are required for estimating a MIMO channel because there are more number of multipaths.

#### 3.1 LS channel estimation

It is a simple estimation technique and very straight forward. The received pilot signal is multiplied with the inverse of the transmitted pilot signal.

$$H_{est}[k] = \frac{A[k]}{B[k]}, k = 0, 1, 2 \dots M-1. \dots\dots\dots (1)$$

LS technique has low complexity and simple to implement. However, LS channel technique doesn't take channel statistics into account and suffer from high mean-square error.

The LS channel estimation the inversion of the channel matrix is done. The cost function is minimized is as follows

$$\begin{aligned} J(H_{est}) &= \|A - BH_{est}\|^2 \dots\dots\dots (2) \\ &= (A - BH_{est})^H (A - BH_{est}) \\ &= A^H A - A^H B H_{est} - H_{est}^H B^H A + H_{est}^H B^H B H_{est} \end{aligned}$$

For error to be minimum, the derivative of the cost function must be equated to zero with respect to  $H_{est}$ .

$$\frac{\partial J(H_{est})}{\partial H_{est}} = -2(B^H B)^* + 2(B^H B H_{est}^H)^* = 0;$$

$$B^H B H_{est}^H = B^H A$$

Therefore the LS solution is given by,

$$H_{est} = (B^H B)^{-1} B^H A = B^{-1} A \dots\dots\dots (3)$$

For each component,  $k=0, 1, 2 \dots M-1$ .

$$H_{est}[k] = \frac{A[k]}{B[k]}, k = 0, 1, 2 \dots M-1. \dots\dots\dots (4)$$

The mean square error is given by

$$\begin{aligned} MSE^{LS} &= E \{(H - H_{est})^H (H - H_{LS})\} \dots\dots\dots (5) \\ &= E \{(H - B^{-1}A)^H (H - B^{-1}A)\} \\ &= E \{(B^{-1}W)^H (B^{-1}W)\} \\ &= E \{W^H (B B^H) W\} \\ &= \frac{\sigma_w^2}{\sigma_b^2} \dots\dots\dots (6) \end{aligned}$$

Where  $\frac{\sigma_w^2}{\sigma_b^2}$  is the SNR. From equation 6, MSE decrease as SNR increases. This is the disadvantage of LS.

#### 3.2 MMSE channel estimation

The Mean Square Error (MSE) can be reduced by using MMSE technique. The time domain representation of the received signal is

$$\begin{aligned} b(n) &= \text{IFFT} \{B(k)\} \quad n = 0, 1, 2 \dots N-1 \dots\dots\dots (7) \\ &= \sum_{k=0}^{N-1} B(k) e^{j(\frac{2\pi kn}{N})} \end{aligned}$$

Where N is the FFT length. The time domain received signal is given by

$$a(n) = b(n) \otimes h(n) + w(n) \dots\dots\dots (8)$$

Where h(n) is the channel impulse response which can be represented as follows

$$h(n) = h_i e^{j(\frac{2\pi kn}{N})}$$

The LS solution in frequency domain is given by  $H_{est}^{LS} = B^{-1}A = \widetilde{H}_{LS}$   
 $H_{est}^{MMSE} = W_t \widetilde{H}_{LS}$ , is the MMSE estimate,  $W_t$  is weight matrix.

The cost function is defined as

$$J(H_{est}^{MMSE}) = E \{ || e ||^2 \} = E \{ || H - H_{est}^{MMSE} ||^2 \} \dots\dots\dots (9)$$

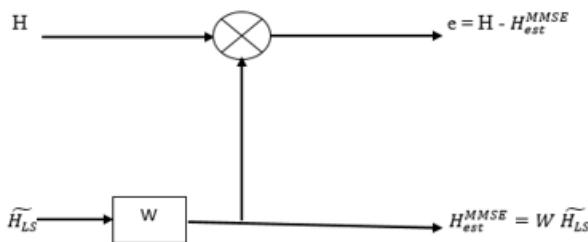


Figure 4: Illustration of MMSE Channel estimation

Better estimate can be obtained using the weight factor  $W_t$  such that MSE is minimized.

### 3.3 CS algorithms

In this section, the greedy algorithms such as SP and CoSaMP are discussed. In SP and CoSaMP, the sparsity is defined priori.

#### 3.3.1 SP algorithm

In Subspace Pursuit (SP) greedy algorithm which has less computational time and better BER. Here instead of selecting one column at each step S columns are selected from the measurement matrix iteratively. LS method is used to select S columns until stopping criteria is met.

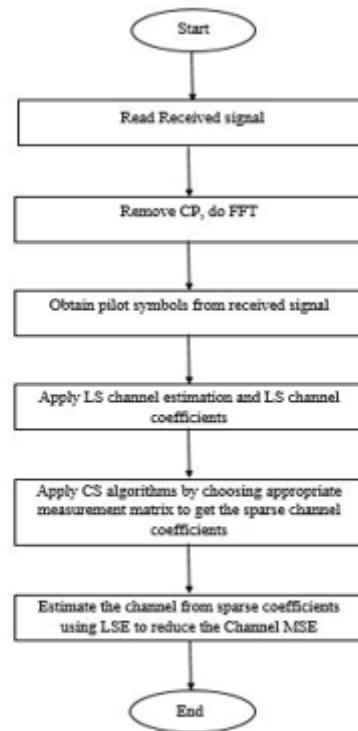
The total computational time of SP is given by  $O(m.N.\log(k))$ . The computational time complexity of SP is reduced compared to OMP because batch selection is done instead of one.

#### 3.3.2 CoSaMP algorithm

CoSaMP first identifies the  $2K$  (where  $K$  is the sparsity level) elements using matched filter and it is combined with the support matrix or set estimated in the earlier iteration. Candidate set is the set of elements which are estimated from the

matched filter. The support set and its union with candidate-set of previous set can be called as union-set. From the union set, a new  $K$ -dimensional subspace is identified from the union-set using least squares. This will reduce the reconstruction error of the sparse signal. The computational time of CoSaMP is given by  $O(m.N)$  which is small compared to OMP.

### 4. IMPLEMENTATION OF CS ALGORITHMS WITH CONVENTIONAL METHODS



Flow chart of LSE channel estimation combined with CS algorithms

Receiver consists of the blocks that are exactly reverse of the transmitted blocks. Initially on the receiver side, Cyclic Prefix (CP) is removed from the received data coming from the channel that eliminates the ISI. The data is then passed through the serial to parallel converter and given to FFT in order to convert from time domain into frequency domain. After converting to frequency domain, LS channel estimation is done to get LS channel estimation coefficients. After this, sparse approximation of the LS channel coefficients is done using CS algorithms such as SP and CoSaMP.

This Sparsity level (K) is varied to get the best estimate of the channel coefficients.

### 5. RESULTS

Number of transmit antennas	2
Number of receive antennas	2
Channel type	Rayleigh
Input sample period	$10^{-7}$
Total number of subcarriers	N=256
Number of pilot subcarriers	$N_p = 24, 12$
Cyclic prefix length	$N_g = 64$
Delay spread	15
Doppler frequency	0.1 Hz
Modulation	QAM

Table 1: System parameters

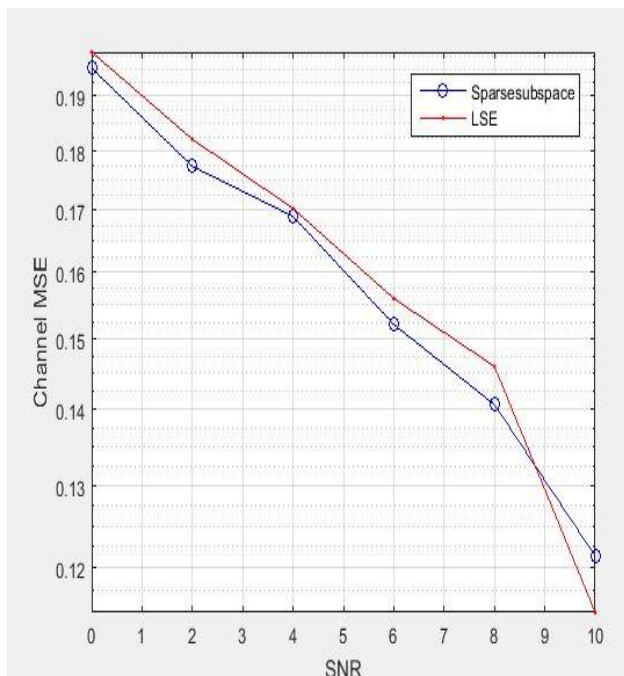


Figure 5 (a): Plot of MSE v/s SNR for channel estimation in 2 x 2 MIMO-OFDM system using LS combined with Subspace Pursuit (SP).

Figure 5 (a) gives the performance of the LS channel estimation using Subspace pursuit (SP) algorithms in 2 x 2 MIMO-OFDM systems. The plot are calculated at sparsity level 64. As sparsity level increases, the performance also improves. More the

sparsity level, more the number of non-zero coefficients and less error.

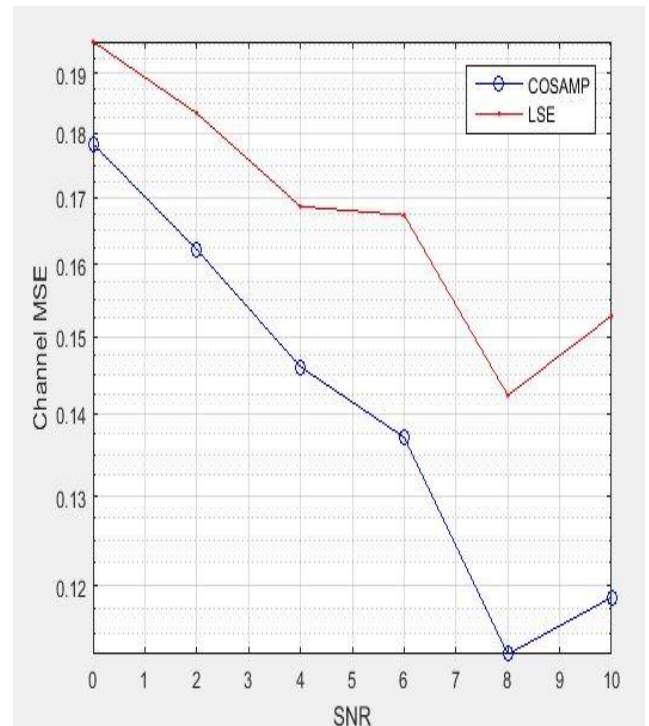


Figure 5(b): Plot of MSE v/s SNR for channel estimation in 2 x 2 MIMO-OFDM system using LS combined with CoSaMP.

Figure 5(b) gives the performance of the LS channel estimation using Compressive Sampling Matching Pursuit (CoSaMP) algorithm in 2 x 2 MIMO-OFDM systems. The plot is calculated at sparsity level 64. As sparsity level increased, the performance is also improved. More the sparsity level, more the number of non-zero coefficients and less error. As sparsity level increases, the performance of SP approaches CoSaMP. In CoSaMP, first calculates the sparse approximation using 2K (sparsity level) highest magnitude columns obtained by dot product of the received pilot signal with the measurement matrix. CoSaMP assures proper selection of columns than SP. Random pilot placement is done for CS algorithms.

### 6. CONCLUSION

In this paper the sparse recovery algorithms for pilot assisted MIMO-OFDM channel estimation is implemented. The channel coefficients obtained after conventional LSE technique were subjected to



sparse approximation using CS algorithms. The results show that conventional technique combined with CS algorithms have better performance compared to normal technique in terms of Bit Error Rate (BER) and Mean Square Error (MSE).

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