Finite Element Solution On Effects Of Viscous Dissipation & Diffusion Thermo On Unsteady Mhd Flow Past An Impulsively Started Inclined Oscillating Plate With Mass Diffusion &Variable Temperature

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Abstract - *The aim of this is paper is to investigate on the effects of viscous dissipation and diffusion thermo on an unsteady MHD flow with an inclined oscillating plate started impulsively. The effects with a variable temperature and mass diffusion are observed. Considered fluid is gray, absorbing-emitting radiation, but a non-scattering medium. Solutions of the nonlinear differential equation are obtained by finite element method. The effects of different flow parameters on the flow variables are discussed. The results have been analyzed graphically.*

Key Words: *Unsteady, variable temperature and mass diffusion, MHD, FEM, Viscous Dissipation*

1. INTRODUCTION

The study of the hydromagnetic flow of an electrically conducting fluid has many applications in science and engineering problems such as magnetohydrodynamic (MHD) generator, plasma studies, nuclear reactors, aerodynamic heating, etc. Soundalgekar *et al* [1] investigated the problem of free convection effects on Stokes problem for a vertical plate with transverse applied magnetic field. Elbasheshy [2] studied MHD heat and mass transfer problem along a vertical plate under the combined buoyancy effects on of thermal and spices diffusion. Ibrahim [3] has investigated analytical solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufor-Soret effect and Hall effect.MHD flow Past an Impulsively started vertical plate with variable temperature and mass diffusion was studied by Rajput and Surender kumar [4]. Rao and Shivaiah [5] studied chemical reaction effects on unsteady MHD flow past semi- infinite vertical porous plate viscous dissipation. P.K Sing [6] showed heat and mass transfer in MHD boundry layer flow past an inclined plate with variable temperature and mass diffusion. T.Arun kumar and L Anand Babu [7] has analyzed the study of Radiation effect of MHD flow past an impulsive started vertical plate with variable temperature and uniform mass diffusion – A finite element method.
P.Srikanth Rao and D.Mahendar [8] investigated Soret effect on unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate. D.Chennakesavaiah and P V Satyanarayana[9] studied the radiation absorption and dufour effect to MHD flow in vertical surface. Dufour effects on unsteady MHD free convection and mass transfer flow fast through a porous medium in slip regime with heat source/ sink was studied by K.Sharmilaa and S.Kaleeswari[10]. K.Anitha [11] has analyzed chemical reaction and radiation effects on unsteady MHD natural convection flow of rotating fluid past a vertical porous flat plate in the presence of a viscous dissipation. The effect of Hall current on an unsteady MHD free convective flow along a vertical plate with the thermal radiation was studied by P.Srikanth Rao and D.Mahendar [12].

2. MATHEMATICAL ANALYSIS

In this paper we have considered MHD flow between two parallel electrically non conducting plates inclined at an angle α from vertical. x' axis is taken along the plate and y' normal to it. A transverse magnetic field B_0 of uniform strength is applied on the flow. The viscous dissipation and induced magnetic field has been neglect due to its small effect. Initially it has been considered that the plate as well as the fluid is at same temperature T'_∞ and the Concentration level C'_∞ everywhere in the fluid is same as in stationary condition. At time $t > 0$, the plate starts oscillating in its own plane with frequency ω and temperature of the plate is raised to T'_w and the concentration level near the plate is raised linearly with respect to time.

The flow modal is as follows:

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial t'^2} + \cos\alpha\left(g\beta\left(T'-T_\infty'\right)+g\beta^*\left(C'-C_\infty'\right)\right) - \left(\frac{\sigma B_0^2 u'}{\rho}\right) \quad - (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{D_m k_T}{C_s C_p}\frac{\partial^2 C'}{\partial y'^2} + \frac{\upsilon}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \quad - (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_r'\left(C'-C_\infty'\right) \quad - (3)$$

With the corresponding initial and boundary conditions:

$$\left.\begin{array}{l} t' \le 0 : u'=0, T'=T'_\infty, C'=C'_\infty, \; for \;\; all \; y' \\ t' > 0 : u' = U_0 \cos\omega' t', T' = T'_\infty + (T'_w - T'_\infty)\dfrac{U_0'^2 t'}{\upsilon}, \\ C' = C'_\infty + (C'_w - C'_\infty)\dfrac{U_0'^2 t'}{\upsilon} \quad at \quad y'=0 \\ u' \to 0, T' \to T', C' \to C'_\infty \quad as \quad y \to \infty \end{array}\right\} \quad - (4)$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non dimensional quantities are introduced.

$$\left.\begin{array}{l} t = \dfrac{t' U_0^2}{\upsilon}, y = \dfrac{U_0 y'}{\upsilon}, \; u = \dfrac{u'}{U_0}, Sc = \dfrac{\upsilon}{D}, \\ \theta = \dfrac{T'-T'_\infty}{T'_w - T'_\infty}, C = \dfrac{(C'-C'_\infty)}{(C'_w - C'_\infty)}, \Pr = \dfrac{\mu c_p}{k}, \\ \mu = \rho\upsilon, \omega = \dfrac{\omega'\upsilon}{U_0^2}, Gr = \dfrac{g\beta\nu(T'_w - T'_\infty)}{U_0^3}, \\ Gm = \dfrac{g\beta^*\upsilon(C'_w - C'_\infty)}{U_0^3}, M = \dfrac{\sigma B^2 \upsilon}{\rho U_0^2}, \\ Du = \dfrac{D_m k_T (C'_w - C'_\infty)}{\upsilon c_S c_P (T'_w - T'_\infty)}, Kr = \dfrac{K'_r \upsilon}{U_0^2}, \\ Ec = \dfrac{U_0^2}{c_P (T'_w - T'_\infty)} \end{array}\right\} \quad - (5)$$

With the non dimensional quantities equations (1),(2) and (3) reduces to the following dimensionless form:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \cos\alpha\left(G_r\,\theta + G_m\,C\right) - Mu \quad - (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Df\frac{\partial^2 C}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 \quad - (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - KrC \quad - (8)$$

With the initial and boundary conditions in dimensionless form are:

$$\left.\begin{array}{l} u=0, \theta=0, C=0, \; for \;\; all \;\; y, t \le 0 \\ t>0 : u = \cos\omega t, \theta = t, C = t, \; at \;\; y=0, \\ u \to 0, \theta \to 0, C \to 0 \quad as \;\; y \to \infty \end{array}\right\} \quad - (9)$$

3. SOLUTION OF THE PROBLEM

Using finite element method with Crank-Nikolson discretization taking $h = 0.1, k = 0.01$.The element equation for the typical element (e) $y_j \le y \le y_k$ for the boundary value problem can be written as:

$$\int_{y_j}^{y_k} N^{(e)^T}\left[\frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} - Mu^{(e)} + P\right]dy = 0 \quad - (10)$$

Where $P = G_r \cos\alpha\,\theta + G_m \cos\alpha\, C$

$$\left\{N^{(e)^T}\frac{\partial u^{(e)}}{\partial y}\right\}_{y_j}^{y_k} - \int_{y_j}^{y_k} N^{(e)^T}\frac{\partial u^{(e)}}{\partial y}dy - \int_{y_j}^{y_k} N^{(e)^T}\left[\frac{\partial u^{(e)}}{\partial t} + Mu^{(e)} - P\right]dy = 0$$

Neglecting the first term in the above equation, we get

$$\int_{y_j}^{y_k}\left\{N^{(e)^T}\frac{\partial u^{(e)}}{\partial y} + N^{(e)^T}\left[\frac{\partial u^{(e)}}{\partial t} + Mu^{(e)} - P\right]\right\}dy = 0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ be the finite element approximation solution $(y_j \le y \le y_k)$ where

$$N^{(e)} = \left[N_j \;\; N_k\right], \phi^{(e)} = \left[u_j \;\; u_k\right]^T \;,\; N_j = \frac{y_k - y}{y_k - y_j}, N_k = \frac{y - y_j}{y_k - y_j}$$

are the basis functions.

$$\int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j' N_j' & N_j' N_k' \\ N_j' N_k' & N'N' \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix}\right\}dy + \int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix}\begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix}\right\}dy +$$
$$M\int_{y_j}^{y_k}\left\{\begin{bmatrix} N_j N_j & N_j N_k \\ N_j N_k & N_k N_k \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix}\right\}dy = P\int_{y_j}^{y_k}\begin{bmatrix} N_j \\ N_k \end{bmatrix}dy$$

Where " ′ "denotes the differentiation with respect to 'y' and denotes the differentiation with respect to 't'. Here

$$N_j' = \frac{-1}{h}, N_k' = \frac{1}{h}, h = y_k - y_j$$

$$\frac{1}{l^{(e)}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{l^{(e)}}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} + \frac{Ml^{(e)}}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{Pl^{(e)}}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_j\\ \dot{u}_k\end{bmatrix}+\frac{M}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}u_j\\ u_k\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 1\end{bmatrix}$$

We write the element equation for the elements $y_{i-1} \le y \le y_i$ and $y_i \le y \le y_{i+1}$ assemble three elements equations, we obtain

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}+\frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{u}_{i-1}\\ \dot{u}_i\\ \dot{u}_{i+1}\end{bmatrix}+ \quad -(11)$$

$$\frac{M}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}u_{i-1}\\ u_i\\ u_{i+1}\end{bmatrix}=\frac{P}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}$$

Now put row corresponding to the node i to zero, from the above equation the difference scheme is

$$\frac{1}{l^{(e)^2}}\left[-u_{i-1}+2u_i-u_{i+1}\right]+\frac{1}{6}\left[\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1}\right]+$$
$$\frac{M}{6}\left[u_{i-1}+4u_i+u_{i+1}\right]=P$$

Put $l^{(e)^2}=h^2$, $r=\frac{k}{h^2}$.

$$\frac{1}{h^2}\left[-u_{i-1}+2u_i-u_{i+1}\right]+\frac{1}{6}\left[\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1}\right]+$$
$$\frac{M}{6}\left[u_{i-1}+4u_i+u_{i+1}\right]=P$$

Applying the trapezoidal rule, the following equations in Crank-Nicholson method are obtained:

$$A_1u_{i-1}^{n+1}+A_2u_i^{n+1}+A_3u_{i+1}^{n+1}=A_4u_{i-1}^{n}+A_5u_i^{n}+A_6u_{i+1}^{n}+P^* \quad -(12)$$

Where

$A_1=2+Mk-6r$ $\quad A_4=2-Mk+6r$

$A_2=8+12r+4Mk,$ $\quad A_5=8-12r-4Mk$

$A_3=2+Mk-6r,$ $\quad A_6=2-Mk+6r$

$P^*=12Pk=12k(G_r\cos\alpha\,\theta_i^n+G_m\cos\alpha\,C_i^n$

Similarly applying the applying Galerikin finite element method for equations (7) and (8) the following equations are obtained:

$$B_1\theta_{i-1}^{n+1}+B_2\theta_i^{n+1}+B_3\theta_{i+1}^{n+1}=B_4\theta_{i-1}^{n}+B_5\theta_i^{n}+B_6\theta_{i+1}^{n}+P^{**} \quad -(13)$$

$$C_1C_{i-1}^{n+1}+C_2C_i^{n+1}+C_3C_{i+1}^{n+1}=C_4C_{i-1}^{n}+C_5C_i^{n}+C_6C_{i+1}^{n} \quad -(14)$$

Where

$B_1=2\Pr-6r$ $\quad B_4=2\Pr+6r$ $\quad C_1=2Sc+kScKr-6r$

$B_2=8\Pr+12r$ $\quad B_5=8\Pr-12r$ $\quad C_2=8Sc+4kScKr+12r$

$B_3=2\Pr-6r$ $\quad B_6=2\Pr+6r$ $\quad C_3=2Sc+kScKr-6r$

$C_5=8Sc-4kScKr-12r$

$C_4=2Sc-kScKr+6r$

$C_6=2Sc-kScKr+6r$

$$P^{**}=12P_1k=12k\left(\Pr Df\frac{\partial^2 C_i}{\partial y_i^2}+\Pr Ec\left(\frac{\partial u}{\partial y}\right)^2\right)$$

Here h,k are the mesh sizes along $y-$ direction and $t-$ direction respectively. Index i refers to the space and n refers to the time. In equation (12), (13) and (14), taking $i=1(1)m$ and using (9), the following systems of equations are obtained:

$$A_iX_i=B_i \qquad i=1(1)m \quad -(15)$$

Where A_i's are the matrices of order m and X_iB_i's column matrices having $m-$ componenets. The solutions of the above system of equations are obtained by using Thomas algorithm for the velocity(u), temperature(θ), concentration(C). Also numerical solutions are obtained by C-program. Computations are carried out until the steady state is reached. In order to prove the convergence of the Galerkin finite element method, the computations are carried out for slight changed values of h,k by running same C program, no significant changes was observed in the values of velocity(u), temperature(θ), concentration(C). Hence, the finite element method is stable and convergent.

4. RESULTS AND DISCUSSION

In order to assess the effects of the dimensionless thermo physical parameters on the regme calculations have been carried out on velocity, temperature, concentration fields for various physical parameters like, mass Grashof number (Gm), thermal Grashof number (Gr), magnetc field parametrt **(**M**)**, Dufour number (Df), Prandtl number problem (Pr), Schmidt number (Sc) and time (t).The results are represented through graphs in figures 1 to 16 for various parameters

Figure 1 show that the velocity profile for different angle of inclination (α) of the oscillating plate. The numerical results shows that the effect of increasing values of angle of inclination result in decreasing velocity.

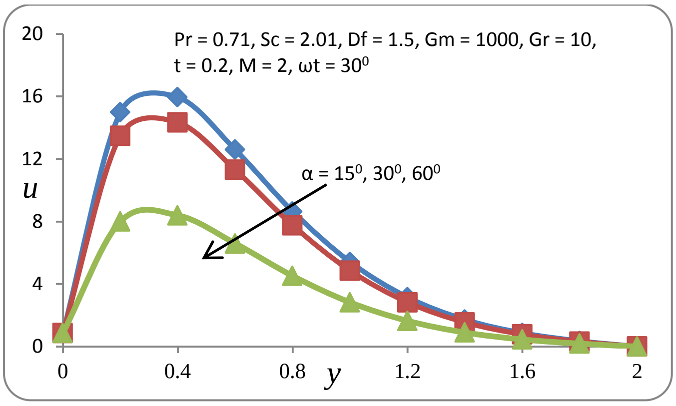

Fig .1.Velocity profile for different values of α

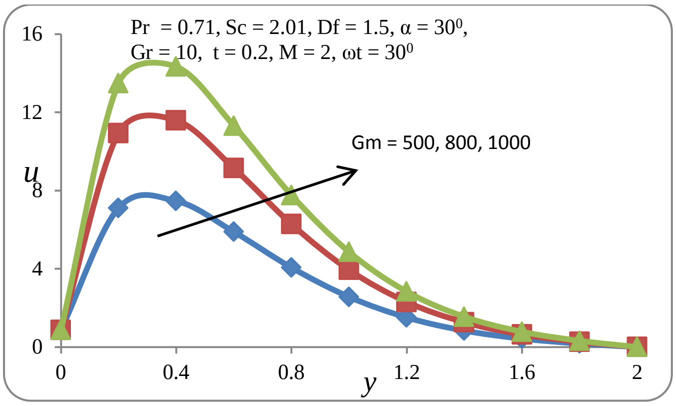

Fig.2.Velocity profile for different values of Gm

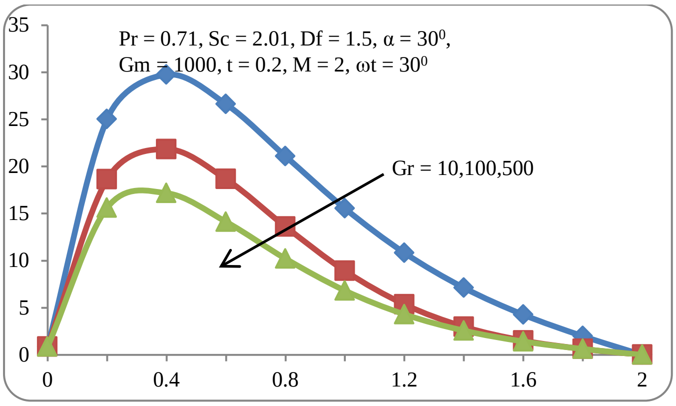

Fig. 3.Velocity profile for different values of Gr

Figure 2 describes that an increase in mass Grashof number cause an increase in velocity of the fluid under consideration. It is observed that from figure 3, the velocity is increased when the thermal Grashof number is increased.

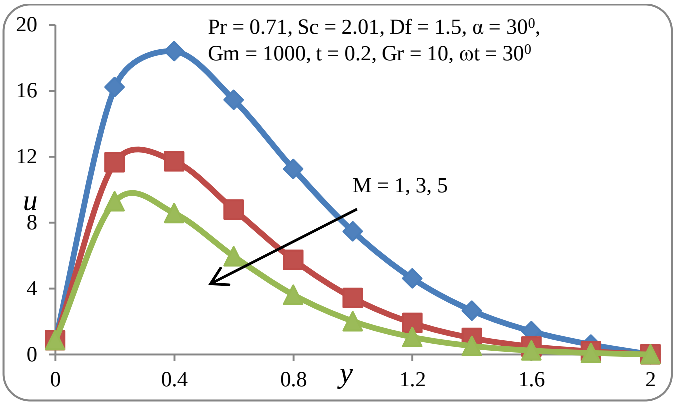

Fig.4. Velocity profile for different values of M

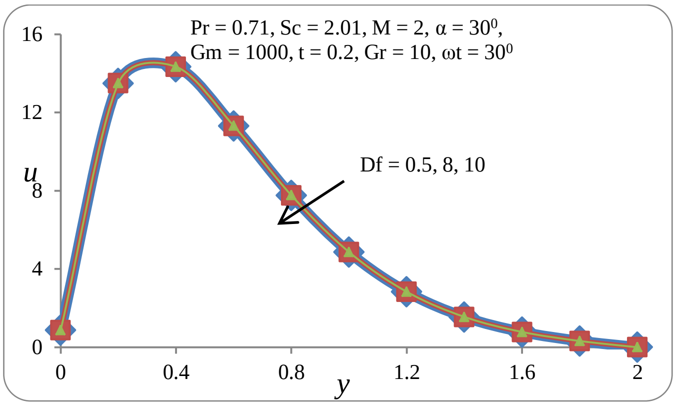

Fig. 5.Velocity profile for different values of Df

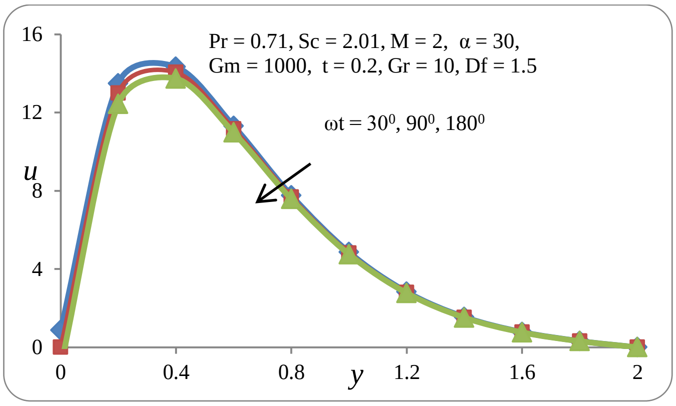

Fig.6. Velocity profile for different values of ωt

Figure 4 details the effect of increasing the values of magnetic parameter M resulting a decrease in velocity. From Figure 5 show that the decrease in velocity is caused by increase in Dufour number

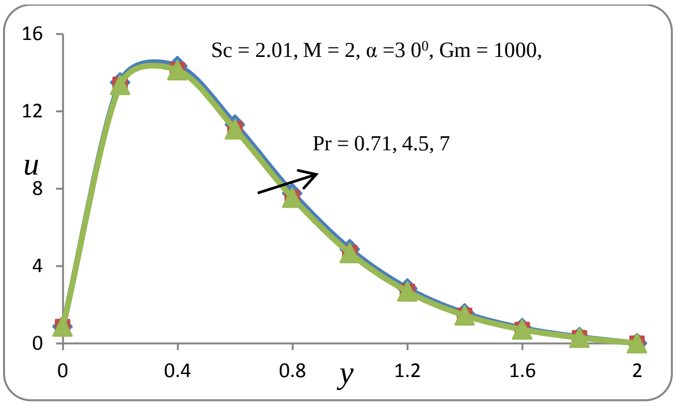

Fig.7.Velocity profile for different values of Pr

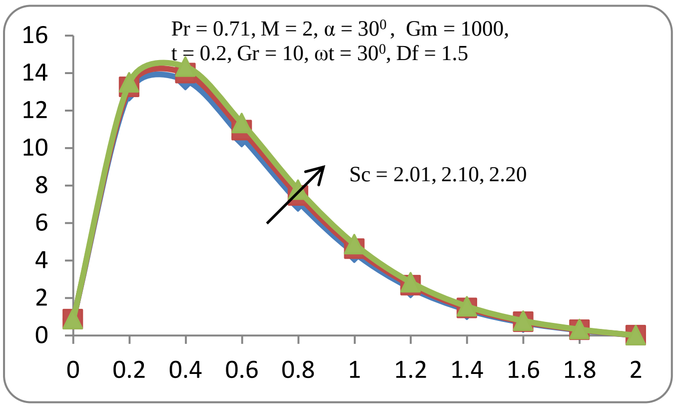

Fig.8.Velocity profile for different values Sc

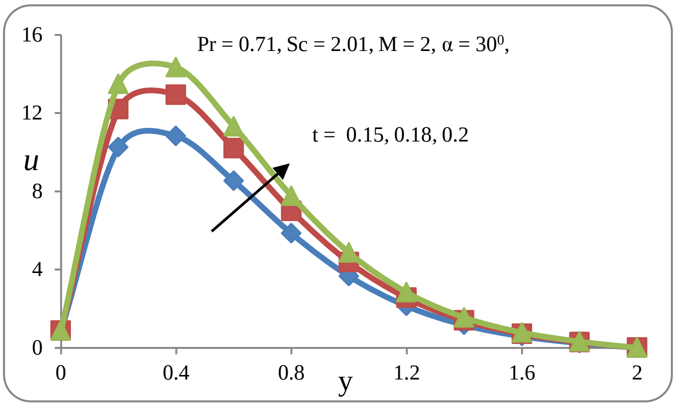

Fig.9.Velocity profile for different values of t

From figure 9, when time is increased then the velocity increased. Figures 10 to 13 describes that the temperature profile for parameters Dufour number, Prandtl number, Schmidt number, time respectively.

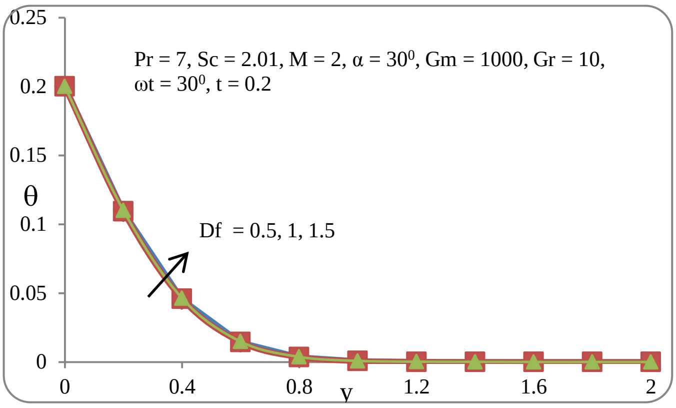

Fig.10.Temperature profile for different values of Df

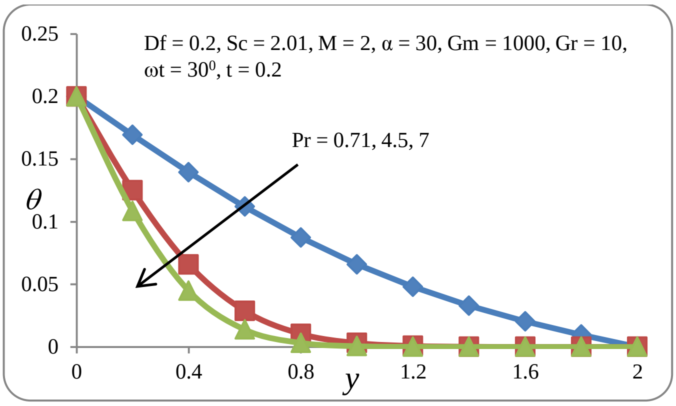

Fig.11.Temperature profile for different values of Pr

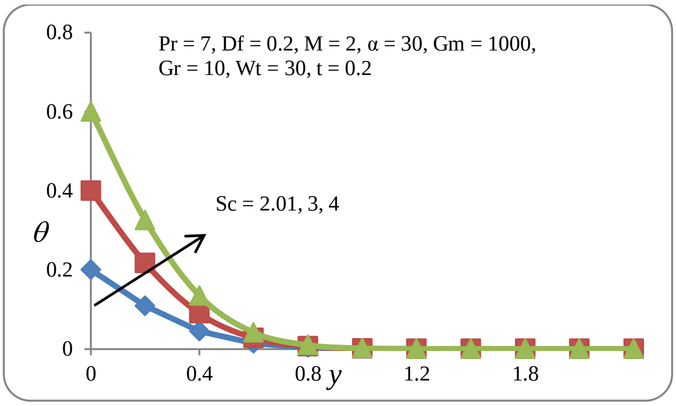

Fig.12.Temperature profile for different of Sc

An increase in the above mentioned parameters lead to increased temperature. Figure 14-16 displays the effects of the Dufour number, Prandtl number and time on concentration profiles. We observe that concentration profiles increases with increasing Df, Pr, t.

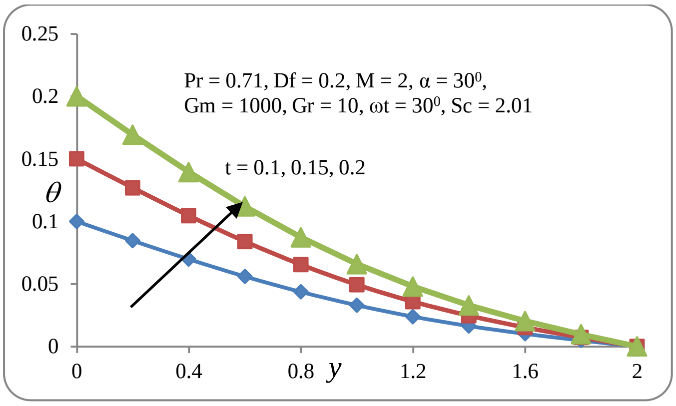

Fig.13.Temperature profile for different values of *t*

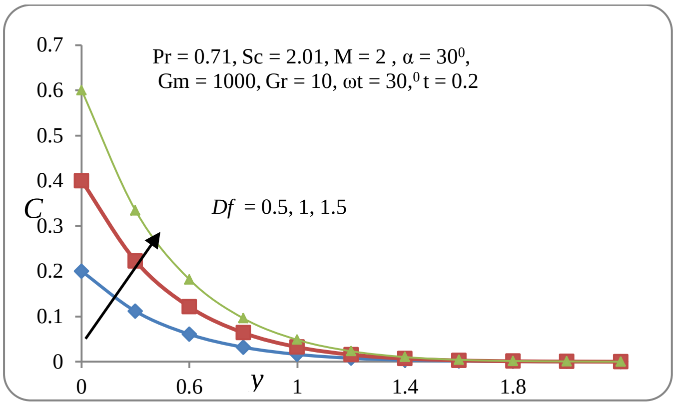

Fig. 14. Concentration profile for different values of *Df*

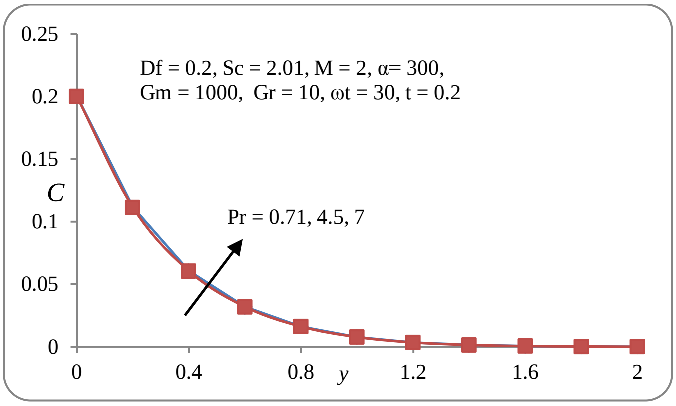

Fig.15. Concentration profile for different values of *Pr*

Observation of Figure 6 depicts that an increase in phase angle, decreases velocity. Figure 7 demonstrates that the increase in velocity with an increase in Prandtl number. Figure 8 depicts an increase in velocity due to an increase in Schmidt number.

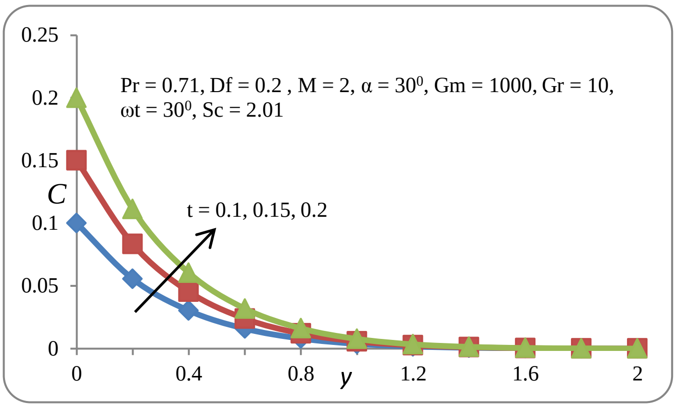

Fig.16. Concentration profile for different values of *t*

5. CONCLUSIONS

In this article a mathematical model has been presented for the effects of the viscous dissipation and diffusion thermo on unsteady magnetohydrodynamics flow past an impulsively started inclined oscillating plate with mass diffusion and variable temperature. Solutions for the model have been derived by finite element method. The conclusions of the study are as follows:

1. Velocity increases with the increase in mass Grashof number, Prandtl number, Schmidt number and time.
2. Velocity decrease with increase in the angle of inclination of plate, thermal Grasof number , the magnetic field, Dufour number and phase angle.
3. Temperature profiles increases with the increase in Dufour number , Prandtl number, Schmidt number and time.

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