Finite element analysis of a crowned spur gear pair to study the effect of shaft misalignment

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Abstract*— Modification of gear tooth such as providing circular crowning is recommended many times to compensate for gear tooth misalignment. Crowning essentially is modifying the tooth face from a straight plane to a circular profile. This shifts the load from the edge of the face to the center of the gear tooth. Since crowning makes the tooth center slightly thicker than the edges, the gear can sustain the contact pressure better and improves the service life. In this study the modified gear profile with crowning is studied and compared with traditional gear tooth profile. For several cases contact pressure is calculated and compared such as when the pair of gear is crowned and misaligned, crowned and not misaligned, not crowned and misaligned and not crowned and not misaligned. The results were compared and it shows that when there is misalignment while the gear pair with traditional profile is in contact, the contact pressure region shifts towards the edge of the gear flank. But when the gear face is crowned and there is misalignment, the contact region remains at the center of the lead profile. However, this modification leads to slight increase in contact pressure which can be considered while determining strength of the gear tooth beforehand.

***Keywords*—spur gear; crowning; misalignment; Hertz contact pressure; FEA**

1. INTRODUCTION

Gears are toothed cylindrical wheels used for transmitting mechanical power from one rotating shaft to another and is one of the most critical components in a mechanical power transmission system. [1] Due to their high degree of compactness and reliability it is expected that gears will predominate as the most effective means of power transmission in near future. Increase in demand for accurate and quiet power transmission in machines, generators and vehicles has resulted in the need for more precise analysis of gear characteristics. In auto industry as lighter vehicles continue to be in demand, highly reliable and light weight gears have become necessary. Designing highly loaded spur gears for power transmission systems that are both strong and quiet, requires analysis methods that can easily be implemented and also provide information on contact and bending stresses, along with transmission errors. Transmission error results due to two main factors i.e. manufacturing inaccuracy or mounting errors and elastic deflection under load. Hence, compensation for transmission errors by modifying gear teeth needs to be considered by gear designers. Transmission error is considered to be one of the main contributors to noise and vibration in a gear set. If a pinion and gear have ideal involute profiles running with no loading torque they should theoretically run with zero transmission error. [2]

It was not until recently that humans began to use mathematics and engineering to more accurately and safely design these gears. Wilfred Lewis introduced a method to calculate the amount of stress at the base of a gear tooth in 1892. Heinrich Hertz began his own work on contact pressures around the same time in 1895. His research on the elastic contact of two cylindrical bodies allowed engineers to calculate the contact pressure between a gear and a pinion. With these tools engineers were able to better predict the bending stress and contact pressures of gear pairs to allow for more robust design. [17] Spur gears are relatively simple in design and most designers prefer to use them wherever design requirements permit. Spur gears are ordinarily thought of as slow speed gears, while helical gears are thought of as high-speed gears. If noise is not a serious design problem, spur gears can be used at almost any speed that can be handled by other types of gears. Aircraft gas-turbine precision spur gears sometimes operate at pitch-line speeds above 50 m/s.

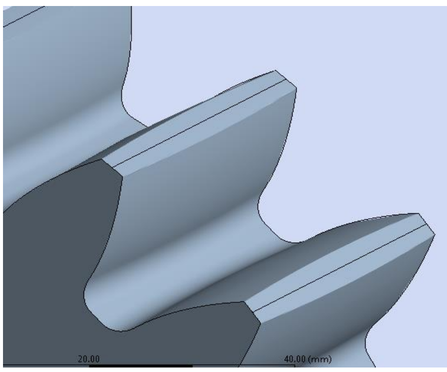


Fig. 1 Spur gear tooth with crowned teeth profile

Many organizations, including the American Gear Manufacturer's Association (AGMA) have sought to standardize the gear design process by developing their own formulas for gear design. The major changes over Lewis' original equations are the ability to take into account the geometrical complexity of the gear tooth, as well as the actual location of the contact. With the advent of Finite Element Analysis and Computer Aided Design (FEA and CAD) the ability to quickly and accurately design gears has been greatly improved upon. With modern CAD programs a typical gear and pinion can be modelled relatively easy. With better computing power, FEA software can quickly and efficiently analyse the stresses and contact pressures in gear pairs. These tools make the design and analysis stage much cheaper and faster for the design engineer. Because actual experiments are costly and take large amounts of time, a repeatable and accurate design tool is crucial for real world application. As the distance between the gear and pinion's axis of rotation increases the bending stress and contact pressure rise.

A relatively small rise in bending stress can cause a gear tooth to fail at lower cycle numbers than the design calls for. With an increase in the contact pressure between a gear and pinion wear will be increased as well. With increased wear, gear failure may occur sooner by pitting, corrosion, and adhesive wear. Although much study has been done on the mechanisms of gear wear and the major contributors to that wear, there is still a lack of understanding when it comes to the tolerances and how they affect the wear characteristics. [17]

2. PROBLEM FORMULATION

In this work a spur gear pair in mesh that is used to transmit power is used. The gear and pinion in mesh has variable number of teeth. The moment of 100 Nm is applied to the pinion and the effects are studied. Table 1 provides the basic parameters of gear as well as the pinion along with the material used. Using these properties a spur gear pair is modeled using SolidWorks modeling software. The finite element analysis was carried using ANSYS v14.

Table 1. Properties of spur gear pair

	Pinion	Gear
Material	Case hardened alloy steel (17CrNiMo6)	
Teeth	25	76
Face width	44 mm	44 mm
Root Dia.	137.982	438.0180
Root Radius	2.28	2.28
Tip Radius	164.982	465.018
Dedendum	7.5	7.5
Addendum	6	6
Crowning radius	0.688 m	0.688 m
Normal module	6	6
Pressure angle	20°	20°
Centre distance	303 mm	

Contact pressure calculation is essential for the determination of service life of the gear. Contact pressure exceeding the limits of maximum value can result in pitting of the gear and eventually gear failure. Following are the general equations used to calculate contact pressure when two curved bodies are in contact. This represents the condition where two gears with crowned teeth are in contact.

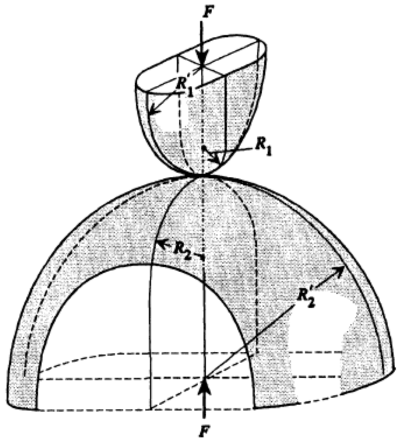


Fig. 2 Two non-conforming bodies in contact

$$(\sigma_c)_{max} = \frac{1.5F}{\pi c d}$$

$$C_E = \frac{1-\nu_1}{E_1} + \frac{1-\nu_2}{E_2}$$

$$c = \alpha \sqrt[3]{P K_D C_E}$$

$$d = \beta \sqrt[3]{P K_D C_E}$$

$$K_D = \frac{1.5}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R'_1}+\frac{1}{R'_2}}$$

$$\cos\theta = \frac{K_D}{1.5}\sqrt{\left(\frac{1}{R_1}-\frac{1}{R'_1}\right)^2 + \left(\frac{1}{R_2}-\frac{1}{R'_2}\right)^2 + 2\left(\frac{1}{R_1}-\frac{1}{R'_1}\right)\left(\frac{1}{R_2}-\frac{1}{R'_2}\right)\cos 2\phi}$$

F=Total Load, N

$(\sigma_c)_{max}$=*max. contact pressure, N/mm*^{2}

c = semi major axis

d= semi minor axis of elliptical contact area

ν=*Poisson's ratio;*

E = modulus of elasticity

R_1 *and* R_2=*minimum radii of curvature for bodies 1 and 2*

R_1' *and* R_2' = *maximum radii of curvature for bodies 1 and 2*

ϕ=*Angle between plane of radius of curvature of body 1 with the corresponding radius of curvature of body 2*

$$C_E = \frac{1-\nu_1}{E_1} + \frac{1-\nu_2}{E_2} = \frac{1-0.3}{2\times10^{11}} + \frac{1-0.3}{2\times10^{11}} = 9\times10^{-12}\ m^2/N$$

$$R_1 = \frac{2\pi R_b \theta_M}{360} = \frac{2\pi\times68.991\times90}{360} = 0.1m$$

Similarly, using above relation and R_B=0.438 m,

R_2=0.688 m, R_1=0.1 m, R_2=0.688 m, R_2'=R_1'=∞.

$$K_D = \frac{1.5}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_1'}+\frac{1}{R_2'}} = \frac{1.5}{\frac{1}{0.1}+\frac{1}{0.688}+\frac{1}{0.1}+\frac{1}{0.688}} = 0.13$$

$$\cos\theta = \frac{K_D}{1.5}\sqrt{\left(\frac{1}{0.1}-\frac{1}{\infty}\right)^2+\left(\frac{1}{0.688}-\frac{1}{\infty}\right)^2+2\left(\frac{1}{0.1}-\frac{1}{\infty}\right)\left(\frac{1}{0.688}-\frac{1}{\infty}\right)\cos2\phi} = 0.99$$

$$c = \alpha\sqrt[3]{PK_DC_E} = 7.774\sqrt[3]{1.2\times0.13\times9.1\times10^{-12}} = 9.048\times10^{-4}$$

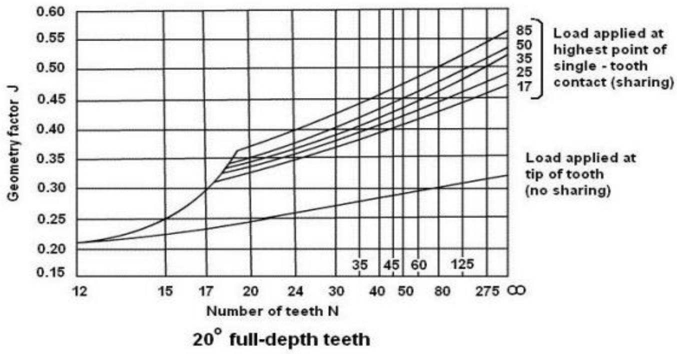
$$d = \beta\sqrt[3]{PK_DC_E} = 0.287\sqrt[3]{1.2\times0.13\times9.1\times10^{-12}} = 3.34\times10^{-5}$$

$$(\sigma_c)_{max} = \frac{1.5P}{\pi cd} = \frac{1.5\times1.33}{\pi cd} = 21.055\ MPa$$

$$\sigma_b = \frac{F_t}{b\times m\times J}K_v\times K_O\times K_m = \frac{100}{10\times1\times0.29}\times1\times1\times1 = 34.66\text{ MPa}$$

$$\sigma_b = \frac{F_t}{b\times m\times J}K_v\times K_O\times K_m = \frac{100}{2.5\times0.3\times0.35}\times1.3\times1\times1 = 247.61\text{ MPa}$$

Table 2 Graph for AGMA (J) Factor [40]



Bending stress calculations

$$\sigma_b = \frac{F_t}{b\times m\times J}K_v\times K_o\times K_m$$

σ_b= *Bending stress*

F_t =*Tangential force=200 N*

b=Face width=8 mm

m=Module= 1

J=Geometry factor=0.24

K_v=*Velocity factor=1*

K_o=*Overload factor=1.3*

K_m=*Load distribution factor=1.3*

3. METHOD OF ANALYSIS

A. Meshing and element type

Selection of type of material is done on the basis of general gear material used in practice. Meshing for contact analysis is complex and requires more refined meshing tools for accurate solution. To reduce the processing time, meshing is refined only around the tooth meshing region. Eight node hexahedral elements are used for the meshing.

Case a

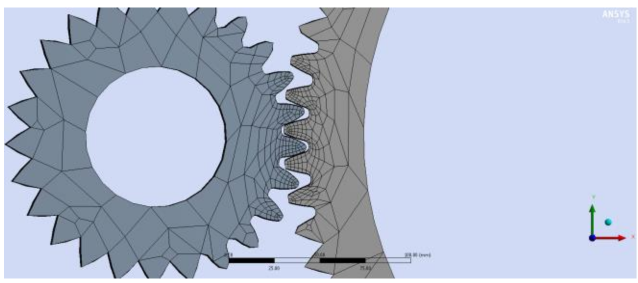


Fig. 3 Gear mesh refined around region of contact

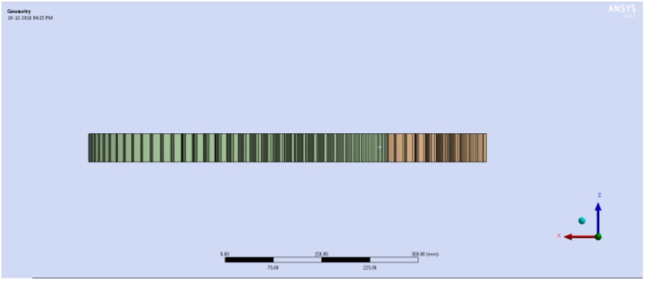


Fig. 4 Showing gear pair with no misalignment

Case b

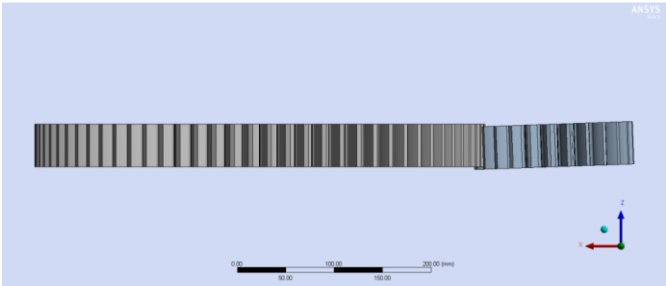


Fig. 5 Showing gears in mesh with misalignment

Case c

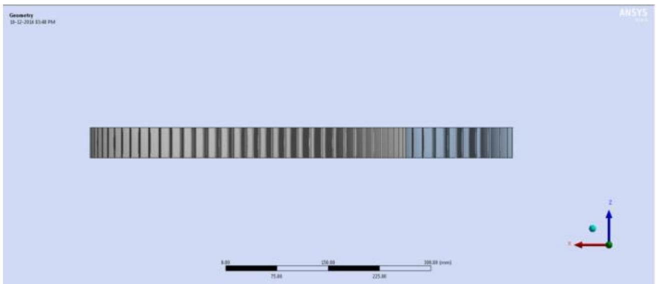


Fig. 6 Showing crowned gear pair with no misalignment

Case d

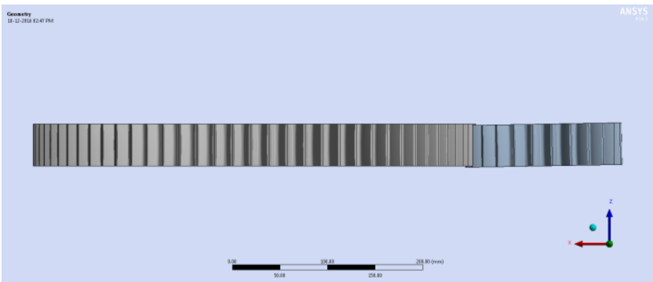


Fig. 7 Showing crowned gear pair with misalignment

B. Applying boundary conditions and load

Initially rigid support is applied to the gear and frictionless support is added to the pinion. Load in terms of moment is applied to the pinion.

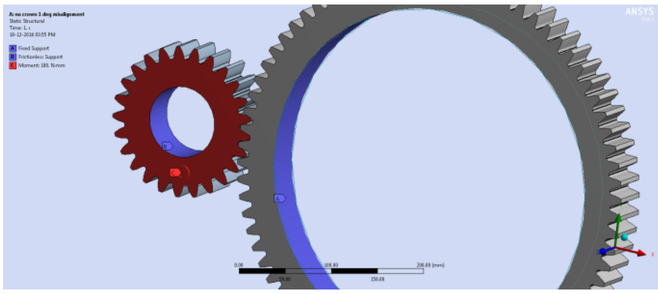


Fig. 8 Gear pair with applied load and boundary conditions

C. Static stress analysis

Following cases shows the distribution of the contact pressure over the gear tooth flank. In case a pressure distribution is a straight plane. In case b it gets shifted towards the edge of theflank. For case c the pressure distribution forms an elliptical shape and for case d the pressure distribution is still elliptical and very slightly shifted to the edge of the flank.

Case a

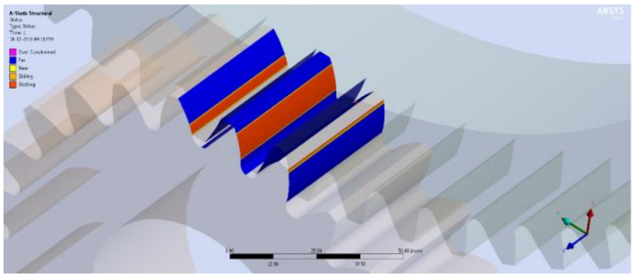


Fig. 9 Contact stress distribution (no crown, no misalignment)

Case b

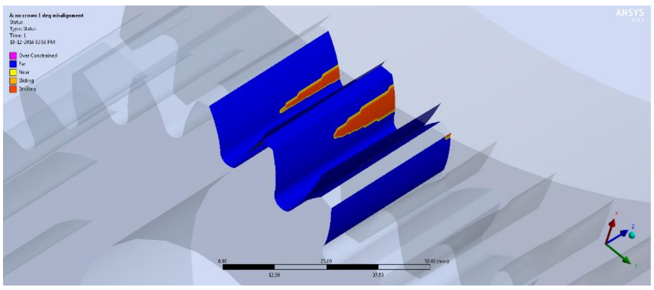


Fig. 10 Contact stress distribution (no crown, misalignment)

Case c

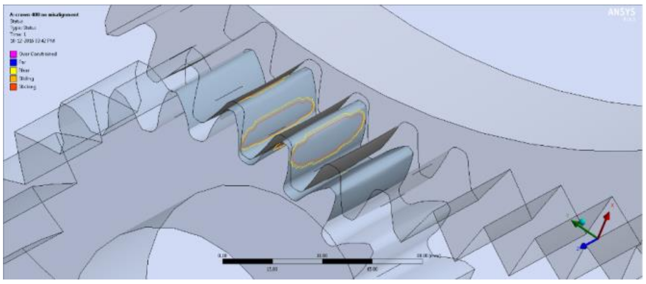


Fig. 11 Contact stress distribution (crown, no misalignment)

Case d

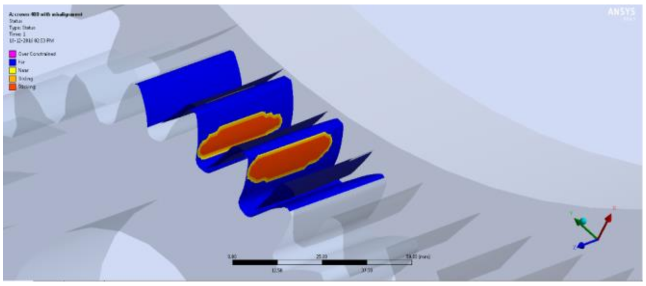


Fig. 12 Contact stress distribution (crown, misalignment)

4. RESULTS

When the gear pair with no crown is compared, the contact stresses are considerably more than the gear pair with no crown and misalignment. Whereas when the gear faces are crowned the increase in contact stressed after misalignment is not significant. The results are evident from Table 3. The second condition of no crown and misalignment shows more increase in contact stress when the spur gear pair misaligns. The fourth condition where the gear pair is misaligned and crowned, the contact stress changes negligibly as compared to condition three.

Fig. 9 shows the contact pressure and nature of contact when the gears have no misalignment and no crowning. The contact region is a line with stress evenly distributed along the gear face. For the same gear pair with no crowning, if there is any misalignment with the gear axis, the contact stress rises and region of contact shifts to one side of the face. On the other hand when the gear has crowned profile, the contact stress increases but if the conditions of misalignment and alignment are considered the change in contact stress is very small if the gear pair is misaligned. Moreover, the contact region in both the cases is elliptical and stays at the center of the gear face and even for considerable misalignment the contact region does not shift to the end of the gear face.

Table 3 Contact stresses for crowning and non-crowning condition

Sr. No.	Condition	Theoretical values, MPa	FEA values, MPa
1	No crown, no misalignment	21.055	19.856
2	No crown, misalignment, 7°	24.175	20.255
3	Crown, no misalignment	32.297	28.314
4	Crown, misalignment, 7°	32.397	28.487

5. CONCLUSION

This work studies the effect of the misalignment on a spur gear pair used to transmit power. Often due to various reasons such as vibrations or improper mounting of bearings, the misalignment occurs. Here from the results in Table 3 it is clear that for the condition of no crown the contact pressure increases considerably when there is misalignment. Also the region of contact shifts to the edge of the gear flank which could result in pitting. For the condition of crowning of the flank the contact pressure rises slightly when the flank is crowned but if the conditions of misalignment and no misalignment are considered there is very small change in the contact pressure. The region of contact also stays at the center of the flank even after the misalignment. This gear tooth modification thus help counter the issues of misaligned gear shafts.

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