# Review of Synthesis of Four BAR Mechanism 

Siddharth Shete ${ }^{1}$, Nitesh Kumar ${ }^{2}$, Praphulla Nalawade ${ }^{3}$, Prakash Tripathi ${ }^{4}$<br>Assistant Professor, Dept. of Mechanical Engineering, Dr. D Y Patil Institute of Engineering, Management \& Research, Akurdi, Pune, Maharashtra, India


#### Abstract

This Paper briefly covers some commonly used graphical and analytical techniques and also different optimization algorithms. This is to review the important aspects which are essentially required in the analysis throughout. The methods are briefly reviewed as they are selectively used to cross verify the results of the proposed algorithm.


Key Words: Graphical Methods, Two Position Synthesis, Three Position Synthesis, Objective Function, Evolutionary Algorithm.

## 1. GRAPHICAL METHODS FOR DIMENSIONAL SYNTHESIS

This method [8] is employed in the path generation problem. The path generation problem is one of the type or subset of motion generation problem. In the path generation problem certain points are prescribed for successive motion of coupler link, these points are known as a precision points. The basic path synthesis starts with two precision positions. This type of synthesis is used to solve up to four precision positions. The methodology for two position synthesis is given below.

### 1.1 Two position synthesis

The objective is to move a crank from point A 1 to A 2 so that output link should move from point B1 to B2 as shown in Fig -1 . To achieve this objective following geometrical method is used.


Fig -1: Two Position Synthesis

The construction lines are drawn to connect A1 to A2 and B1 to B2. The lines A1A2 and B1B2 are then bisected and extended in the convenient directions as shown. Then 02 \& 04 are conveniently selected as fixed pivots. The link 2 is 02 connected with A1 and the link 4 is 04 connected to B 1 . The line A1B1 is link-3, while 0204 is link 1.

### 1.2 Three position synthesis

The objective is to move a crank through three points A1, A2 and A3 so that output link should move through points B1, B2 and B3 as shown in Fig -2. To achieve this objective following geometrical method is used.


Fig -2: Three Position Synthesis

The construction lines are drawn to connect B2 to D and B3 to $D$. The line $B 2 D$ is rotated through angle $\phi 12$ to get the point B2' and the line B3D is rotated through angle $\phi 13$ to get the point B3'. The lines B2B2' and B3B3' are then bisected and extended in the convenient directions as shown. The intersection point is C 1 . The lines are drawn to connect A to B1, B1 to C1 and C1 to D to form the four bar mechanism.

The graphical procedure employed for the two-position synthesis problem can be extended up to the four position synthesis. As the number of precision points to be traced increases, the graphical method fails to give a correct solution.

## 2. ANALYTICAL METHODS FOR DIMENSIONAL SYNTHESIS

There are two analytical methods widely used for dimensional synthesis, the first method is the Freudenstein's equation and the second is chebyshev spacing equation [8].

Freudenstein's equation is,
$\cos (\theta-\phi)=K 3+K 1 \cos \phi-K 2 \cos \theta$
Where, $d a=\mathrm{k} 1 ; d c=\mathrm{k} 2$; and $(a 2-b 2+c 2+d 2) 2 a c=\mathrm{k} 3$
If the input $(\theta)$ and output angles $(\phi)$ are given for three position, then by solving freudenstein's equation for three positions simultaneously one can get the values of $\mathrm{k} 1, \mathrm{k} 2$ and k 3 and then the dimensions of the links $a, b, c, d$ can be calculated. The analytical method is simple up to three or four precision points because it is straight forward and it can be solved by converting those to system of linear simultaneous equations which are very easy to calculate. For more than four precision points, the system of equations becomes nonlinear and therefore to solve those equations computer is required.

There is one more method which is called as chebychev spacing equation. The structural error is the difference between generated path and desired path therefore the position of the precision points should be spaced in such a way that it minimises the structural error and the chebychey space equation gives good precision positions. According to the Freudenstein and sandor, the chebyshev spacing for n positions in the range $x f \leq x \leq x s$ is given by,
$x j=12(x f+x s)-12(x f-x s) \cos [\pi(2 j-1) 2 n]$
Where,
$x j=$ Precision Positions
$x s=$ Starting Position
$x f=$ Finishing Position $\mathrm{j}=1,2,3, \ldots . . . . . \mathrm{n}$
$\mathrm{n}=$ Number of Precision Position
By calculating precision positions from above equation and using those values as input to the Freudenstein equation, one can get the dimensions of the four bar mechanism to achieve the desired path. The chebyshev spacing of precision positions is also obtained by graphical method shown in Fig -3.

The description of the methodology is as follows. The circle is drawn with diameter equal to the range $\Delta x=x f-x s$ i.e. if $x f=3$ and $x s=1$ then $\Delta x=2$. The polygon is drawn having number of sides equal to 6 as the number of precision positions require are 3 . The perpendicular is drawn from
each corner which intersects the diagonal of circle at precision positions $x 1, x 2, x 3$ as shown in Fig -3.


Fig - 3: Graphical Method for Chebyshev Spacing
The classical analytical and graphical methods for synthesis are discussed above. In every method there must be some assumption to be made and also there is restriction on precision points and also in some cases one has to give the positions of input and output angles for those precision points. The intention behind using optimization technique is that, while using optimization methods, the major advantage is no need of assuming anything and because it is random and logical based technique, it is simple to implement.

Many authors have employed different optimization techniques for dimensional synthesis which are surveyed briefly.

## 3. OBJECTIVE FUNCTION

Cabrera et al. [1] applied Euclidean distance error function. This error function is the sum of squares of the difference between the desired path point and generated path point. The objective function is the addition of two parts, the first part contains the Euclidean distance error function and the second part consists of the penalties if the applied constraints are not satisfied. The constraints applied are grashof's criteria and sequence of input crank angle.

Laribi et al. [2] formulated the error function called as the orientation structural error of fixed link Es. The Fig -4 shows the four bar mechanism where the coupler point M1 is allowed to move through desired trajectory and the point D allows to float from $\psi \min$ to $\psi \max$ then the dimensions of the links are finalized to get minimum orientation structural error, $E s=\psi \max +\psi \min$.

Matekar et al. [5] formulated new objective function and methodology for optimization. The modified distance error function is formulated based on longitudinal and transverse
errors between prescribed path points and obtained path points.


Fig -4: Orientation of Structural Error
The constraints applied are grashof's criteria, sequence of input crank angle, shortest link must be the input link or crank and non-violation transmission angle. The limitation of using this objective function is that it cannot be used without prescribed timing. The crank angle for particular point must be known to solve the problem.

## 4. EVOLUTIONARY ALGORITHMS

Cabrera et al. [1] employed GA having three operators namely selection, crossover and mutation on three cases, in the first case objective is to form a straight line through which the coupler point should pass, i.e. all target points should be aligned and not with prescribed timing. In other two examples the objective is to form a mechanism whose coupler point should pass through the described points which are in a particular curvature pattern with prescribed timing. The purpose is to compare the efficiency of GA with other algorithms.

Laribi et al. [2] implemented the fuzzy logic controller with GA. The fuzzy logic controller is used to monitor the variations in the design variables during the first iteration of genetic algorithm and then this fuzzy logic controller modifies initial bounding intervals or boundary values which will form the new boundary values for the second run of the genetic algorithm. The constraints considered are sequence of input crank angle and deviation of transmission angle from 90 degrees. The investigation in this case is the influence of boundary intervals on the results. On that basis the fuzzy logic controller is implemented which monitors the results of the different variables during optimization and then rearrange the boundary values of each design variable and start the second round of optimization to improve the results. This process converts to optimal solution rapidly and within less number of generations.

Acharya et al. [3] also implemented evolutionary algorithm for the synthesis of four bar mechanism. The different optimization techniques such as genetic algorithm, differential evolution and particle swarm optimization are considered. Author analyses that, the performance of differential evolution is better than that of the other two optimization methods.

The objective is to analyse the best algorithm from three algorithms Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Differential Evolution (DE) which are quite popular in optimization world. The main interest is to find out strength and limitations of these three evolutionary algorithms. The concept of particle swarm optimization is based on simulation of birds flocking in N-dimensional space. The bunch of particles is defined in the space and each particle in that space can be the solution of the optimization problem. The algorithm is so farmed that each particle is displaced to another position to get better solution for optimization. The author applied one more optimization algorithm or methodology called as constriction factor approach which is the modification of simple particle swarm optimization.

The author also implemented differential evolution algorithm, the basic difference between GA and DE algorithm is, in the DE, mutation parameter is adjusted in such a way that it is automatically scaled to a correct value while the algorithm is getting towards the correct solution. There are many different strategies of using DE algorithm based on number of parameters. The objective function and constraints used by the author is same as that of Cabrera [1]. Author also considered the same three cases and concluded as $D E$ algorithm gives the best solution.

Cabrera et al. [4] again implemented an algorithm which is the modification of previous algorithm presented by himself and the name of this algorithm is Malaga University Mechanism Synthesis Algorithm (MUMSA). This algorithm gives the best solution compared with the other algorithms. The approach presented is based on differential evolutionary techniques whose description is as follows. The first population is created by random considering upper and lower bonds of each variable. Then by giving the weightage to every solution, the best possible solution is found which is considered as a reference and other two individuals from the population are randomly selected, which are then combined with the best individual to form a vector which is crossover with every individual vector of the population. After crossover if the new individual has better solution than that of old then the old one is replaced by the new individual otherwise it remains same and then the mutation operator changes the value of genes of individual depending on theirs ranges and also the mutation probability.

Matekar et al. [5] defined the longitudinal and transverse error considering with factor of equivalence. The factor of
equivalence is multiplied with transverse error and this product is added with longitudinal error to form the objective function. The intention is to check the change in the output coupler curve by varying the value of factor of equivalence and set the optimization value of the same to get desired results. The results shows that up to certain limit of factor of equivalence it gives better results but beyond that value error is again going to increase with increase in factor of equivalence.

Jianyou et al. [6] applied analytical method for synthesis of $\lambda$ - formed four bar mechanism in order to produce straight line. From this $\lambda$ - formed four bar mechanism, there could be the possibility of formation of 3 pairs of linkages. 3 constraints are formulated in which there are 2 separate position constraints and a velocity constraint. The displacement matrix method is used. The $\lambda$ - formed four bar mechanism is capable of producing extraordinary good approximations of the desired coupler path. There are some geometrical methods also developed for synthesis of mechanism but there are very much complications occur while solving with this process and also one should know very much knowledge about the kinematical geometry theory.

The coupler curve produced by this analytical method may be asymmetrical or symmetrical. There is a restriction while solving by this analytical method that is one has to assume certain variables because there are many variable and equations so in order to simplify the optimization problem using analytical methods, the assumption is must.

## 5. CONCLUSIONS

The classical graphical and analytical methods are analyzed briefly. The assumption is must while solving with these methods. Six papers are analyzed briefly in which most of the authors have employed different evolutionary algorithms and comparative study between those algorithms is done. Three objective functions are implemented by authors which are Euclidean distance error function, modified distance error function and orientation of structural error.

## 6. REFERENCES

[1] J.A. Cabrera, A. Simon, M. Prado, April 2002, "Optimal synthesis of mechanisms with genetic algorithms", Mechanism and Machine Theory 37 (2002) 1165-1177.
[2] M. A. Laribi, A. Mlika, L. Romdhane, S. Zeghloul, February 2004, "A combine genetic algorithm-fuzzy logic method (GA-FL) in mechanisms synthesis", Mechanism and Machine Theory 39 (2004) 717-735.
[3] S.K. Acharyya, M. Mandal, April 2009, "Performance of EAs for four-bar linkage synthesis", Mechanism and Machine Theory 44 (2009) 1784-1794.
[4] J.A. Cabrera, A. Ortiz, F. Nadal, J.J. Castillo, October 2010, "An evolutionary algorithm for path synthesis of mechanisms", Mechanism and Machine Theory 46 (2011) 127-141.
[5] Sanjay B. Matekar, Gunesh R. Gogate, February 2012, "Optimum synthesis of path generating four-bar mechanisms using differential evolution and a modified error function", Mechanism and Machine Theory 52 (2012) 158-179
[6] Han Jianyou, Qian Weixiang, Zhao Huishe, June 2007, "Study on synthesis method of $\lambda$ - formed four bar linkages approximating a straight line", Mechanism and Machine Theory 44 (2009) 57-65.
[7] D. E. Goldberg, "Genetic Algorithm in Search, Optimization, and Machine Learning", Person Education Pte Ltd.
[8] Joseph Edward Shigley, "Theory of Machines and Mechanisms", The MaGraw Hill Company.
[9] Kalyanmoy Deb, "Optimization for Engineering Design Algorithms and Examples", Prentice-Hall of India Pvt Ltd.
[10] Dan B. Marghitu, "Mechanisms and Robots Analysis with MATLAB®", Springer.

