Analytical Solution of Compartment Based Double Pipe Heat Exchanger Using Differential Transform Method

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Abstract – *This paper discussed analytical solution of the compartment based double pipe heat exchanger model obtained using Differential Transform Method for parallel flow with theoretical varying initial and boundary condition. The working fluid is transformer oil i.e. hot fluid and water act as coolant. Convergence analysis of solution is also discussed.*

***Key Words*:** Compartment Model, Double Pipe Heat Exchanger, Differential Transform Method, Convergence.

1. INTRODUCTION

Heat transfer continues to be a field of major interest to engineering and scientific researchers, as well as designers, developers, and manufacturers. A heat exchangers are one of the mostly used device used to transfer heat between one or more fluids in which fluids are separated by a solid wall to prevent mixing. Heat exchangers has many applications like petroleum refineries, power stations, refrigeration, air conditioning, chemical plants, petrochemical plants, natural-gas processing, and sewage treatment. One can realize their usage that any process which involve cooling, heating, condensation, boiling or evaporation will require a heat exchanger for these purpose. Process fluids, usually are heated or cooled before the process or undergo a phase change. Heat exchangers are typically classified according to flow arrangement and type of construction. The simplest heat exchanger is one for which the hot and cold fluids flow in the same or opposite directions in a concentric-tube (or double pipe) construction. In the parallel-flow arrangement, the hot and cold fluids enter at the same end, flow in the same direction, and leave at the same end. In the counter flow arrangement, the fluids enter at opposite ends, flow in opposite directions, and leave at opposite ends.

A typical concentric tube heat exchanger consists of one pipe placed concentrically in side another of larger diameter with appropriate fittings to direct the flow from one section to the another section. One fluid flows through the inner pipe and other fluid flows through the annular space. The major use of double pipes exchangers for sensible heating or cooling of process fluids where small heat transfer area required.

In this paper double pipe heat exchanger is treated as two compartment model to find the analytical solution to study the temperature profile of hot fluid. Section 2 deals with the literature review of the numerical and analytical solution of double pipe heat exchanger. Section 3 describes the mathematical model of compartment based double pipe heat exchanger model with theoretical varying initial and constant boundary condition. It also discusses an analytical solution of compartment based double pipe heat exchanger using two-dimensional differential transform method with theoretical varying initial and constant boundary condition. It also discusses the convergence analysis of analytical solution of the mathematical model. Section 4 discusses result and its analysis.

2. LITERATURE REVIEW

This section deals with the literature review of numerical and analytical solution found to study the thermal performance of the double pipe heat exchanger.

Khoddamrezaee, F et al. [5] used finite volume method to study the thermal performance of heat exchanger using Nano fluids. They found that the effect of increasing in shear stress in comparison with the extreme increasing of heat transfer coefficient (325%) is negligible. C. Z. Patel et al. [6] validated CFX model by comparing it to the experimental results. Thermal performance of a shell and tube heat exchanger operated with Nano fluids has been analytically investigated at different mass flow rates and compared with water as the base fluid was done by Shahrul I. M et al [7]. Suspensions of ZnO, CuO, Fe_3O_4, TiO_2, and Al_2O_3 nanoparticles in water (W) at 0.03 volumetric fractions have been considered. Khairul M. A et al. [8] proposed a fuzzy logic expert system model for the prediction of heat transfer coefficient and friction factor. Verification of the developed fuzzy logic model was carried out through various numerical error criteria. A computational fluid dynamics (CFD) study has been carried out to study the heat transfer and pressure drop characteristics of water-based Al_2O_3 Nano fluid flowing inside coiled tube-in-tube heat exchangers by Aly et al [9]. Forced convection flows of Nano fluids consisting of water with AL2O3 nanoparticles in a double-tube counter flow heat exchanger are investigated numerically by Borazjani M. S et al [10]. Demir, H., et al. [11] studied forced convection flows of Nano fluids consisting of water with TiO_2 and Al_2O_3 nanoparticles in a horizontal tube with constant wall temperature are investigated numerically. Mohammed, H. A et al. [12] studied the effect of using louvered strip inserts placed in a circular double pipe heat exchanger on the thermal and flow fields utilizing various types of Nano fluids is studied numerically. The thermal performance of heat exchanger has studied by many researchers and for more literature reader can refer [12].

3. COMPARTMENT BASED DOUBLE PIPE HEAT EXCHANGER MODEL

This section describe the two compartment based heat exchanger model with concurrent flow as shown in the figure -1. Considering the two compartment heat exchanger model of length L with inner diameter of inner tube as D_i and outer diameter of inner tube as D_o with parallel flow. The process of heat transfer in double pipe heat exchanger by means of convection and conduction is shown in figure 1.

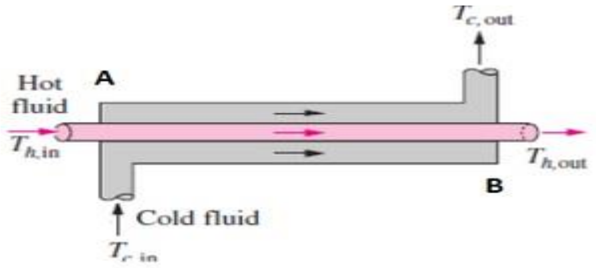

Figure-1: Compartment based double pipe heat exchanger with the concurrent flow.

The energy balance of the heat exchanger, after classical simplifying hypotheses [1,2], gives the following partial differential equation for the internal tube:

$$\frac{\partial T_C}{\partial t}=\omega v_C(t)\frac{\partial T_C}{\partial x}+\frac{UA}{M_C C_{pc}}(T_h-T_C) \quad (1)$$

And for the annulus, the partial differential equation is given by

$$\frac{\partial T_h}{\partial t}=-v_h(t)\frac{\partial T_h}{\partial x}-\frac{UA}{M_h C_{ph}}(T_h-T_c) \quad (2)$$

Where v_h denotes the velocity of hot fluid , v_c denotes the velocity of cold fluid, T_c denotes the temperature of hot fluid, T_h denotes the temperature of cold fluid M_c denotes the total mass inside the annulus , M_h denotes the total mass inside the inner tube , C_{pc} denotes the specific heat of the cold fluid /coolant C_{ph} denotes the specific heat of the hot fluid.

Initial and boundary condition for cold Fluid

$$T_c(x,0)=c_1+c_2e^{\alpha x} \quad ; \qquad T_c(0,t)=c_2. \quad (3)$$

Initial and boundary condition for Hot Fluid

$$T_h(x,0)=d_1+d_2e^{-\gamma x}; \qquad T_h(0,t)=d_2. \quad (4)$$

3.1. SOLUTION OF TWO COMPARTMENT BASED DOUBLE PIPE HEAT EXCHANGER MODEL USING DIFFERENTIAL TRANSFORM METHOD

The transformed equation obtained by applying two-dimensional DTM to coupled system of PDE (1) and (2) with initial and boundary condition (3) and (4), the system of recursive formula is given as follows:

$$(h+1)T_c(k,h+1)=\omega v_c(k+1)T_c(k+1,h)+\frac{UA}{M_cC_{pc}}\left(T_h(k,h)-T_c(k,h)\right) \quad (5)$$

$$(h+1)T_h(k,h+1)=-v_h(k+1)T_h(k+1,h)-\frac{UA}{M_hC_{ph}}\left(T_h(k,h)-Tc(k,h)\right) \quad (6)$$

Transformed initial and boundary condition for cold fluid on applying two-dimensional DTM to initial and boundary condition (3) is given by

Transform Initial Value Transform Boundary value

$$T_c(0,0)=c_1+c_2 \qquad T_c(0,0)=c_1 \quad (7)$$

In general for k $\geq 1, and\ h \geq 1$

$$T_c(k,0)=\frac{c_2(\alpha)^k}{k!}. \quad (8)$$

Transformed initial and boundary condition for hot fluid on applying two-dimensional DTM to initial and boundary condition (4) is given by

Transform Initial Value Transform Boundary value

$$T_h(0,0)=d_1+d_2 \qquad T_h(0,0)=d_1 \quad (9)$$

In general k $\geq 1, and\ h \geq 1$

$$T_h(k,0)=\frac{c_2(-\gamma)^k}{k!}. \quad (10)$$

Utilizing system of recursive formula (5) and (6) and initial and boundary transformed coefficient (7)-(10), the coefficients of T_c and T_h of the series solution can be obtained.

3.2. NUMERICAL SIMULATION

The analysis of heat transfer is carried out by using water as coolant and Transformer oil as hot fluid for varying theoretical initial and constant boundary condition, the temperature profile of hot and cold fluid is studied for 10000 seconds. The inlet temperature of the water and transformer oil is considered as 293.15 K and 398.15 K. The geometry of double pipe heat exchanger consists of a length of 1 m made of copper material. Concentric tube of inner tube inner diameter 0.1m, inner tube outer diameter 0.14m and outer tube inner diameter 0.24 m. [3] The analysis of heat transfer is done for copper material. The material properties are mention below in the Table 1 and 2.

Table -1: Thermophysical properties used in the model.

Material	Density(kg/m^3)	Thermal Conductivity(W/mK)	Specific Heat (j/Kg K)
Copper	8978	387.6	381

Table -2: Thermophysical properties fluid used in the model.

Fluid	Density ρ (kg/m^3)	Prandtl Number P_r	Thermal conductivity (K)W/m K	Specific Heat C_p (j/kg K)	Dynamic Viscosity μ (kg/m s)
Trans	826	159	0.134	2328	0.0091

Following parameters are used to evaluate the analytic solution of compartment based double pipe heat exchanger model: c_1=d_1=0, c_2= 293.15, d_2=398.15, α =0.01, $\beta = 0$, γ=0.00001.

Substituting the thermo physical properties mention in table 1 and 2 in (5)-(10) the coefficient for coupled system of equation (1) and (2) with initial and boundary condition (3) -(4) is obtained and the analytical approximate series solution for cold and hot fluid is given by

Compartment –I: Transformer oil as **Hot Fluid**

$$\begin{aligned} T_h(x,t) &= 398.15 - 31.852x + 1.27408x^2 - 0.03398x^3 + \ldots \\ &-1.80\times10^{-5}x^2t + 4.90\times10^{-7}x^3t - 9.77\times10^{-9}x^4t + \ldots \\ &-4.56\times10^{-12}x^3t^2 + 9.09\times10^{-14}x^4t^2 - 1.46\times10^{-15}x^5t^2 + \ldots \\ &-4.98\times10^{-19}x^4t^3 + 7.98\times10^{-21}x^5t^3 1.52\times10^{-19}xt^4 + \ldots \end{aligned} \tag{11}$$

Compartment –II: Water as Coolant

$$\begin{aligned} T_c(x,t) &= 293.15 + 2.9315x + 0.014658x^2 + 0.0000489x^3 + .. \\ &-9.73\times10^{-5}xt + 3.40\times10^{-6}x^2t - 9.24\times10^{-8}x^3t + .. \\ &+5.02\times10^{-10}xt^2 - 1.52\times10^{-11}x^2t^2 + 4.21\times10^{-13}x^3t^2 + \ldots \\ &-4.10\times10^{-15}xt^3 + 1.22\times10^{-16}x^2t^3 - 3.39\times10^{-18}x^3t^3 + \ldots \\ &+1.78\times10^{-20}xt^4 - 4.36\times10^{-22}x^2t^4 + 1.26\times10^{-23}x^3t^4 + \ldots \end{aligned} \tag{12}$$

3.2.1 CONVERGENCE OF SERIES SOLUTION

Equation (11) and (12) represents the analytic approximated solution of coupled system of differential equation (1) and (2) with initial and boundary condition (3) to (4) with specific parameter value mention above. The convergence of this series solution is discussed for m=n=1, m=n=2, m=n=3, m=n=4 and the error analysis table for hot and cold fluid is given below:

Table -2: Error analysis for equation (11).

(x_i,t_i)	$E_{1,1}(x_i,t_i)$	$E_{2,2}(x_i,t_i)$	$E_{3,3}(x_i,t_i)$
(0,0.5)	0	0	0
(0.2,1.5)	0.00058654	0.000000391	0.0000000002
(0.4,2)	0.00234643	0.00000313	0.000000000322
(0.6,2.5)	0.00528012	0.0000106	0.0000000164
(0.8,3)	0.00938803	0.0000025	0.00000000522

Table -2: Error analysis for equation (12).

(x_i,t_i)	$E_{1,1}(x_i,t_i)$	$E_{2,2}(x_i,t_i)$	$E_{3,3}(x_i,t_i)$
(0,0.5)	0	0	0
(0.2,1.5)	0.0509631	0.00027	0.0000011
(0.4,2)	0.2038522	0.00217	0.000017
(0.6,2.5)	0.4586672	0.00734	0.000088
(0.8,3)	0.8154078	0.0174	0.00028

From the Table-1 and Table-2, it is clear that $E_{m,n}(x_i, y_i) \geq E_{p,q}(x_i, y_i)$, for some (x_i, y_i) .Thus, the series solution (7.90) converges when $m,n \to \infty$ [4].

4. RESULT AND DISCUSSION

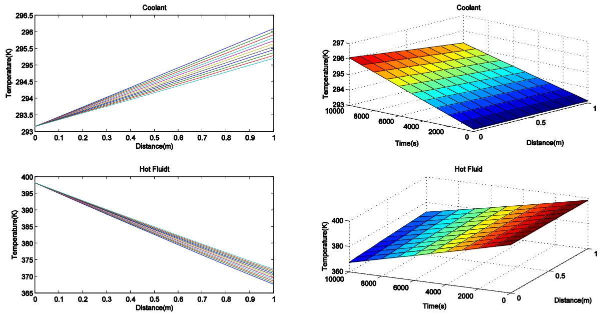

Figure-2: Temperature profile of Coolant and Hot fluid for coupled system of differential equation (1) and (2) with initial and boundary condition (3) – (4).

The inlet temperature of the coolant is 293.15 K whereas for hot fluid it is 398.15 K. From the above Figure 2 it is clear that the as time and distance increases the temperature of the coolant increases whereas the temperature of the hot fluid decreases when both coolant and hot fluid are moving with velocity of 0.0001m/s when overall heat coefficient is 0.7687 W/k and heat transfer coefficient pipe is 5.2873773w/m^{2}k.The outlet temperature of the coolant and hot fluid is 295.11K and 372.13 K for initial and boundary condition (3) -(4).

5. CONCLUSION

Differential Transform Method proves to be one of the powerful method to find the analytical solution of compartment based double pipe heat exchangers.

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