Bending Analysis of Sandwich Beam with Soft Core

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Abstract - *In this paper, the refined beam theory (RBT) is analyzed for the bending of simply supported isotropic, laminated composite and sandwich light beams. The axial displacement field uses parabolic function in conditions of thickness ordinate to incorporate the effect of slanted shear deformation. The transverse displacement includes folding and shears components. the current theory satisfies the traction force free conditions on the top and lower floors of the beam without using problem dependent shear correction factors of Timoshenko. Governing differential equations and border conditions associated with the assumed displacement field are obtained utilizing the rule of virtual work. To prove the credibility of the present theory, we applied it to the bending analysis of light beams. A simply supported isotropic, laminated composite and sub beams are analyzed using navier approach. The statistical results of non-dimensional Displacements and stresses obtained utilizing the present theory are presented and compared with those of other processed theories available in the literature along with the elasticity solution*

Key Words: composite plate; finite element analysis; isotropic; laminated; orthotropic; transverse shear strain

1. INTRODUCTION

This document is template. Strength components made of fibrous ceramic material are significantly being used {in a variety of executive applications due to their attractive properties in power stiffness, and lightness. The effect of transverse shear deformation much more pronounced in thick beams made of fibrous composite material which has a high extensional modulus to shear modulus ratio. The classical light beam theory (CBT) does not the proper bending tendencies thick beams made of fibrous ceramic materials. The first order shear deformation beam theory (FSDT) developed by Timoshenko (1921) includes the effect of transverse shear deformation but does not gratify the zero shear stress conditions on the top and bottom surfaces of the beam, hence, it requires shear correction factor.

A large number of higher order theories are available in the materials for the bending, attachment and free vibration evaluation of laminated composite light beams which take into accounts the a result of transverse shear deformation and don't require shear correction factor. The third order theory of Reddy (1984) is the most widely frequently used higher order theory for light beams as well as for plates. A recent overview of higher order theories available for the analysis of laminated composite beams has been presented by Ghugal and Shimpi (2001). Kadoli et al. (2008) applied the third order theory of Reddy for the static analysis of functionally graded beams. An total analytical model was developed by Lee (2005) using the shear deformable column theory and was applied to the flexural evaluation of thin walled I-shaped laminated composite beams. Chen and Wu (2005) developed a new higher-order shear deformation theory depending on global-local superposition technique. Reddy (2007) reformulated various beam ideas using nonlocal elasticity and applied them to the bending, buckling and oscillation analysis of beams. Wang et al. (2008) also presented some work on beam bending solutions centered on nonlocal Timoshenko light beam theory. Mechab et approach (2008) performed an evaluation of parabolic and rapid shear deformation theories on bending of short laminated composite beams subjected to mechanical and thermal charge. Carrera and Giunta (2010) presented refined beam hypotheses based on an specific formulation and applied these to the static analysis of beams made of isotropic materials. Karama et approach . (2008) did the processing of Ambartsumian multi-layer light beam theory considering an great function in conditions of thickness coordinate. Chakrabarti et al. (2011) presented a new finite aspect model based on the zig-zag theory for the research of sandwich beams which is further extended by Chalak et al. (2011) for free vibration research of laminated sandwich light beams having soft core. Gherlone et al. (2011) performed the finite aspect research of multilayered composite and sandwich beams based on the refined zigzag theory.

Sayyad and Ghugal (2011) developed a trigonometric shear and normal deformation theory for the bending research of laminated composite light beams exposed to various static charge. Sayyad (2011) presented a refined shear deformation theory for the static flexure and free vibration analysis of thick isotropic beams considering parabolic, trigonometric, hyperbolic and exponential functions in terms of thickness co-ordinate associated with transverse shear deformation effect. This kind of theory is further prolonged by Sayyad et approach. (2014) for the flexural analysis of single split composite beams. Chen et al. (2011) carried away bending analysis of laminated composite resin plates considering first order shear deformation structured on modified couple stress theory.

Aguiar et approach. (2012) accomplished static examination of composite beams of different cross-sections using combined and displacement based models. Ghugal and Shinde (2013) extended the layerwise trigonometric shear deformation theory of Shimpi and Ghugal (2001) for the bending evaluation of two layered anti-symmetric laminated composite beams with various boundary conditions. Lately Sayyad et al. (2015) developed a new trigonometric shear deformation theory for the bending analysis of laminated composite and sub beams.

The theory used in the present research is formerly produced by Shimpi and Patel (2006) for the bending analysis of orthotropic plates. With this newspaper, this theory is applied to the bending research of laminated composite and sandwich beams. Governing equations and boundary conditions of the presented theory are obtained using the theory of virtual work. The Navier's solution technique is used for the simply recognized boundary conditions. The statistical the desired info is obtained for isotropic, laminated composite and sub beams subjected to sinusoidal load.

1.1 The Development of the Theory

A laminated composite beam of length '*L*', width '*b*' and overall thickness '*h*' as shown in Fig. 1 is considered. The beam consists of '*N*' number of layers made up of linearly elastic orthotropic material. The beam occupies the region $0 \le x \le L$, $-b/2 \le y \le b/2$ and $-h/2 \le z \le h/2$ in Cartesian coordinate system.

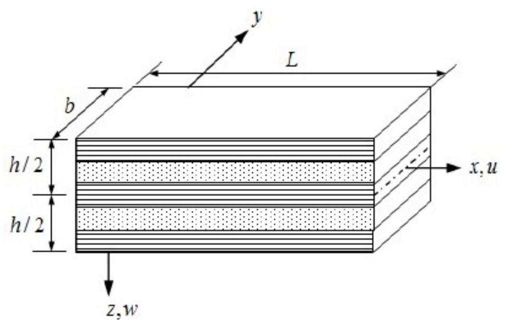

Fig. 1: Geometry and coordinate system of laminated composite beam

1.1.1 The Displacement field

In the present theory, the axial displacement *u* in *x* direction consists of extension, bending and shear components, whereas transverse displacement *w* in the *z*-direction.

$$u = u_0 - z\frac{\partial w_0}{\partial x} + f(z)\phi_x \quad (1)$$

$$w = w_0 + f'(z)\psi_x \quad (2)$$

1.1.2 Strains

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} + f(z)\frac{\partial \phi_x}{\partial x} \quad (3)$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = f''(z)\psi \quad (4)$$

Where,

$$f'(z) = 1 - \left[\frac{\cosh\left(\frac{\pi z}{h}\right) - 1}{\cosh\left(\pi/2\right) - 1}\right]$$

$$f''(z) = -\left[\frac{\left(\frac{\pi}{h}\right)\sinh\left(\frac{\pi z}{h}\right)}{\cosh\left(\pi/2\right) - 1}\right]$$

1.1.3 Stresses

Direct stresses and shear stress can be given by following relationship

$$\sigma_x = Q_{11}\varepsilon_x + Q_{13}\varepsilon_z \quad (5)$$

$$\sigma_x = Q_{13}\varepsilon_x + Q_{33}\varepsilon_z \quad (6)$$

$$\tau_{xz} = Q_{55}\gamma_{xz} \quad (7)$$

Where,

Q11, Q13 Q33 are the Young's modulus in the axial direction of the laminated composite beam, while *Q55* is the shear modulus. The principle of virtual work is used to obtain the governing equations of equilibrium and associate boundary conditions.

1.1.4 Principle of Virtual Work

The analytical form of the Principle of virtual work is,

$$\int_{-h/2}^{h/2}\int_0^L (\sigma_x \delta\varepsilon_x + \sigma_z \delta\varepsilon_z + \tau_{xz}\gamma_{xz})\,dxdz = \int_0^L q\delta w_0 dx \quad (8)$$

Substituting expressions for strains and stresses from Eqs. (3) - (7) into Eq. (8) and integrating throughout the thickness the following equations are obtained:

$$N_x = A_{11}\frac{\partial u_0}{\partial x} - B_{11}\frac{\partial^2 w_0}{\partial x^2} + C_{11}\frac{\partial \phi_x}{\partial x} + D_{13}\psi \quad (9)$$

$$M_x^b = B_{11}\frac{\partial u_0}{\partial x} - E_{11}\frac{\partial^2 w_0}{\partial x^2} + F_{11}\frac{\partial \phi_x}{\partial x} + G_{13}\psi \quad (10)$$

$$M_x^s = C_{11}\frac{\partial u_0}{\partial x} - E_{11}\frac{\partial^2 w_0}{\partial x^2} + H_{11}\frac{\partial \phi_x}{\partial x} + I_{13}\psi \quad (11)$$

$$Q_x = J_{55}\phi + J_{55}\frac{\partial \psi}{\partial x} \quad (12)$$

$$Q_z = D_{13}\frac{\partial u_0}{\partial x} - G_{13}\frac{\partial^2 w_0}{\partial x^2} + I_{13}\frac{\partial \phi_x}{\partial x} + L_{33}\psi \quad (13)$$

1.1.5 Boundary Conditions

Integrating Eq. (8) by parts and setting the coefficients of $\partial u_0, \delta w_0, \delta\phi, \delta\psi$ zero, the following governing differential equations and associated boundary conditions are obtained:

$$\left.\begin{aligned} \partial u_0 &= -\frac{dN_x}{dx} = 0 \\ \delta w_0 &= -\frac{d^2 M_x^b}{dx^2} = q \\ \delta\phi &= Q_x - \frac{dM_x^s}{dx} = 0 \\ \delta\psi &= Q_z - \frac{dQ_x}{dx} = 0 \end{aligned}\right\} \text{........(14)}$$

1.1.6 The Governing Differential Equations

The governing differential equations in terms of unknown displacement variables (u_0, w_0, ϕ_x, ψ_z) are rewritten as:

$$-A_{11}\frac{\partial^2 u_0}{\partial x^2} + B_{11}\frac{\partial^3 w_0}{\partial x^3} - C_{11}\frac{\partial^2 \phi_x}{\partial x^2} - D_{13}\frac{\partial \psi}{\partial x} = 0\text{....................(15)}$$

$$-B_{11}\frac{\partial^3 u_0}{\partial x^3} + E_{11}\frac{\partial^4 w_0}{\partial x^4} - F_{11}\frac{\partial^3 \phi_x}{\partial x^3} - G_{13}\frac{\partial^2 \psi}{\partial x^2} = q\text{....................(16)}$$

$$-C_{11}\frac{\partial^2 u_0}{\partial x^2} + F_{11}\frac{\partial^3 w_0}{\partial x^3} - H_{11}\frac{\partial^2 \phi_x}{\partial x^2} + (J_{55} - I_{13})\frac{\partial \psi}{\partial x} + J_{55}\phi = 0\text{..(17)}$$

$$D_{13}\frac{\partial u_0}{\partial x} - G_{13}\frac{\partial^2 w_0}{\partial x^2} + (I_{13} - J_{55})\frac{\partial \phi_x}{\partial x} - J_{55}\frac{\partial^2 \psi}{\partial x^2} + K_{33}\psi = 0\text{....(18)}$$

Where,

$$A_{11} = Q_{11}\int_{-h/2}^{h/2} dz,\quad B_{11} = Q_{11}\int_{-h/2}^{h/2} z\,dz,$$

$$C_{11} = Q_{11}\int_{-h/2}^{h/2} f(z)dz,\quad D_{13} = Q_{13}\int_{-h/2}^{h/2} f''(z)dz,$$

$$E_{11} = Q_{11}\int_{-h/2}^{h/2} z^2 dz,\quad F_{11} = Q_{11}\int_{-h/2}^{h/2} z f(z)dz,$$

$$G_{13} = Q_{13}\int_{-h/2}^{h/2} z f''(z)dz,\quad H_{11} = Q_{11}\int_{-h/2}^{h/2} f(z)^2 dz,\quad (19)$$

$$I_{13} = Q_{13}\int_{-h/2}^{h/2} f(z) f''(z)dz,\quad J_{55} = Q_{55}\int_{-h/2}^{h/2} f'(z)^2 dz,$$

$$K_{33} = Q_{33}\int_{-h/2}^{h/2} f''(z)^2 dz.$$

1.1.7 Navier Solution

The closed form solution is obtained using the Navier's solution technique. A beam as shown in Fig. 1 is considered for the detailed numerical study. The following simply-supported boundary conditions are considered at $x = 0, x = L$

$$N_x = M_x^b = M_x^s = Q_x = Q_z = 0 \text{ .... (20)}$$

The beam is subjected to sinusoidal load $q(x)$ on the top surface, *i.e.* $z = -h/2$. The load q is expanded in single trigonometric series:

$$q(x) = q_m \sin\alpha \text{ .... (21)}$$

Where, q_m denotes the intensity of the load at the center of the beam. The following expansions of the unknown displacement variables (u_0, w_0, ϕ_x, ψ_z) satisfy the boundary conditions in Eq. (20):

$$u_0 = u_m\cos\alpha x,\ w_0 = w_m\sin\alpha x$$

$$\phi_x = \phi_{xm}\cos\alpha x,\ \psi_z = \psi_{zm}\sin\alpha x \qquad (22)$$

Where, $u_m, w_m, \phi_{xm}, \psi_{zm}$ are arbitrary parameters. Substituting unknown displacement variables (u_0, w_0, ϕ_x, ψ_z) from Eq. (22) and the load from Eq. (21) into the Eqs. (15) - (18), the closed-form solutions can be obtained from the following equations:

$$\begin{bmatrix} A_{11}\alpha^2 & -B_{11}\alpha^3 & C_{11}\alpha^2 & -D_{13}\alpha \\ -B_{11}\alpha^3 & E_{11}\alpha^4 & -F_{11}\alpha^3 & G_{13}\alpha^2 \\ C_{11}\alpha^2 & -F_{11}\alpha^3 & (H_{11}\alpha^2 + J_{55}) & (J_{55} - I_{13})\alpha \\ -D_{13}\alpha & G_{13}\alpha^2 & (J_{55} - I_{13})\alpha & (J_{55}\alpha^2 + K_{33}) \end{bmatrix} \begin{Bmatrix} u_m \\ w_m \\ \phi_m \\ \psi_m \end{Bmatrix} = \begin{bmatrix} 0 \\ q_m \\ 0 \\ 0 \end{bmatrix}$$

(23)

2. NUMERICAL RESULTS AND DISCUSSION

To assess the efficiency of the present theory, the bending analysis of simply supported beams is considered. The numerical results are obtained for displacements and stresses for isotropic, laminated composite and sandwich beams. The values of transverse shear stress presented in the tables are obtained by using equilibrium equations of the theory of elasticity to satisfy interface continuity.

The following non-dimensional forms are used to present the displacements and stresses:

$$\begin{aligned} &\overline{u}(0,-h/2) = u \times n_1, \overline{w}(L/2,0) = w \times n_2 \\ &\overline{\sigma_x}(0,-h/2) = \sigma_x \times n_3, \overline{\tau_{xz}}(0,0) = \tau_{xz} \times n_3 \end{aligned} \qquad (24)$$

Where,

$$n_1 = \frac{b}{100q_0h}, n_2 = \frac{100h^3}{q_0a^4}, n_3 = \frac{b}{10q_0}$$

2.1 Bending analysis of isotropic beam

The simply supported isotropic beams subjected to sinusoidal load are considered with the following material properties:

$Q11$ = 210GPa and $Q55$ = 80.77GPa

Table 1 shows the maximum displacements and stresses for the simply supported isotropic beams with L/h = 4, 10, 20, 50 and 100. The present results are compared with the elasticity solution provided by Ghugal (2006), the higher order shear deformation theory (HSDT) of Reddy (1984), the first order shear deformation theory (FSDT) of Timoshenko (1921) and the classical beam theory (CBT). From the examination of Table1 it is observed that the present theory accurately predicts the values of axial (u) and transverse (w) displacements. For L/h = 4, 10 and 20, these displacements (u and w) are identical to those obtained by the HSDT of Reddy (1984). The bending stress predicted by the present theory is in excellent agreement with that of the exact solution.

3. CONCLUSIONS

This paper, the refined light beam theory has been requested laminated composite and very soft core sandwich beams. The mathematical formulation and putting on the present theory to bending analysis of light beams generated the following a conclusion:

1. The idea complies with the zero transverse shear conditions on top and bottom surfaces of the beam. The transverse stress continuity is satisfied using equilibrium equations of the theory of elasticity.

2. The governing equations and boundary conditions are variationally consistent.

3. The theory obviates the requirement of shear a static correction factors which can be associated with the first order shear deformation theory.

4. The current results are in excellent agreement with those of the exact solution and the HSDT of Reddy.

5. The CBT and the FSDT show inaccurate results as opposed} with the current theory and the HSDT of Reddy.

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