Comparison of Non-linear and Linear Model Predictive control of Point Absorber Wave Energy Converter

Thara V Mohan[1], Manju Sreekumar[2]

[1]PG Student, Dept. of Electrical & Electronics Engineering, Mar Baselios college of Engineering and Technology, Kerala, India
[2]Assistant Professor, Dept. of Electrical & Electronics Engineering, Mar Baselios college of Engineering and Technology, Kerala, India

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Abstract - *Ocean wave energy is a promising renewable energy source which can be converted into electrical energy by using wave energy converters (WECs). For the WECs to become a commercially viable alternative in the established methods of energy generation, operating WEC in an optimal fashion is a key task. One subclass of Wave Energy Converters is the buoy type point absorber which uses a linear generator as the power take-off (PTO) system. This work aims to control WEC power potential while respecting the constraints on motions and forces using Model predictive control (MPC). Due to possible nonlinear effects such as the mooring forces and radiation forces a nonlinear model predictive controller (NMPC) is also proposed, whose performance is compared to that of linear MPC. The MPC and NMPC controllers are compared through simulation for regular sea states. On the basis of the simulated results, the NMPC coupled forecasting shows the system to optimize energy capture while respecting system constraints*.

Key Words: Wave energy converter, Point absorber wave energy converter, Model predictive control, Non-linear model predictive control

1. INTRODUCTION

The Ocean wave energy is enormous and it is almost untapped source of energy. The various form of ocean energy exists are tidal and marine currents, thermal, salinity and wave energy. This work mainly focuses on wave energy. Wave energy basically originates from wind, which in turn originates from the sun. When the wind blows over the ocean, the friction gives rise to water movements and thus waves are generated. The total energy of waves on earth is much lower than the total solar energy but it is much denser, specifically wave energy is about five times denser than solar energy and it has power density up to 2-3kw/m on the surface. The device which converts ocean wave energy into electricity is called a Wave Energy Converter (WEC). A WEC comes in various shapes and sizes. The one that is modelled here is a point absorber wave energy converter. Figure 1 shows Wave energy Converter. It consists of float, spar, and a power taking off device which is the linear generator.

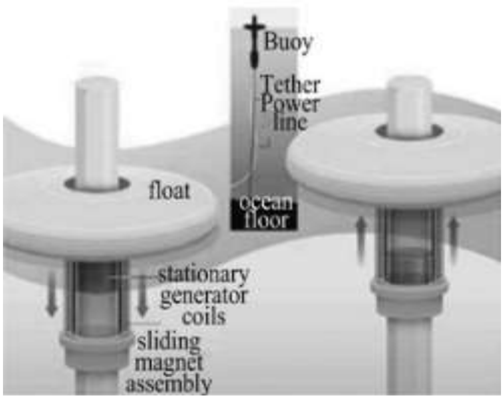

Figure 1 Wave Energy Converter

In order for the Wave Energy Converter to become a commercially viable alternative, the WEC has to be operated in an optimal fashion. The optimization of Wave Energy Converter has led to various control laws such as phase and amplitude control. The Controllers for Wave Energy Converter needs to perform two main tasks i.e., the generated power needs to be maximized, and the WEC's motion needs to be respected. The Controllers can also be used to predict the wave data since wave prediction is generally possible and it is promising. Hence Model Predictive Control (MPC) is a promising control approach, since it can exploit the entire power potential of WEC on one hand, while respecting the system constraints and forces on the other hand. Moreover prediction data can be included. In order to deal with nonlinear effects such as mooring forces and radiation forces, a nonlinear model predictive control is proposed and its performance is compared to that of linear MPC, also controlling the nonlinear system. The proposed controllers, Model predictive control and Non-linear model predictive control are validated and compared through simulations for regular waves. The average power obtained by MPC and NMPC are compared through MATLAB simulations.

1.1 Point Absorber Model

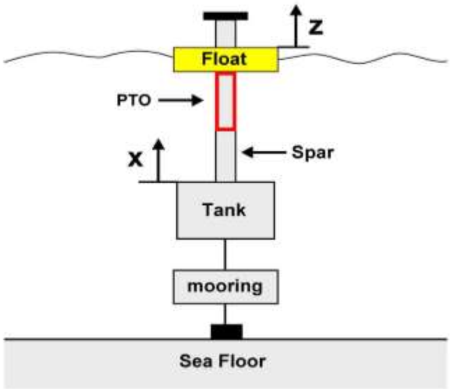

Figure 2 Schematic diagram of WEC

Figure 2 shows schematic diagram of Wave energy Converter. It consists of a float, also called buoy which is floating on the ocean surface and a second body consisting of spar and ballast tank, where the spar's motion is damped through mooring. The relative motion of the two bodies is converted into usable energy through a power take off (PTO) system. Here z and x represents the positions of float and the spar, respectively.

A. Equations of Motion

For modeling, linear hydrodynamics and frequency independent WEC parameters are usually considered. These simplifications are reasonable for simple body shapes and small ranges of motion. Based on Newton's law, the dynamic equations can be written as given in equation (1).

$$m\ddot{z}(t) = F_e + F_r + F_h + F_{gen} \qquad (1)$$

where $\ddot{z}$(t) -Buoy acceleration , m - mass of device, F_{gen} - force produced by the power take-off system. It also presents the manipulable input to control the system. The other forces which act on the body are mainly F_r, the radiation force, created by the moving of the float and thus radiating waves , F_h ,the hydrodynamic force which represents the restoring force of the water and F_e the excitation force, which is the force the incoming wave exerts on the body. The radiation force and the hydrodynamic force can be written as

$$F_r = -A\ddot{z}(t) - B\dot{z}(t) \qquad (2)$$

$$F_h = -g\rho\pi r_{float}^2 z(t) = -kz(t) \qquad (3)$$

where A is the added mass of the body, B is the viscous damping, g is the acceleration of gravity, ρ is the density of seawater, r float is the radius of the float and k is then called the hydrostatic stiffness. By means of the Morison approach the wave motion can be linearized and the excitation force denotes the un manipulable system disturbance and is expressed as,

$$F_e = A\ddot{\eta}(t) + B\dot{\eta}(t) + k\eta(t) \qquad (4)$$

Where $\eta(t)$ - water surface elevation

The wave motion can be linearized and it is given in equation (5)

$$F_e + F_{gen} = (A + m)\ddot{z} + B\dot{z} + kz \qquad (5)$$

where η(t) is the water surface elevation. This then gives a simple model for determining the excitation force, and the float motion profile from the water surface elevation η(t).

2. MPC Formulation

The WEC equation motion given in (1) can be expressed in time-domain state-space form as

$$\begin{bmatrix}\dot{z}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{k}{m+A} & -\frac{B}{m+A}\end{bmatrix}\begin{bmatrix}z\\ \dot{z}\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m+A}\end{bmatrix}F_{gen} + \begin{bmatrix}0\\ \frac{1}{m+A}\end{bmatrix}F_e \qquad (6)$$

This model can be discretized into the form

$$x(k+1) = Ax(k) + B_u u(k) + B_v v(k) \qquad (7)$$

Where x(k) is the current state vector i.e., the WEC float velocity and position, u(k) is the generator force-Fgen, and v(k) is the estimated excitation force from the waves Fe.

A sequential model can be found by solving the system forward in time and back-substituting in (7)

$$x(k+2) = Ax(k+1) + B_u u(k+1) + B_v v(k+1) \qquad (8)$$

$$= A\big(A(x) + B_u u(k) + B_v v(k)\big) + B_u u(k+1) + B_v v(k+1) \qquad (9)$$

This can be expressed in matrix form

$$\begin{bmatrix}x(k+1)\\ x(k+2)\end{bmatrix} = \begin{bmatrix}A\\ A^2\end{bmatrix}[x(k)] + \begin{bmatrix}B_u & 0\\ AB_u & B_u\end{bmatrix}\begin{bmatrix}u(k)\\ u(k+1)\end{bmatrix} + \begin{bmatrix}B_v & 0\\ AB_v & B_v\end{bmatrix}\begin{bmatrix}v(k)\\ v(k+1)\end{bmatrix} \qquad (10)$$

The output of the system is equal to the states of the system

$$y = x = \begin{bmatrix}z\\ \dot{z}\end{bmatrix} \qquad (11)$$

A. Optimization formulation

Making the assumptions that the wave climate is monochromatic and there is a steady state wave input is applied, phasors can be used, since all equations are linear. Then, the position and the excitation force can be written as

$$\eta(t) = \eta_A \cos(\omega t + \varphi_n) \approx \hat{H} = \frac{\eta_A}{\sqrt{2}} e^{j\varphi n} \qquad (12)$$

$$F_e = F_{e,A}\cos(\omega t + \varphi F_e) \approx F_e = \frac{F_{eA}}{\sqrt{2}} e^{j\varphi F_e} \qquad (13)$$

The excitation force F_e can be considered as the measured or estimated disturbance. In the prediction model, the current and future values of the disturbance are required and hence it can be calculated by auto-regressive method. The control objective is to find the control action ΔU(k) over the prediction horizon Hp that will minimize the quadratic sum of tracking error and controller effort. The sum is weighted by the Q and R matrices.

$$J(k) = (Y(k) - T(k))^T Q(Y(k) - T(k)) + (\Delta U(k))^T R(\Delta U(k)) \quad (14)$$

The free evolution of the system γ(k), that is, the evolution of the outputs if no control action is taken (i.e., ΔU(k)=0) is expressed as

$$\gamma(k) = S_x x(k) + S_{u1} u(k-1) + H_v V(k) \quad (23)$$

$$= \mathrm{Y}(k) - S_u \Delta u(k) \quad (24)$$

The free evolution error E(k), that is, the evolution of the tracking error if no control action is taken is expressed as

$$E(k) = T(k) - \gamma(k) \quad (25)$$

$$= T(k) - Y(k) + S_u \Delta U(k) \quad (26)$$

This is then substituted into the cost function so that it can be expressed in terms of the control action ΔU(k).

$$J(k) = E(k)^T QE(k) - 2\Delta U(k)^T S_u^T QE(k) + \Delta U(k)^T (S_u^T Q S_u + R)\Delta U(k) \quad (27)$$

The minimization of equation (27) yields the optimal control action vector ΔU(k), such that the tracking error and control action are minimized with consideration of the weights given by the Q and R matrices. Here, the first control action of this vector, Δu(k), is chosen as the control action for time k and the entire process is repeated the next sample time. The Q matrix can be used to emphasize certain outputs. Here, the L10 float velocity, the first of the two states of (6), is desired to be regulated, and all samples are weighted equally. Therefore Q is a block diagonal matrix, in which the first state is emphasized as.

$$Q = \begin{bmatrix} Q_{10} & 0 & \dots & 0 \\ 0 & Q_{10} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Q_{10} \end{bmatrix} \quad Q_{10} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (28)$$

The R matrix is used to penalize the rate of change of control action. In this research the rate of change of generator force is electromagnetic in nature and is not directly penalized in the cost function. It is however accounted for in the cost function solution constraints, covered in the next section.

$$R = 0 \quad (29)$$

3. NMPC Formulation

The need for the NMPC arises, since there is systems which are inherently nonlinear which cannot be described adequately by a linear model. Here, the control and process requirements have increased tremendously. For these cases, the use of linear MPC is inadequate and the use of NMPC is a promising alternative to deal with inherently nonlinear systems. The discretization of the nonlinear system and the implementation of the NMPC for a point absorber wave energy converter are done here. By means of simulation results of controlling the nonlinear model, the proposed NMPC is validated and compared to the linear MPC. The Equations of Motions for the bodies are given by the equations (31) and (32).

$$F_{PTO} + F_{e1} - F_{r1} - F_{h1} - F_{r12} = m_1 \ddot{z} \quad (31)$$

$$-F_{PTO} + F_{e2} - F_{r2} - F_{r21} - F_m = m_1 \ddot{x} \quad (32)$$

where F_r denotes the radiation forces, F_h denotes the hydrodynamic force, F_m denotes the mooring force and F_{PTO} denotes the generator force. The following forces are represented by:

$$F_{r1} = -A_1 \ddot{z}(t) + b_1 \dot{z}(t) + \int_{-\infty}^{t} f_{r1}(t-\tau)\, \dot{z}(\tau) \mathrm{d} \quad (33)$$

$$F_{h1} = -g\rho\pi r_{bouy}^2 z(t) = k_1 z(t) \quad (34)$$

$$F_{r12} = A_{12}\ddot{x}(t) + \int_{-\infty}^{t} f_{r12}(t-\tau)\dot{x}(t) d\tau \quad (35)$$

$$F_{r2} = A_2 \ddot{x}(t) + b_2 \dot{x}(t) + \int_{-\infty}^{t} f_{r2}(t-\tau) x(\dot{\tau})\, \mathrm{d}\tau \quad (36)$$

$$F_{r21} = A_{21}\ddot{z}(t) + \int_{-\infty}^{t} f_{r21}(t-\tau)\dot{z}(\tau) d\tau \quad (37)$$

$$F_m = 8K_{n1} x(t) \left(1 - \frac{L_{n1}}{\sqrt{L^2{}_{n1} + x(t)^2}}\right) \quad (38)$$

The impulse response functions for the different radiation forces are denoted by F_r.

A. Optimization Formulation

The system is described as a discrete- time nonlinear state space model in the form

$$x_{k+1} = f_k(x_k, u_k, s_k) \quad (39)$$

where, f_k: maps the current state x_k , u_k controllable input, s_k uncontrollable input

The optimization problem can be defined as

$$min_{x_k, u_k} J(x_k, u_k) \quad (40)$$

$$J(x_k, u_k) = \frac{1}{2}\sum_{k=1}^{N} [q(x_{k(2)} - x_{k,(4)})u_{k-1} + r{u_{k-1}}^2] \quad (41)$$

Subject to

$$x_{k+1} = f_k(x_k, u_k)$$

$$|x_{k(1)} - x_{k(3)}| \le c_p , \quad k = 1 \dots N \quad (42)$$

$$|x_{k(2)} - x_{k(4)}| \leq c_v, \quad k=1.....N \tag{43}$$

$$|u_k| \leq c_u, \quad k=0....N-1 \tag{44}$$

4. Simulation results

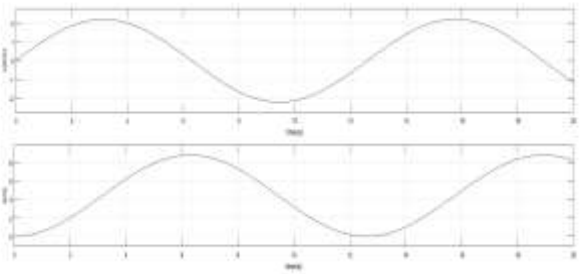

Figure 3. Simulation results of position and velocity during sinusodial wave as input signal.

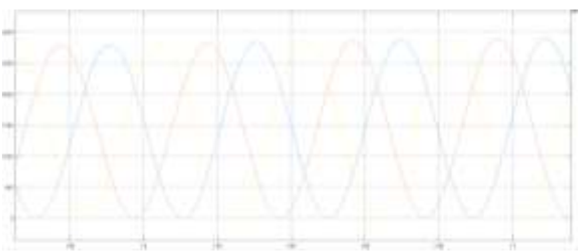

Figure 4. Simulation result of power with linear MPC.

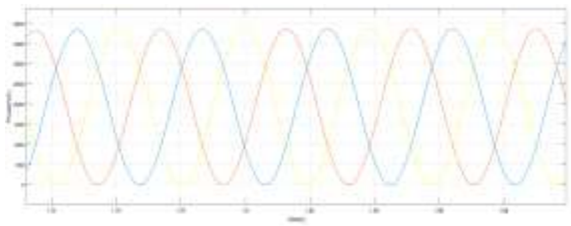

Figure 5. Simulation result of power with NMPC

The formulation of Model predictive control and nonlinear model predictive control of point absorber wave energy converter is done. The prediction algorithm is used to implement an MPC that tracks the optimal velocity trajectory while respecting system velocity, position and generator limits is done and the simulation result obtained is shown. The optimal trajectory yields good results and power maximization is also done. The average power obtained for WEC by Model predictive control is 2.7 KW. In order to deal with the nonlinear effect which occurs in the WEC, such as mooring forces an NMPC approach is used here. The average power obtained from NMPC is 3.8KW.It is shown that the NMPC is able to maximize the power generation while considering the WEC position and constraints.

5. CONCLUSIONS

A point absorber wave energy converter is controlled by Model predictive control and nonlinear model predictive control. The goal was to maximize the generated power while satisfying generator force as well as position and velocity constraint of the buoy. The state space model and nonlinear model for the point absorber has been derived. The implementation of MPC algorithm has shown that it can track optimal velocity trajectory while respecting velocity and position limits and generator force limits while maximizing the power. The MPC formulations respect the constraints as desired and that the power formulation yields almost the same results as the trajectory formulation for the monochromatic wave data. The better mooring of the spar leads to a higher energy generation hence accounting the highly nonlinear mooring force and radiation force nonlinearities an NMPC control approach is proposed which yields better control action. In order to evaluate the performance, the results were compared with those of MPC. It has been demonstrated that the NMPC is able to keep the point absorber states within limits while maximizing the power generation.

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