ENTIRE DOMINATION IN JUMP GRAPHS

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ABSTRACT:

The vertices and edges of a graph J(G) are called the element of J(G). A set X of elements in J(G) is an entire dominating set if every element not inX ix either adjacent or incident to at least one element in X The entire domination number $\mathfrak{E}$(J(G)) is the order of a smallest entire dominating set in J(G) In this paper exact values of $\mathfrak{E}$(J(G)) for some standard graphs are obtained Also, bounds on $\mathfrak{E}(J(G))$ and Nordhaus- Gaddam type results are established.

INTRODUCTION;

The graph considered here are finite, connected, undirected without loops or multiple edges. We denote by √(J(G)) and $\mathfrak{E}(J(G))$ the vertex set and the edge set of J(G) respectively. For any undefined term or notation in this paper see Harary[3]. The study of dominating sets in graph was begun by Ore[7] and Berge[5]. The entire domination number was defined by Kulli[4].

The open neighborhood N(v) (N(e)) of a vertex (an edge e) is the set of vertices (edges) adjacent to v(e). The closed neighborhood N[v] 9 N[e]) of a vertex 9an edge e0 is N(v) ∪ {v} (N(e) ∪ {e}). The open entire neighborhood n(x) of an edge x is the set of elements either adjacent or incident to x. the closed entire neighborhood n[x] of an element x is n(x) ∪ {x}. Δ (J(G)) denoes the maximum degree of J(G). The degree of an edge e=uv is defined as deg u +deg v -2.The maximum edge degree of J(G) is denoted by Δ'(J(G)), we will employ the following notation ┌ x ┐ (└ x ┘) to denote the smallest (largedst) integer greater(lesser) than equal tox

A set D of vertices in J(G) is a dominating set if every vertex not in D is adjacent to atleast one vertexin V(J(G)) – D. The domination number √(J(G)) is the order of a smallest dominating set in J(G).

A set F of edges of J(G) is an edge dominating set if every edge not in F is adjacent to at least one edge in E(J(G)) – F. The edge domination number √'(J(G)) of J(G) is the smallest edge dominating set in J(G)..

We now obtained a relation between the domination, edge domination and entire domination number of a graph.

Theorem 1; For any graph J(G) (√(J(G)) + √'(J(G))) /2 ≤ $\mathfrak{E}$ (J(G)) ≤ √(J(G)) + √'(J(G)).

Further the upper bound attains if there exists a minimum entire dominating set X= D ∪ f satisfying.

i) N[D} = V(J(G)), N[F] = E(J(G)) and ∩ N[v] = ∩ N[e] = ∅
ii) Deg v = Δ(J(G)), deg e = Δ' (J(G)) for all in D and e in F.

Proof; First we establish the lower bound . Let X =D ∪ F be a minimum entire dominating set of J(G). for each edge e=uv in F Choose a vertex u or v, not both and F' be the collection of such vertices Clearly D ∪ F' is a dominating set ,

There fore

√(J(G)) ≤ | E ∪ F' |

= | D ∪ F|

= $\mathfrak{E}$(J(G))……(1)

Now for each vertex u in D choose exactly one edge e incident with u and let D' be the collection of such edges. Clearly D' ∪ F is an edge dominating set. Therefore

$\surd'(J(G)) \leq | D' \cup F |$

$= |D \cup F |$

$= \mathfrak{E}(J(G))$ ……..(2)

From (1) and (2) follows

$\surd(J(G)) + \surd'(J(G)) \leq 2\, \mathfrak{E}(J(G))$.

Therefore

$\surd(J(G)) + \surd'(J(G)) /2 = \mathfrak{E}(J(G))$

Now for the upper bound, let D and f be the minimum dominating and edge dominating sets respectively.

Then D∪ F is an entire dominating set. Thus

$\mathfrak{E}(J(G)) \leq | D \cup F |$

$= \surd(J(G)) + \surd'(J(G))$.

Theorem 2; For any connected jump graph J(G).

$P - q \leq \mathfrak{E}(J(G)) \leq p - \lceil \frac{\Delta(n)}{2} \rceil$

For the lower bound is attained if and only if J(G) is a star.

Proof; First we establish the upper bound. Let v be a vertex of degree Δ(J(G)).Let F be the set of independent edges in <N(v)>. Then V(J(G))∪ F – N(v) is an entire dominating set. Also $|F| \leq \lfloor \frac{\Delta(n)}{2} \rfloor$ Therefore

$\mathfrak{E}(J(G)) \leq | V(J(G)) \cup F - N(v) |$

$\leq p + \lfloor \frac{\Delta(n)}{2} \rfloor - \Delta(J(G))$

$\leq p - \lceil \frac{\Delta(n)}{2} \rceil$

Now for the lower bound, let X be a minimum entire dominating set of J(G). Then

$P + q - |X| = | V(J(G)) \cup E(J(G)) - X|$

$\leq | V(J(G)) \cup E(J(G)) | - 1$

$\leq p + q-(p-q)$

$\leq 2q$.

Then $\mathfrak{E}(J(G)) \geq p-q$.

Suppose $\mathfrak{E}(J(G)) = p-q$ Then $p-q \geq 1$ and from the above inequalities it follows that $p-q=1$ This shows that J(G) is a star.

Conversely, suppose J(G) is a star obliviously $\mathfrak{E}(J(G)) = p-q$.

Theorem 3; For any jump graph J(G)

$\mathfrak{E}(J(G)) \geq \frac{(p+q)}{(2\Delta(J(G)+1)}$

Further equality holds if there excists a minimum entire dominating set X such that.

i) X is an entire independent set
ii) For any elemrnt x in (V ∪ E)_X there is anelement y in X such that
n(x) ∩ X ={y}
iii) |n(x)| = 2, Δ(J(G)) for every x in X

Proof; This follows from Theorem A and the notation of totalgraph if there exists a minimum entire dominating set satisfying (i) (ii) and (iii) the bound is attained.

Theorem 4; For any connected J(G) of order p

$$\mathfrak{E}(J(G)) \leq \lceil \frac{p}{2} \rceil$$

Proof; We prove the result by induction on p if p≤ 4 then the result can be verified. Assume the result is true for all connected graphs J(G) and p-2 vertices. Let J(G) be a connected graph then p vertices. Let u and v denote either two adjacent vertices or two non adjacent vertices having a common neighbor w such that J(G)= J(G') – {u v} is connected. Let X be the minimum entire dominating set of J(G). Then either X∪ {w] or X ∪ {u v} is an entire dominating set of J(G'). Then,

$$\mathfrak{E}(J(G')) \leq |X| + 1$$

$$\leq \lceil \frac{p-2}{2} \rceil + 1$$

$$= \lceil \frac{p}{2} \rceil$$

Finally we establish Nordhaus-Gaddum type results.

Theorem 5 ; For any connected graph J(G) with p vertices

$$\mathfrak{E}(J(G)) + \mathfrak{E}(J(\bar{G})) \leq \lceil \frac{3p}{2} \rceil$$

$$\mathfrak{E}(J(G)) + \mathfrak{E}(J(\bar{G})) \leq p \lceil \frac{p}{2} \rceil$$

Proof; J(G) is complete, then J ($\bar{G}$) is totally disconnected $\mathfrak{E}(J(\bar{G})) = p$

There fore

$$\mathfrak{E}(J(G)) + \mathfrak{E}(J(\bar{G})) = \lceil \frac{p}{2} \rceil + p$$

$$= \lceil \frac{3p}{2} \rceil$$

And $\mathfrak{E}(J(G)) . \mathfrak{E}(J(\bar{G})) = p \lceil \frac{p}{2} \rceil$

Theorem6; Let J(G) and J($\bar{G}$) be connected complete graph then,

$$\mathfrak{E}(J(G)) + \mathfrak{E}(J(\bar{G})) \leq p + 1$$

$$\mathfrak{E}(J(G)) . \mathfrak{E}(J(\bar{G})) \leq (p+1)^2 / 4$$

Proof; This follows from Theorem 4.

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