A Mathematical Programming Approach to Cellular Manufacturing System Design

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Abstract – *Cellular manufacturing is an application of group technology, in this the manufacturing industry has been converted to manufacturing cells. A manufacturing cell is a group of dissimilar machines of closeness and dedicated to manufacture a family of parts. In this paper a mathematical programming model is proposed to design cellular manufacturing to minimize inter-cell material handling by considering the demand and machine capacity. The developed model is solving using optimization software.*

Key Words: Cellular manufacturing, machine capacity.

1. INTRODUCTION

In cellular manufacturing system, parts are grouped together according to the similarities in design or processing requirements. These groups of parts are called part families. There formed machine cells in manufacturing systems, in each machine cell, there are dissimilar machines dedicated to manufacture one or more part families. The benefits of CMS is to reduce the throughput time and setup times, material handling, reduced inventory, maximizing flexibility, improves productivity and quality, and improved human relations.

Cellular manufacturing can also provide companies with the flexibility to vary product type or features on the production line in response to specific customer demands. The approach seeks to minimize the time it takes for a single product to flow through the entire production process. The cell design is a very complex exercise. Cell design is the identification of a set of part types that are suitable for manufacture on a group of machines. However, there are a number of other strategic level issues such as level of machine flexibility, cell layout, type of material handling equipment, types and number of tools and fixture that should be considered as a part of cell design problem.

2. LITERATURE REVIEW

Cellular manufacturing system design is a complex and there are many studies are done in CMS. The model incorporated probabilistic demand in designing the CMS. In this approach, probabilities are assigned to each product mix and their related part-machine matrix. Then the best cell configuration is determined for each product mix being considered. Subsequently, the expected inter-cell material handling cost will be calculated for possible product mixes. At the end, the cell configuration with the least expected inter-cell material handling cost is chosen as the preferred CMS [1]. Later a simulation study was conducted on a CMS where the part mixes change. This simulation aimed at analyzing the sensitivity of the CMS to the changes in part mixes. This sensitivity analysis can help the decision maker in predicting the performance of a CMS under uncertainty [2].

An efficient mathematical programming formulation is proposed to form the machine cells and minimize the cost of eliminating the exceptional elements simultaneously. A fuzzy linear programming model was then developed to deal with the fuzziness embodied in the CF problem [3]. Discuss the methods of cell formation in group technology. A mathematical formulation is derived to form the cell by considering the work load and tooling other than grouping parts and machines in to cells [4].

In CMS designing, cell formation is the first stage of designing an effective CM system. A multi-functional MP model is used in this study [5]. A Mathematical programming model is used for the cell formation, where considering the demands, upper and lower limits, sequence of operations, machine capacity, or machine cost, and machine replication. The model was solved by means of LINGO optimization software. In order to evaluate the performance of the proposed model, it was compared with some existing well-known models [6].

3. METHODOLOGY

In this, first developed a mathematical model and then solves the model using optimization software, AMPLDev 3.0 with a numerical data. The aim of the proposed model is to minimize total cost of CMS including the machine cost, material handling cost, inventory holding cost.

3.1 Mathematical Formulation

3.1.1 Indexing sets

p: Part type index: p = 1……P.

m: Machine type index: m = 1….M.

k: index of cell: k = 1….C.

3.1.2 Parameters

ω_{pm} - 1 if part *p* needs machine type *m*; = 0 otherwise

ℓM_k - Lower bound of the number of machine types in cell *k*

νM_k - Upper bound of the number of machine types in cell *k*

ℓP_{ik} - Lower bound of the number of parts assigned in cell *k*

Δ_m - Available time for machine *m*

T_{pm} - Processing time o part *p* on machine type *m*

D - Demand of part *p*

φ_p^{inter} - Unit material handling cost between cells

α_p - Unit holding cost of part type *p*

β_m - Maintenance and overhead cost of machine type *m*

Ω - Number of products per batch

A - An arbitrary big positive number

n - Number of machine type

3.1.3 Decision variables

N_{pk} - 1 if part *p* is processed in cell *k;* = 0 otherwise

X_{pmk} - 1 if part *p* is processed on machine type *m* in cell *k*;

= 0 otherwise

γ_{mk} - 1 if machines type *m* is assigned in cell *k*

Q_p - Number of parts *p* produced.

K_p - 1 if part *p* is produced; = 0 otherwise

H_p - Inventory of part *p* at the end of the planning horizon

3.1.4 Objective function

$$\text{Minimize } Z = \sum_{p=1}^{P} \alpha_p H_p + \sum_{k_i=1}^{C}\sum_{m=1}^{M} \beta_m \gamma_{mk} + \sum_{p=1}^{P}\left[\left(\sum_{k=1}^{C} N_{pk_i}.K_p\right) - K_p\right]\varphi_p^{inter}$$

3.1.5 Constraints

$$\sum_{p=1}^{P} X_{pmk} \cdot T_{pm} \cdot Q_p \le \gamma_{mk}\Delta_m \qquad \forall m,k \qquad \text{......} (1)$$

$$\sum_{m=1}^{M} X_{pmk} \le A * N_{pk} \qquad \forall\ p,k \qquad \text{......} (2)$$

$$\sum_{m=1}^{M} X_{pmk} \ge N_{pk} \qquad \forall\ p,k \qquad \text{......} (3)$$

$$\sum_{p=1}^{P} N_{pk} \ge \ell P_k \qquad \forall\ k \qquad \text{......} (4)$$

$$\sum_{k=1}^{C} N_{pk} = 1 \qquad \forall\ p \qquad \text{......} (5)$$

$$\sum_{k=1}^{C} X_{pmk} = K_p \omega_{pm} \qquad \forall\ p,m \qquad \text{......} (6)$$

$$\sum_{m=1}^{M} \gamma_{mk} \ge \ell M_k \qquad \forall\ k \qquad \text{......} (7)$$

$$\sum_{m=1}^{M} \gamma_{mk} \le \nu M_k \qquad \forall\ k \qquad \text{......} (8)$$

$$\sum_{k=1}^{C} \gamma_{mik} \le 1 \qquad \forall m \qquad \text{......} (9)$$

$$\sum_{k=1}^{C}\sum_{m=1}^{M} \gamma_{mk} = n \qquad \text{......} (10)$$

$$K_p, N_{pk}, X_{pmk}, \gamma_{mk} \in \{0,1\}. \qquad \forall\ \ p,m,k \text{ ......} (11)$$

$$Q_p, H_p \ge 0 \text{ and integer.} \qquad \forall\ k \qquad \text{......} (12)$$

The objective function includes three costs such as the holding cost which is the cost associated with inventory, machine cost is the maintenance and overhead cost of machines and inter-cell material handling cost.

The equations (1) to (12) are the constraints of the model. Constraints (1) enforce that the assigned time-capacity of machine does not exceed the available capacity. Constraint (2) & (3) to ensure that part is processed within the cell. Constraints (4) specify the lower bound for the number of parts assigned to each cell. Constraint (5) ensures that each part is assigned only to a single cell. Equation (6) implies that only one part is processed in a machine at a time. Constraint (7) & (8) defines the upper and lower bound for the number of machines assigned to each cell. Constraint (9) restricts the multiple allocation of machine. Equation (10) implies that cell must be utilizing all kinds of machines. Constraint (11) and (12) gives the logical binary, non-negativity integer and continuities necessity for decision variables.

3.2 Illustrative Example

To verify the performance of the proposed model, consider a hypothetical data of a manufacturing industry, which consists of five machines and eight parts and can be formed into 2 cells. The minimum and maximum cell sizes for each cell in each plant are 2 and 4, respectively. Table 1 lists the machine information and table 2 lists the processing times of each part, part-machine incidence matrix, part demand, holding cost and material handling cost. The proposed model is solved by using the software AMPLDev 3.0.

Solve the model for the numerical values given in the following tables, Table – 1 &2.

Table -1: Machine Information

	Machine 1	Machine 2	Machine 3	Machine 4	Machine 5
β_m	11	13	15	17	14
Δ_m	500	500	500	500	500

Table -2: Part Information

	Machine	Part							
		1	2	3	4	5	6	7	8
T_{pm}	1	0.38	0.34	0.46	0	0	0	0	0.37
	2	0.39	0.38	0	0.29	0	0.50	0	0.19
	3	0.44	0	0.29	0.52	0.38	0	0.27	0
	4	0	0.50	0	0	0.14	0.36	0	0
	5	0	0	0.49	0.25	0	0.54	0.19	0
ω_{pm}	1	1	1	1	0	0	0	0	1
	2	1	1	0	1	0	1	0	1
	3	1	0	1	1	1	0	1	0
	4	0	1	0	0	1	1	0	0
	5	0	0	1	1	0	1	1	0
φ_{pi}^{inter}		8	7	9	6	3	8	5	10
α_{pi}		8	13	18	10	12	17	9	10
D_{pj}		110	250	200	160	280	120	100	150

4. COMPUTATIONAL RESULTS

The proposed model is solve using the optimization software and following results is obtained. The given table shows that the cell formed in the cellular manufacturing systems with a minimization of holding cost, machine cost and inter-cell material handling cost. There are two cells formed and cell # 1 includes the part types 3, 5, 6, 7, 8 and machine types 1, 2, 5. The part type 1, 2, 4 and remaining machines 3, 4 are assigned in cell # 2.

Table -3: Results of cell formation

Cell	Part assigned	Machine assigned
1	3, 5, 6, 7, 8	1, 2, 5
2	1, 2, 4,	3, 4

5. CONCLUSIONS

The developed model tries to form cellular design which is to minimize the material handling cost. This mathematical model is form the cell to optimize the result.

This model is extended to worker assignment, part routing etc.

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