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BIPOLAR INTERVAL VALUED MULTI FUZZY GENERALIZED SEMIPRE CONTINUOUS MAPPINGS

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ABSTRACT: *In this paper, bipolar interval valued multi fuzzy generalized semi-precontinuous mappings is defined and introduced. Using this definitions, some theorems are introduced.*

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KEYWORDS: Bipolar interval valued multi fuzzy subset, bipolar interval valued multi fuzzy topological space, bipolar interval valued multi fuzzy interior, bipolar interval valued multi fuzzy closure, bipolar interval valued multi fuzzy continuous mapping, bipolar interval valued multi fuzzy generalized semi-preclosed set, bipolar interval valued multi fuzzy generalized semi-preopen set, bipolar interval valued multi fuzzy generalized semi-preclosed mapping, bipolar interval valued multi fuzzy generalized semi-preopen mapping.

1. INTRODUCTION: The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [19] in the year 1965, the subsequent research actvities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper C.L.Chang [4] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces many researchers like, and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] has introduced semipreclosed sets and Dontchev [5] has introduced generalized semipreclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by saraf and khanna [15]. Tapas kumar mondal and S.K.Samantha [12] have introduced the topology of interval valued fuzzy sets. Jeyabalan.R and Arjunan [8, 9] have introduced interval valued fuzzy generalized semi pre continuous mapping. After that interval valued fuzzy generalized semi pre continuous mapping has been generalized into interval valued intuitionistic fuzzy generalized semi pre continuous mapping by S.Vinoth and K.Arjunan[17, 18]. The interval valued fuzzy set has been extended into the bipolar interval valued multi fuzzy topological spaces. R.Selvam et.al [16] have defined and introduced the bipolar interval valued multi fuzzy generalized semipreclosed sets. In this paper, we introduce bipolar interval valued multi fuzzy generalized semi-precontinuous mappings and some properties are investigated.

2. PRELIMINARIES:

Definition 2.1[19]. Let X be a non-empty set. A **fuzzy subset A** of X is a function $A : X \to [0, 1]$.

Definition 2.2[14]. A **multi fuzzy subset** A of a set X is defined as an object of the form $A = \{ \langle x, A_1(x), A_2(x), A_3(x), \ldots, A_n(x) \rangle / x \in X\}$, where $A_i : X \to [0, 1]$ for all i and i = 1, 2, …, n.

Definition 2.3[19]. Let X be any nonempty set. A mapping $A : X \to D[0, 1]$ is called an interval valued fuzzy subset (briefly, IVFS) of X, where $D[0,1]$ denotes the family of all closed subintervals of [0, 1].

Definition 2.4[19]. A interval valued multi fuzzy subset A of a set X with degree n is defined as an object of the form $A = \{ \langle x, A_1(x), A_2(x), A_3(x), \ldots, A_n(x) \rangle / x \in X\}$, where $A_i : X \to D[0, 1]$ for all i and i = 1, 2, …, n.

Definition 2.5[10]. A bipolar valued fuzzy set A in X is defined as an object of the form $A = \{\langle x, M(x), N(x) \rangle / x \in X\}$, where $M : X \to [0, 1]$ and $N : X \to [-1, 0]$. The positive membership degree M(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree N(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 2.6. $A = \{ \langle a, 0.7, -0.5 \rangle, \langle b, 0.3, -0.8 \rangle, \langle c, 0.2, -0.4 \rangle \}$ is a bipolar valued fuzzy subset of $X = \{ a, b, c \}$.

Definition 2.7[10]. A bipolar valued multi fuzzy set A in X with degree n is defined as an object of the form A = {⟨x, $M_1(x)$, $M_2(x)$, $M_3(x)$, …, $M_n(x)$, $N_1(x)$, $N_2(x)$, $N_3(x)$, …, $N_n(x)$ ⟩ / x∈X}, where $M_i : X\to [0, 1]$ and $N_i : X\to [-1, 0]$ for all i and i = 1, 2, …, n.. The positive membership degrees $M_i(x)$ denotes the satisfaction degrees of an element x to the property corresponding to a bipolar valued multi fuzzy set A and the negative membership degrees $N_i(x)$ denotes the satisfaction degrees of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set A.

Example 2.8. A = { ⟨ *k*, 0.5, 0.4, 0.7, −0.2, −0.5, −0.8 ⟩, ⟨ *l*, 0.3, 0.7, 0.3, −0.3, −0.4, −0.6 ⟩, ⟨ *m*, 0.5, 0.8, 0.4, −0.5, −0.2, −0.9 ⟩ } is a bipolar valued multi fuzzy subset of X = { *k, l, m* } with degree 3.

Definition 2.9[16]. A bipolar interval valued fuzzy set A in X is defined as an object of the form A = {⟨ x, M(x), N(x) ⟩/ x∈X}, where $M : X\to D[0, 1]$ and $N : X\to D[-1, 0]$. The positive membership interval degree M(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar interval valued fuzzy set A and the negative membership interval degree N(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar interval valued fuzzy set A.

Example 2.10. A = { ⟨ *k*, [0.3, 0.9], [−0.5, −0.4] ⟩, ⟨ *l*, [0.2,0.9], [−0.9, −0.5] ⟩, ⟨ *m*, [0.5, 0.8], [−0.8, −0.6] ⟩} is a bipolar interval valued fuzzy subset of X ={ a, b, c }.

Definition 2.11[16]. A bipolar interval valued multi fuzzy set A in X with degree n is defined as an object of the form A = {⟨ x, $M_1(x)$, $M_2(x)$, $M_3(x)$, …, $M_n(x)$, $N_1(x)$, $N_2(x)$, $N_3(x)$, …, $N_n(x)$ ⟩ / x∈X}, where $M_i : X\to D[0, 1]$ and $N_i : X\to D[-1, 0]$ for all i and i = 1, 2, …, n.. The positive membership degrees $M_i(x)$ denotes the satisfaction degrees of an element x to the property corresponding to a bipolar interval valued multi fuzzy set A and the negative membership degrees $N_i(x)$ denotes the satisfaction degrees of an element x to some implicit counter-property corresponding to a bipolar interval valued multi fuzzy set A.

Example 2.12. A = { ⟨ *k*, [0.3, 0.7], [0.2, 0.6], [0.5, 0.9], [−0.3, −0.2], [−0.6, −0.3], [−0.8, −0.3] ⟩, ⟨ *b*, [0.6, 0.9], [0.1, 0.9], [0.5, 0.5], [−0.3, −0.2], [−0.5, −0.3], [−0.6, −0.4] ⟩, ⟨ *m*, [0.5, 0.8], [0.3, 0.6], [0.4, 0.9], [−0.5, −0.2], [−0.8, −0.5], [−0.9, −0.7] ⟩ } is a bipolar interval valued multi fuzzy subset of X = { a, b, c } with degree 3.

Definition 2.13[16]. Let A = ⟨ M_i, N_i ⟩ and B = ⟨ O_i, P_i ⟩ be any two bipolar interval valued multi fuzzy subsets of a set X with degree n. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $M_i(x) \le O_i(x)$ and $N_i(x) \ge P_i(x)$ for all x in X and for all i.

(ii)A =B if and only if $M_i(x) = O_i(x)$ and $N_i(x) = P_i(x)$ for all x in X and for all i.

(iii) $(A)^c$ = { ⟨ x, $(M_i)^c(x)$, $(N_i)^c(x)$ ⟩ / x∈X }.

(iv) $A \cap B$ = { ⟨ x, rmin{ $M_i(x)$, $O_i(x)$ }, rmax{ $N_i(x)$, $P_i(x)$ } ⟩ / x∈X }.

(v) $A\cup B$ = { ⟨ x, rmax { $M_i(x)$, $O_i(x)$}, rmin{ $N_i(x)$, $P_i(x)$ }⟩ / x∈X }.

Remark 2.14. 0_S = { ⟨ x, [0, 0], [0, 0], …[0, 0] ⟩ : x∈X} and $\overline{1}$ = { ⟨ x, [1, 1], [1, 1],..,[1, 1], [−1, −1], [−1, −1],…, [−1, −1] ⟩ : $x\in X$ }.

Definition 2.15[16]. Let S be a set and ϑ be a family of bipolar interval valued multi fuzzy subsets of S. The family ϑ is called a bipolar interval valued multi fuzzy topology (BIVMFT) on S if ϑ satisfies the following axioms

(i) $0_S, 1_S \in \vartheta$ (ii) If { A_i ; i∈I } ⊆ ϑ, then $\bigcup_{i=1}^{\infty} A_i \in \vartheta$

(iii) If $A_1, A_2, A_3, \ldots A_n \in \vartheta$, then $\bigcap_{i=1}^{n} A_i \in \vartheta$.

The pair (S, ϑ) is called a bipolar interval valued multi fuzzy topological space (BIVMFTS). The members of ϑ are called bipolar interval valued multi fuzzy open sets (BIVMFOS) in S. An bipolar interval valued multi fuzzy subset A in S is said to be bipolar interval valued multi fuzzy closed set (BIVMFCS) in S if and only if $(A)^c$ is a BIVMFOS in S.

Definition 2.16[16]. Let (S, ϑ) be a BIVMFTS and A be a BIVMFS in S. Then the bipolar interval valued multi fuzzy interior and bipolar interval valued multi fuzzy closure are defined by $bivmfint(A) = \cup \{H : H$ is a BIVMFOS in X and $H \subseteq A\}$, $bivmfcl(A) = \cap\{$ K: K is a BIVMFCS in S and $A \subseteq K\}$. For any BIVMFS A in (S, ϑ), we have $bivmfcl(A^c) = (bivmfint(A))^c$ and $bivmfint(A^c) = (bivmfcl(A))^c$.

Definition 2.17[16]. A BIVMFS A of a BIVMFTS (S, ϑ) is said to be a

(i) bipolar interval valued multi fuzzy regular closed set (BIVMFRCS for short) if

$A = bivmfcl(bivmfint(A))$

(ii) bipolar interval valued multi fuzzy semiclosed set (BIVMFSCS for short) if $bivmfint(bivmfcl(A)) \subseteq A$

(iii) bipolar interval valued multi fuzzy preclosed set (BIVMFPCS for short) if

$bivmfcl(bivmfint(A)) \subseteq A$

(iv) bipolar interval valued multi fuzzy α closed set (BIVMF**α**CS for short) if $bivmfcl\left(bivmfint(bivmfcl(A))\right) \subseteq A$

(v) bipolar interval valued multi fuzzy β closed set (BIVMF**β**CS for short) if $bivmfint\left(bivmfcl(bivmfint(A))\right) \subseteq A$.

Definition 2.18[16]. A BIVMFS A of a BIVMFTS (S, ϑ) is said to be a

(i) bipolar interval valued multi fuzzy generalized closed set (BIVMFGCS for short) if bivmfcl(A) ⊆ U, whenever A⊆ U and U is a BIVMFOS

(ii) bipolar interval valued multi fuzzy regular generalized closed set (BIVMFRGCS for short) if $bivmfcl(A) \subseteq U$, whenever A⊆ U and U is a BIVMFROS.

Definition 2.19[16]. A BIVMFS A of a BIVMFTS (S, ϑ) is said to be a

(i) bipolar interval valued multi fuzzy semipreclosed set (BIVMFSPCS for short) if there exists a BIVMFPCS B such that $bivmfint(B) \subseteq A \subseteq B$

(ii) bipolar interval valued multi fuzzy semipreopen set (BIVMFSPOS for short) if there exists a BIVMFPOS B such that $B \subseteq A \subseteq bivmfcl(B)$.

Definition 2.20[16]. Two BIVMFSs A and B are said to be not q-coincident if and

only if $A \subseteq B^c$.

Definition 2.21[16]. Let B be a BIVMFS in a BIVMFTS (S, ϑ). Then the bipolar interval valued multi fuzzy semipre interior of A ($bivmfspint$(B) for short) and the bipolar interval valued multi fuzzy semipre closure of A ($bivmfspcl(B)$ for short) are defined by $bivmfspint(B) = \cup\{H : H$ is a BIVMFSPOS in S and $H \subseteq B\}$, $bivmfspcl(B) = \cap\{K : K$ is a BIVMFSPCS in S and $B \subseteq K\}$. For any BIVMFS B in (S, ϑ), we have $bivmfspcl(B^c) = (bivmfspint(B))^c$ and $bivmfspint(B^c) = (bivmfpcl(B))^c$.

Definition 2.22[16]. A BIVMFS B in BIVMFTS (S,ϑ) is said to be a bipolar interval valued multi fuzzy generalized semipreclosed set (BIVMFGSPCS for short) if $bivmfspcl(B) \subseteq U$ whenever $B \subseteq U$ and U is a BIVMFOS in (S,ϑ).

Example 2.23. Let $S = \{k, l\}$ and ϑ = {0, H ,1 } is a BIVMFT on S, where H = { ⟨ k, [0.5, 0.5], [0.6, 0.6], [0.4, 0.4], [−0.4, −0.4], [−0.5, −0.5], [−0.3, −0.3] ⟩, ⟨ l, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [−0.3, −0.3], [−0.4, −0.4], [−0.2, −0.2] ⟩}. And the BIVMFS B = { ⟨ k, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [−0.3, −0.3], [−0.4, −0.4], [−0.2, −0.2] ⟩, ⟨ l, [0.2, 0.2], [0.3, 0.3], [0.1, 0.1], [−0.1, −0.1], [−0.2, −0.2], [−0.05, −0.05] ⟩ } is a BIVMFGSPCS in (S,ϑ).

Definition 2.24[16]. The complement B^c of a BIVMFGSPCS B in a BIVMFTS (S, ϑ) is called a bipolar interval valued multi fuzzy generalized semi-preopen set (BIVMFGSPOS) in S.

Definition 2.25[16]. A BIVMFTS (S, ϑ) is called a bipolar interval valued multi fuzzy semi-pre $T_{1/2}$ space (BIVMFSP$T_{1/2}$), if every BIVMFGSPCS is a BIVMFSPCS in S.

Definition 2.26. Let (S, ϑ) and (T, ψ) be BIVMFTSs. Then a map h: S→T is called a (i) bipolar interval valued multi fuzzy continuous (BIVMF continuous) mapping if $h^{-1}(B)$ is BIVMFOS in S for all BIVMFOS B in T.

(ii) a bipolar interval valued multi fuzzy closed mapping (BIVMFC mapping) if h(B) is a BIVMFCS in T for each BIVMFCS B in T.

(iii) bipolar interval valued multi fuzzy semi-closed mapping (BIVMFSC mapping) if $h(B)$ is a BIVMFSCS in T for each BIVMFCS B in S.

(iv) bipolar interval valued multi fuzzy preclosed mapping (BIVMFPC mapping) if $h(B)$ is a BIVMFPCS in T for each BIVMFCS B in S.

(v) bipolar interval valued multi fuzzy semi-open mapping (BIVMFSO mapping) if $h(B)$ is a BIVMFSOS in T for each BIVMFOS B in S.

(vi) bipolar interval valued multi fuzzy generalized semi-preopen mapping (BIVMFGSPO mapping) if $h(B)$ is a BIVMFGSPOS in T for each BIVMFOS B in S.

(vii) bipolar interval valued multi fuzzy generalized semi-preclosed mapping (BIVMFGSPC mapping) if $h(B)$ is a BIVMFGSPCS in T for each BIVMFCS B in S.

Theorem 2.27. For any BIVMFS B in (S, ϑ) where S is a BIVMFSP$T_{1/2}$ space, B∈BIVMFGSPO(S) if and only if for every BIVMFP $p_{(\alpha,\beta)}$∈B, there exists a BIVMFGSPOS C in S such that $p_{(\alpha,\beta)} \in C \subseteq B$.

Definition 2.28[14]. Let $\alpha \in D[0,1]$ and $\beta \in D[-1,0]$, where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$. A bipolar interval valued multi fuzzy point (BIVMFP), written as $P_{(\alpha,\beta)}$ is defined to be a BIVMFS of X is given by

$$P_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & \text{if} \quad x = p \\ (0,0) & \text{otherwise.} \end{cases}$$

3. SOME PROPERTIES:

Based on the paper Selvam.R et al.[16], the following Theorems are stated and proved.

Definition 3.1. A mapping j: (S, ϑ) → (T, ψ) is called an bipolar interval valued multi fuzzy generalized semiprecontinuous (BIVMFGSP continuous for short) mapping if $j^{-1}(N)$ is a BIVMFGSPCS in (S, ϑ) for every BIVMFCS N of (T, ψ).

Example 3.2. Let S = { *k, l* }, T = { *m, n* } and H_1 = { ⟨ *k*, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [−0.5, −0.5], [−0.6, −0.6], [− 0.7, −0.7] ⟩, ⟨ *l*, [0.3, 0.3], [0.4, 0.4], [0.5, 0.5], [−0.4, −0.4], [−0.5,−0.5], [−0.6, −0.6] ⟩ } and H_2 = { ⟨ *m*, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.5, −0.5], [−0.6, −0.6], [−0.7, −0.7] ⟩, ⟨ *n*, [0.6, 0.6], [0.7, 0.7], [0.8, 0.8], [−0.7, −0.7], [−0.8, −0.8], [−0.9, −0.9] ⟩ }. Then ϑ = { 0_S, H_1, 1_S } and ψ= { 0_T, H_2, 1_T } are BIVMFTs on S and T respectively. Define a mapping j : (S, ϑ) → (T, ψ) by j(*k*) = *m* and j(*l*) = *n*. Then f is a BIVMFGSP continuous mapping.

Theorem 3.3. Every BIVMF continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let j: (S, ϑ) →(T, ψ) be a BIVMF continuous mapping. Let N be a BIVMFCS in T. Then $j^{-1}(N)$ is a BIVMFCS in S. Since every BIVMFCS is a BIVMFGSPCS, $j^{-1}(N)$ is a BIVMFGSPCS in S. Hence j is a BIVMFGSP continuous mapping.

Remark 3.4. The converse of the above theorem 3.3 need not be true.

Proof. Consider the following example: Let S = { *k, l* }, T = { *m, n* } and H_1 = { ⟨ *k*, [0.3, 0.3], [0.4, 0.4], [0.5, 0.5], [−0.4, −0.4], [−0.5,−0.5],[−0.6, −0.6] ⟩, ⟨ *l*, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩ }, H_2 = { ⟨ *m*, [0.4, 0.4],[0.5, 0.5],[0.6, 0.6],[−0.5, −0.5], [−0.6, −0.6], [−0.7, −0.7] ⟩, ⟨ *n*, [0.6, 0.6], [0.7, 0.7], [0.8, 0.8], [−0.7, −0.7], [−0.8, −0.8], [−0.9, −0.9] ⟩ }. Then ϑ = { 0_S, H_1, 1_S } and ψ = { 0_T, H_2, 1_T } are BIVMFT on S and T respectively. Define a mapping j : (S, ϑ) → (T,ψ) by j(*k*) = *m* and j(*l*) = *n*. Then j is a BIVMFGSP continuous mapping but not a BIVMF continuous mapping.

Theorem 3.5. Every BIVMFG continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let j : (S, ϑ) → (T, ψ) be a BIVMFG continuous mapping. Let N be a BIVMFCS in T. Then j^{-1}(N) is a BIVMFGCS in S. Since every BIVMFGCS is a BIVMFGSPCS, j^{-1}(N) is a BIVMFGSPCS in S. Hence j is a BIVMFGSP continuous mapping.

Remark 3.6. The converse of the above theorem 3.5 need not be true.

Proof. Consider the following example: Let S = { *k, l* }, T = { *m, n* } and H_1 = { ⟨ *k*, [0.2, 0.2], [0.3, 0.3], [0.4, 0.4], [−0.3, −0.3], [−0.4, −0.4], [−0.5, −0.5] ⟩, ⟨ *l*, [0.3, 0.3], [0.4, 0.4], [0.5, 0.5], [−0.4, −0.4], [−0.5, −0.5], [−0.6, −0.6] ⟩ }, H_2 = { ⟨ *m*, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩, ⟨ *n*, [0.6, 0.6], [0.7, 0.7], [0.8, 0.8], [−0.7, −0.7], [−0.8, −0.8], [−0.9, −0.9] ⟩ }. Then ϑ = { 0_S, H_1, 1_S } and ψ = { 0_T, H_2, 1_T } are BIVMFTs on S and T respectively. Define a mapping j : (S, τ) →(T, ψ) by j(*k*) = *m* and j(*l*) = *n*. Then j is a BIVMFGSP continuous mapping but not a BIVMFG continuous mapping. Since H_2^c = { ⟨ *m*, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [−0.4, −0.4], [−0.3, −0.3], [−0.2, −0.2] ⟩, ⟨ *n*, [0.4, 0.4], [0.3, 0.3], [0.2, 0.2], [−0.3, −0.3], [−0.2, −0.2], [−0.1, −0.1] ⟩ } is a BIVMFCS in T and $j^{-1}(H_2^c)$ = { ⟨ *k*, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [−0.4, −0.4], [−0.3, −0.3], [−0.2, −0.2] ⟩, ⟨ *l*, [0.4, 0.4], [0.3, 0.3], [0.2, 0.2], [−0.3, −0.3], [−0.2, −0.2], [−0.1, −0.1] ⟩ }⊆H_1. But bivmfcl ($j^{-1}(H_2^c)$) = $H_1^c \not\subset H_1$. Therefore $j^{-1}(H_2^c)$ is not a BIVMFGCS in S.

Theorem 3.7. Every BIVMFS continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let j : (S, ϑ) →(T, ψ) be a BIVMFS continuous mapping. Let N be a BIVMFCS in T. Then j^{-1}(N) is a BIVMFSCS in S. Since every BIVMFSCS is a BIVMFGSPCS, j^{-1}(N) is a BIVMFGSPCS in S. Hence f is a BIVMFGSP continuous mapping.

Remark 3.8. The converse of the above theorem 3.7 need not be true.

Proof. Consider the following example: Let S = { *k, l* }, T = { *m, n* } and H_1 = { ⟨ *k*, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [−0.5, −0.5], [−0.6, −0.6], [−0.7, −0.7] ⟩, ⟨ *l*, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩ }, H_2 = { ⟨ *m*, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩, ⟨ *n*, [0.2, 0.2], [0.3, 0.3], [0.4, 0.4], [−0.3, −0.3], [−0.4, −0.4], [−0.5, −0.5] ⟩ }. Then ϑ = { 0_S, H_1, 1_S } and ψ = { 0_T, H_2, 1_T } are BIVMFTs on S and T respectively. Define a mapping j : (S, ϑ) → (T, ψ) by j(*k*) = *m* and j(*l*) = *n*. Then j is a BIVMFGSP continuous mapping but not a BIVMFS continuous mapping. Since H_2^c = { ⟨ *m*, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3],[−0.4, −0.4], [−0.3, −0.3], [−0.2, 0.2] ⟩, ⟨ *n*, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [−0.7, −0.7], [−0.6, −0.6], [−0.5, −0.5] ⟩ } is a BIVMFCS in T and $j^{-1}(H_2^c)$ = { ⟨ *k*, [0.5, 0.5],[0.4, 0.4], [0.3, 0.3], [−0.4, −0.4], [−0.3, −0.3], [−0.2, 0.2] ⟩, ⟨ *l*, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [−0.7, −0.7], [−0.6, −0.6], [−0.5, −0.5] ⟩ } is not a BIVMFSCS in S, because bivmfint(bivmfcl($j^{-1}(H_2^c)$)) = bivmfint(1_S) = $1_S \not\subset j^{-1}(H_2^c)$.

Theorem 3.9. Every BIVMFP continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let j : (S, ϑ) →(T, ψ) be a BIVMFP continuous mapping. Let N be a BIVMFCS in T. Then j^{-1}(N) is a BIVMFPCS in T. Since every BIVMFPCS is a BIVMFGSPCS, j^{-1}(N) is a BIVMFGSPCS in S. Hence f is a BIVMFGSP continuous mapping.

Remark 3.10. The converse of the above theorem 3.9 need not be true.

Proof. Consider the following example: Let S = { *k, l* }, T = { *m, n*} and H_1 = { ⟨ *k*, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [−0.5, −0.5], [−0.6, −0.6], [−0.7, −0.7] ⟩, ⟨ *l*, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩ }, H_2 = { ⟨ *m*, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩, ⟨ *n*, [0.2, 0.2], [0.3, 0.3], [0.4, 0.4], [−0.3, −0.3], [−0.4, −0.4], [−0.5, −0.5] ⟩ }. Then ϑ = { 0_S, H_1, 1_S } and ψ = { 0_T, H_2, 1_T } are BIVMFTs on S and T respectively. Define a mapping j : (S, ϑ) → (T, ψ) by j(*k*) = *m* and j(*l*) = *n*. Then j is a BIVMFGSP continuous mapping but not a BIVMFP continuous mapping, since H_2^c = {⟨ *m*, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [−0.4, −0.4], [−0.3, −0.3], [−0.2, 0.2] ⟩, ⟨ *n*, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [−0.7, −0.7], [−0.6, −0.6], [−0.5, −0.5]⟩} is a BIVMFCS in T and $j^{-1}(H_2^c) = \{ \langle k, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [-0.4, -0.4], [-0.3, -0.3], [-0.2, 0.2] \rangle, \langle l, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [-0.7, -0.7], [-0.6, -0.6], [-0.5, -0.5] \rangle \}$ is not a BIVMFPCS in S, because bivmfcl(bivmfint($j^{-1}(H_2^c)$)) = bivmfcl(H_1) = $1_S \not\subseteq j^{-1}(H_2^c)$.

Theorem 3.11. Every BIVMFSP continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let $j : (S, \vartheta) \rightarrow (T, \psi$ be a BIVMFSP continuous mapping. Let n be a BIVMFCS in T. Then $j^{-1}(N)$ is a BIVMFSPCS in S. Since every BIVMFSPCS is a BIVMFGSPCS, $j^{-1}(V)$ is a BIVMFGSPCS in S. Hence j is a BIVMFGSP continuous mapping.

Remark 3.12. The converse of the above theorem 3.11 need not be true.

Proof. Consider the following example: Let S = { *k, l* }, T = { *m, n*} and $H_1 = \{ \langle k, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [-0.5, -0.5], [-0.6, -0.6], [-0.7, -0.7] \rangle, \langle l, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [-0.6, -0.6], [-0.7, -0.7], [-0.8, -0.8] \rangle \}$, $H_2 = \{ \langle m, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [-0.6, -0.6], [-0.7, -0.7], [-0.8, -0.8] \rangle, \langle n, [0.2, 0.2], [0.3, 0.3], [0.4, 0.4], [-0.3, -0.3], [-0.4, -0.4], [-0.5, -0.5] \rangle \}$. Then $\vartheta = \{ 0_S, H_1, 1_S \}$ and $\psi = \{ 0_T, H_2, 1_T \}$ are BIVMFT on S and T respectively. Define a mapping $j : (S, \vartheta) \rightarrow (T, \psi)$ by $j(k) = m$ and $j(l) = n$. Then j is a BIVMFGSP continuous mapping but not a BIVMFSP continuous mapping. Since $H_2^c = \{\langle m, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [-0.4, -0.4], [-0.3, -0.3], [-0.2, 0.2] \rangle, \langle n, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [-0.7, -0.7], [-0.6, -0.6], [-0.5, -0.5] \}$ is a BIVMFCS in T and $j^{-1}(H_2^c) = \{ \langle k, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [-0.4, -0.4], [-0.3, -0.3], [-0.2, 0.2] \rangle, \langle l, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [-0.7, -0.7], [-0.6, -0.6], [-0.5, -0.5] \rangle \}$ is not a BIVMFSPCS in S, because there exist no BIVMFPCS C in S such that bivmfint(C)$\subseteq j^{-1}(H_2^c) \subseteq$ C.

Theorem 3.13. Every BIVMFβ continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let $j : (S, \vartheta) \rightarrow (T, \psi)$ be a BIVMFβ continuous mapping. Let N be a BIVMFCS in T. Then $j^{-1}(N)$ is a BIVMFβCS in S. Since every BIVMFβCS is a BIVMFGSPCS, $j^{-1}(N)$ is a BIVMFGSPCS in S. Hence j is a BIVMFGSP continuous mapping.

Remark 3.14. The converse of the above theorem 3.13 need not be true.

Proof. Consider the following example: Let S = { *k, l* }, T = { *m, n*} and $H_1 = \{ \langle k, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [-0.5, -0.5], [-0.6, -0.6], [-0.7, -0.7] \rangle, \langle l, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [-0.6, -0.6], [-0.7, -0.7], [-0.8, -0.8] \rangle \}$, $H_2 = \{ \langle m, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [-0.6, -0.6], [-0.7, -0.7], [-0.8, -0.8] \rangle, \langle n, [0.2, 0.2], [0.3, 0.3], [0.4, 0.4], [-0.3, -0.3], [-0.4, -0.4], [-0.5, -0.5] \rangle \}$. Then $\vartheta = \{ 0_S, H_1, 1_S \}$ and $\psi = \{ 0_T, H_2, 1_T \}$ are BIVMFT on S and T respectively. Define a mapping $j: (S, \vartheta) \rightarrow (T, \psi)$ by $j(k) = m$ and $j(l) = n$. Then j is a BIVMFGSP continuous mapping but not a BIVMFβ continuous mapping, since $H_2^c = \{\langle m, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [-0.4, -0.4], [-0.3, -0.3], [-0.2, 0.2] \rangle, \langle n, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [-0.7, -0.7], [-0.6, -0.6], [-0.5, -0.5] \}$ is a BIVMFCS in T and $j^{-1}(H_2^c) =\{ \langle k, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [-0.4, -0.4], [-0.3, -0.3], [-0.2, 0.2] \rangle, \langle l, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [-0.7, -0.7], [-0.6, -0.6], [-0.5, -0.5] \rangle \}$ is not a BIVMFβCS in S, because bivmfint(bivmfcl (bivmfint ($j^{-1}(H_2^c)$))) = bivmfint (bivmfcl (H_1)) = bivmfint(1_S) = $1_S \not\subseteq j^{-1}(H_2^c)$.

Theorem 3.15. Every BIVMFα continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let $f : (S, \vartheta) \rightarrow (T, \psi)$ be a BIVMFα continuous mapping. Let N be a BIVMFCS in T. Then $j^{-1}(N)$ is a BIVMFαCS in S. Since every BIVMFαCS is a BIVMFGSPCS, $j^{-1}(N)$ is a BIVMFGSPCS in S. Hence j is a BIVMFGSP continuous mapping.

Remark 3.16. The converse of the above theorem 3.15 need not be true.

Proof. Consider the following example: Let S = { *k, l* }, T = { *m, n*} and $H_1 = \{ \langle k, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [-0.5, -0.5], [-0.6, -0.6], [-0.7, -0.7] \rangle, \langle l, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [-0.6, -0.6], [-0.7, -0.7], [-0.8, -0.8] \rangle \}$, $H_2 = \{ \langle m, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [-0.6, -0.6], [-0.7, -0.7], [-0.8, -0.8] \rangle, \langle n, [0.2, 0.2], [0.3, 0.3], [0.4, 0.4], [-0.3, -0.3], [-0.4, -0.4], [-0.5, -0.5] \rangle \}$. Then $\vartheta = \{ 0_S, H_1, 1_S \}$ and $\psi = \{ 0_T, H_2, 1_T \}$ are BIVMFT on S and T respectively. Define a mapping $j : (S, \vartheta) \rightarrow (T, \psi)$ by $j(k) = m$ and $j(l) = n$. Then j is a BIVMFGSP continuous mapping but not a BIVMFα continuous mapping, since $H_2^c = \{\langle m, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [-0.4, -0.4], [-0.3, -0.3], [-0.2, 0.2] \rangle, \langle n, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [-0.7, -0.7], [-0.6, -0.6], [-0.5, -0.5] \}$ is a BIVMFCS in T and $j^{-1}(H_2^c) = \{ \langle k, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [-0.4, -0.4], [-0.3, -0.3], [-0.2, 0.2] \rangle, \langle l, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [-0.7, -0.7], [-0.6, -0.6], [-0.5, -0.5] \rangle \}$ is not a BIVMFαCS in S, because bivmfcl(bivmfint(bivmfcl($j^{-1}(H_2^c)$))) = bivmfcl(bivmfint(1_S)) = bivmfcl(1_S) = $1_S \not\subseteq j^{-1}(H_2^c)$.

Theorem 3.17. Let j : (S, ϑ) → (T, ψ) be a mapping, where $j^{-1}(N)$ is a BIVMFRCS in S for every BIVMFCS N in T. Then j is a BIVMFGSP continuous mapping.

Proof. Assume that j : (S, ϑ) → (T, ψ) is a mapping. Let B be a BIVMFCS in T. Then $j^{-1}(N)$ is a BIVMFRCS in S, by hypothesis. Since every BIVMFRCS is a BIVMFGSPCS, $j^{-1}(N)$ is a BIVMFGSPCS in S. Hence j is a BIVMFGSP continuous mapping.

Remark 3.18. The converse of the above theorem 3.17 need not be true.

Proof. Consider the following example: Let S = { *k, l* }, T = {*m, n* } and H_1= { ⟨ *k*, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [−0.5, −0.5], [−0.6, −0.6], [−0.7, −0.7] ⟩, ⟨ *l*, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩ }, H_2 = { ⟨ *m*, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [−0.6, −0.6], [−0.7, −0.7], [−0.8, −0.8] ⟩, ⟨ *n*, [0.2, 0.2], [0.3, 0.3], [0.4, 0.4], [−0.3, −0.3], [−0.4, −0.4], [−0.5, −0.5] ⟩ } . Then ϑ = { 0_S, H_1, 1_S } and ψ = { 0_T, H_2, 1_T } are BIVMFT on S and T respectively. Define a mapping j : (S, ϑ) → (T, ψ) by j(*k*) = *m* and j(*l*) = *n*. Then j is a BIVMFGSP continuous mapping but not a mapping as defined in Theorem 2.17, since H_2^c = {⟨ *m*, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [−0.4, −0.4], [−0.3, −0.3], [−0.2, 0.2] ⟩, ⟨ *n*, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [−0.7, −0.7], [−0.6, −0.6], [−0.5, −0.5] } is a BIVMFCS in T and $j^{-1}(H_2^c)$= { ⟨ *k*, [0.5, 0.5], [0.4, 0.4], [0.3, 0.3], [−0.4, −0.4], [−0.3, −0.3], [−0.2, 0.2] ⟩, ⟨ *l*, [0.8, 0.8], [0.7, 0.7], [0.6, 0.6], [−0.7, −0.7], [−0.6, −0.6], [−0.5, −0.5] ⟩ } is not a BIVMFRCS in S, because bivmfcl(bivmfint($j^{-1}(H_2^c)$)) = bivmfcl(H_1)= $1_S \neq j^{-1}(H_2^c)$.

Theorem 3.19. If j : (S, ϑ) → (T, ψ) is a BIVMFGSP continuous mapping, then for each BIVMFP $p_{(\alpha,\beta)}$ of S and each B∈ψ such that j($p_{(\alpha,\beta)}$)∈B, there exists a BIVMFGSPOS C of S such that $p_{(\alpha,\beta)}$ ∈C and j(C) ⊆ B.

Proof. Let $p_{(\alpha,\beta)}$ be a BIVMFP of S and B∈ψ such that j($p_{(\alpha,\beta)}$)∈B. Put C = $j^{-1}(B)$. Then hypothesis, C is a BIVMFGSPOS in S such that $p_{(\alpha,\beta)}$∈C and j(C) = j($j^{-1}(B)$) ⊆ B.

Theorem 3.20. If j : (S, ϑ) → (T, ψ) is a BIVMFGSP continuous mapping. Then j is a BIVMFSP continuous mapping if S is a BIVMFSP$T_{1/2}$ space.

Proof. Let N be a BIVMFCS in T. Then $j^{-1}(N)$ is a BIVMFGSPCS in S, by hypothesis. Since S is a BIVMFSP$T_{½}$ space, $j^{-1}(N)$ is a BIVMFSPCS in S. Hence j is a BIVMFSP continuous mapping.

Theorem 3.21. Let j : (S, ϑ) →(T, ψ) be a BIVMFGSP continuous mapping and let g : (T, ψ)→(Q, ω) be BIVMFG continuous mapping where T is a BIVMF$T_{1/2}$-space. Then g º j : (S, ϑ) →(Q, ω) is a BIVMFGSP continuous mapping.

Proof. Let N be a BIVMFCS in Z. Then $g^{-1}(N)$ is a BIVMFGCS in T, by hypothesis. Since T is a BIVMF$T_{½}$ space, $g^{-1}(N)$ is a BIVMFCS in T. Therefore $j^{-1}(g^{-1}(N))$ is a BIVMFGSPCS in S, by hypothesis. Hence g ° j is a BIVMFGSP continuous mapping.

Theorem 3.22. Let j : (S, ϑ) → (T, ψ) be a BIVMFGSP continuous mapping and let g : (T, ψ) → (Q, ω) be a BIVMF continuous mapping, then g•j : (S, ϑ) → (Q, ω) is a BIVMFGSP continuous mapping.

Proof. Let N be a BIVMFCS in Z. Then $g^{-1}(N)$ is a BIVMFCS in T, by hypothesis. Since f is a BIVMFGSP continuous mapping, $j^{-1}(g^{-1}(N))$ is a BIVMFGSPCS in S. Hence g•j is a BIVMFGSP continuous mapping.

Theorem 3.23. Let j: (S, ϑ) → (T, ψ) be a mapping from a BIVMFT S into a BIVMFT T. Then the following conditions are equivalent if S and T are BIVMFSP$T_{1/2}$ spaces.

(i) j is a BIVMFGSP continuous mapping,

(ii) $j^{-1}(C)$ is a BIVMFGSPOS in S for each BIVMFOS C in T,

(iii) for every BIVMFP $p_{(\alpha,\beta)}$ in S and for every BIVMFOS C in T such that j($p_{(\alpha,\beta)}$) ∈ C, there exists a BIVMFGSPOS B in S such that $p_{(\alpha,\beta)}$∈B and j(B) ⊆ C.

Proof. (i) ⇔ (ii) is an obvious, since $j^{-1}(B^c)$ = ($j^{-1}(B)$)^c.

(ii) ⇒ (iii) Let C any BIVMFOS in T and let $p_{(\alpha,\beta)} \in E^X$. Given $j(p_{(\alpha,\beta)}) \in C$. By hypothesis $j^{-1}(C)$ is a BIVMFGSPOS in S. Take B= $j^{-1}(C)$. Now $p_{(\alpha,\beta)} \in j^{-1}(j(p_{(\alpha,\beta)}))$. Therefore $j^{-1}(j(p_{(\alpha,\beta)})) \in j^{-1}(C) = B$. This implies $(p_{(\alpha,\beta)}) \in B$ and $j(B) = j(j^{-1}(C)) \subseteq C$.

(iii) ⇒ (i) Let B be a BIVMFCS in T. Then its complement, say $C = B^c$, is a BIVMFOS in T. Let $p_{(\alpha,\beta)} \in E^X$ and $j((p_{(\alpha,\beta)}) \in C$. Then there exists a BIVMFGSPOS, say D in S such that $p_{(\alpha,\beta)} \in D$ and $j(D) \subseteq C$. Now $D \subseteq j^{-1}(j(D) \subseteq j^{-1}(C)$. Thus $p_{(\alpha,\beta)} \in j^{-1}(C)$. Therefore $j^{-1}(C)$ is a BIVMFGSPOS in S, by Theorem 2.27. That is $j^{-1}(B^c)$ is a BIVMFGSPOS in S and hence $j^{-1}(B)$ is a BIVMFGSPCS in S. Thus j is a BIVMFGSP continuous mapping.

Theorem 3.24. Let j: $(S, \vartheta) \to (T, \psi)$ be a mapping from a BIVMFT S into a BIVMFT T. Then the following conditions are equivalent if S and T are $BIVMFSPT_{1/2}$ spaces.

(i) j is a BIVMFGSP continuous mapping,

(ii) for each BIVMFP $p_{(\alpha,\beta)}$ in S and for every BIVMFN B of $j(p_{(\alpha,\beta)})$, there exists a BIVMFGSPOS C in S such that $p_{(\alpha,\beta)} \in C \subseteq j^{-1}(B)$,

(iii) for each BIVMFP $p_{(\alpha,\beta)}$ in S and for every BIVMFN B of $j(p_{(\alpha,\beta)})$, there exists a BIVMFGSPOS C in S such that $p_{(\alpha,\beta)} \in C$ and $j(C) \subseteq B$.

Proof. (i) ⇒ (ii) Let $p_{(\alpha,\beta)} \in E^X$ and let B be a BIVMFN of $j(p_{(\alpha,\beta)}$. Then there exists a BIVMFOS D in T such that $j(p_{(\alpha,\beta)}) \in D \subseteq B$. Since j is a BIVMFGSP continuous mapping, $j^{-1}(D) = C$ (say), is a BIVMFGSPOS in S and $p_{(\alpha,\beta)} \in C \subseteq j^{-1}(B)$.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in E^X$ and let B be a BIVMFN of $j(p_{(\alpha,\beta)}$. Then there exists a BIVMFGSPOS C in S such that $p_{(\alpha,\beta)} \in C \subseteq j^{-1}(B)$, by hypothesis. Therefore $p_{(\alpha,\beta)} \in C$ and $j(C) \subseteq j(j^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let C be a BIVMFOS in T and let $p_{(\alpha,\beta)} \in j^{-1}(C)$. Then $j(p_{(\alpha,\beta)}) \in C$. Therefore C is a BIVMFN of $j(p_{(\alpha,\beta)})$. Since C is BIVMFOS, by hypothesis there exists a BIVMFGSPOS B in S such that $p_{(\alpha,\beta)} \in B \subseteq j^{-1}(\, j(B)\,) \subseteq j^{-1}(C)$. Therefore $j^{-1}(C)$ is a BIVMFGSPOS in S, by Theorem 2.27. Hence j is a BIVMFGSP continuous mapping.

Theorem 3.25. Let j: $(S, \vartheta) \to (T, \psi)$ be a mapping from a BIVMFT S into a BIVMFT T. Then the following conditions are equivalent if S is a $BIVMFSPT_{1/2}$ space.

(i) j is a BIVMFGSP continuous mapping,

(ii) if C is a BIVMFOS in T then $j^{-1}(C)$ is a BIVMFGSPOS in S,

(iii) $j^{-1}(\, bivmfint(C)\,) \subseteq bivmfcl(bivmfint(\, bivmfcl(\, j^{-1}(C)\,)))$ for every BIVMFS C in T.

Proof. (i) ⇔ (ii) is obviously true by Theorem 3.23.

(ii) ⇒ (iii) Let C be any BIVMFS in T. Then bivmfint(C) is a BIVMFOS in T. Then $j^{-1}(bivmfint(C)\,)$ is a BIVMFGSPOS in S. Since S is a $BIVMFSPT_{1/2}$ spaces, $j^{-1}(\, bivmfint(C)\,)$ is a BIVMFSPOS in S. Therefore $j^{-1}(bivmfint(C)) \subseteq bivmfcl(bivmfint\, (bivmfcl\, (j^{-1}(bivmfint(C)))))$ $\subseteq bivmfcl(\, bivmfint(\, bivmfcl(\, j^{-1}(C)\,)\,)\,)$.

(iii) ⇒ (i) Let C be a BIVMFOS in T. By hypothesis $j^{-1}(C) = j^{-1}(bivmfint(C)) \subseteq bivmfcl(bivmfint(\, bivmfcl(j^{-1}(C))))$. This implies $j^{-1}(C)$ is a BIVMFβOS in S. Therefore it is a BIVMFGSPOS in S and hence j is a BIVMFGSP continuous mapping, by Theorem 2.23.

Theorem 3.26. Let j : $(S, \vartheta) \to (T, \psi)$ be a mapping from a BIVMFT S into a BIVMFT T. Then the following conditions are equivalent if S and T are $BIVMFSPT_{1/2}$ space.

(i) j is a BIVMFGSP continuous mapping,

(ii) $bivmfint(\, bivmfcl(\, bivmfint(\, j^{-1}(C)\,)\,)\,) \subseteq j^{-1}(\, bivmfspcl(C)\,)$ for each BIVMFCS C in T,

(iii) $j^{-1}(\, bivmfspint(C)\,) \subseteq bivmfcl(\, bivmfint(\, bivmfcl(\, j^{-1}(C)\,)\,)\,)$ for each BIVMFOS C of T,

(iv) $j(\, bivmfint(\, bivmfcl(\, bivmfint(B)\,)\,)\,) \subseteq bivmfcl(\, j(B)\,)$ for each BIVMFS B of S.

Proof. (i) ⇒ (ii) Let C be a BIVMFCS in T. Then $j^{-1}(C)$ is a BIVMFGSPCS in S. Since S is a $BIVMFSPT_{1/2}$ space, $j^{-1}(C)$ is a BIVMFSPCS. Therefore bivmfint(bivmfcl(bivmfint($j^{-1}(C)$))) ⊆ $j^{-1}(C)$ = j^{-1}(bivmfspcl(C)).

(ii) ⇒ (iii) It can be easily proved by taking complement in (ii).

(iii) ⇒ (iv) Let B∈S. Taking C = j(B) we have B ⊆ $j^{-1}(C)$.

Here bivmfint(j(B)) = bivmfint(C) is a BIVMFOS in T. Then (iii) implies that j^{-1}(bivmfspint(bivmfint(C))) ⊆ bivmfcl(bivmfint(bivmfcl(j^{-1}(bivmfint(C))))) ⊆ bivmfcl(bivmfint(bivmfcl($j^{-1}(C)$))). Now we have (bivmfcl (bivmfint (bivmfcl (B^c))) $)^c$ ⊆ (bivmfcl (bivmfint (bivmfcl ($j^{-1}(C))^c$)) $)^c$ ⊆ (j^{-1}(bivmfspint (bivmfint (C^c))) $)^c$. This implies bivmfint (bivmfcl (bivmfint (B))) ⊆ j^{-1} (bivmfspcl (bivmfcl (C))). Therefore j (bivmfint(bivmfcl(bivmfint(B)))) ⊆ j(j^{-1} (bivmfspcl (bivmfcl (C)))) ⊆ bivmfcl(C) = bivmfcl (j(B)).

(iv) ⇒ (i) Let C be any BIVMFCS in T. Then j^{-1} (C) is a BIVMFS in S. By hypothesis j(bivmfint (bivmfcl (bivmfint ($j^{-1}(C)$)))) ⊆ bivmfcl (j ($j^{-1}(C)$)) ⊆ bivmfcl(C) = C. Now bivmfint (bivmfcl (bivmfint($j^{-1}(C)$))) ⊆ j^{-1}(j (bivmfint (bivmfcl (bivmfint ($j^{-1}(C)$))))) ⊆ $j^{-1}(C)$. This implies $j^{-1}(C)$ is a BIVMFβCS and hence it is a BIVMFGSPCS in S. Thus j is a BIVMFGSP continuous mapping.

Theorem 3.27. A mapping j : (S, ϑ) → (T, ψ) is a BIVMFGSP continuous mapping if bivmfcl(bivmfint(bivmfcl($j^{-1}(B)$))) ⊆ j^{-1}(bivmfcl(B)) for every BIVMFS B in T.

Proof. Let B be a BIVMFOS in T. Then B^c is a BIVMFCS in T. Therefore bivmfcl(B^c) = B^c. By hypothesis, bivmfcl (bivmfint (bivmfcl (j^{-1}(B^c)))) ⊆ j^{-1}(bivmfcl (B^c)) = j^{-1}(B^c). Now (bivmfint(bivmfcl(bivmfint(j^{-1}(B)))) $)^c$ = bivmfcl(bivmfint(bivmfcl($j^{-1}(B^c)$))) ⊆ $j^{-1}(B^c)$ = $(j^{-1}(A))^c$. This implies $j^{-1}(B)$ ⊆ bivmfint(bivmfcl(bivmfint($j^{-1}(B)$))). Hence $j^{-1}(B)$ is a BIVMFαOS in S and hence it is a BIVMFGSPOS in S. Therefore j is a BIVMFGSP continuous mapping, by Theorem 3.23.

Theorem 3.28. Let j : (S, ϑ) → (T, ψ) be a mapping from a BIVMFT S into a BIVMFT T. Then the following conditions are equivalent if S is a $BIVMFSPT_{1/2}$ space.

(i) j is a BIVMFGSP continuous mapping,

(ii) j^{-1} (C) is a BIVMFGSPCS in S for every BIVMFCS C in T,

(iii) bivmfint(bivmfcl(bivmfint($j^{-1}(B)$))) ⊆ j^{-1}(bivmfcl(B)) for every BIVMFS B in T.

Proof. (i) ⇔ (ii) is obvious from the Definition 3.1. (ii) ⇒ (iii) Let B be a BIVMFS in T. Then bivmfcl(B) is an BIVMFCS in T. By hypothesis, j^{-1}(bivmfcl(B)) is a BIVMFGSPCS in S. Since S is a $BIVMFSPT_{1/2}$ space, j^{-1}(bivmfcl(B)) is an BIVMFSPCS in (S, ϑ). Therefore we have bivmfint (bivmfcl (bivmfint(j^{-1}(bivmfcl(B))))) ⊆ j^{-1}(bivmfcl(B)). Now bivmfint(bivmfcl(bivmfint($j^{-1}(B)$))) ⊆ bivmfint(bivmfcl(bivmfint(j^{-1}(bivmfcl(B))))) ⊆ j^{-1}(bivmfcl(B)). (iii) ⇒ (i) Let B be an BIVMFCS in T. By hypothesis bivmfint(bivmfcl(bivmfint($j^{-1}(B)$))) ⊆ j^{-1}(bivmfcl(B)) = $j^{-1}(B)$. This implies $j^{-1}(B)$ is a BIVMFβCS in S and hence it is a BIVMFGSPCS. Thus j is a BIVMFGSP continuous mapping.

4. CONCLUSION:

We conclude that, every BIVMF continuous mapping, BIVMFα continuous mapping, BIVMFβ continuous mapping, BIVMFP continuous mapping, BIVMFSP continuous mapping are an BIVMFGSP continuous mapping. Also some equivalent condition Theorems are proved in this paper. Using this concept, we can develop some new theorems and properties.

REFERENCES:

[1] Andrijevic.D, "Semipreopen sets", Mat.Vesnic, 38 (1986), 24 –32.

[2] Azhagappan. M and M.Kamaraj, "Notes on bipolar valued fuzzy rw-closed and bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological spaces", International Journal of Mathematical Archive, 7(3), 30-36, 2016.

[3] Bhattacarya.B., and Lahiri. B.K., "Semi-generalized closed set in topology", Indian Jour.Math., 29 (1987), 375 –382.

[4] Chang.C.L., "Fuzzy topological spaces", JI. Math. Anal. Appl., 24(1968), 182 –190.

[5] Dontchev.J., "On generalizing semipreopen sets", Mem. Fac. sci. Kochi. Univ. Ser. A, Math.,16 (1995), 35 – 48.

[6] Ganguly.S and Saha.S, "A note on fuzzy Semipreopen sets in fuzzy topological spaces", Fuzzy sets and system, 18 (1986), 83 – 96.

[7] Indira.R, Arjunan.K and Palaniappan.N, "Notes on interval valued fuzzy rw-closed, interval valued fuzzy rw-open sets in interval valued fuzzy topological space", International Journal of Computational and Applied Mathematics.,Vol.3, No.1(2013), 23 –38.

[8] Jeyabalan.R and K. Arjunan, "Notes on interval valued fuzzy generalized semipreclosed sets", International Journal of Fuzzy Mathematics and Systems, Volume 3, Number 3 (2013), 215 –224.

[9] Jeyabalan,R and Arjunan.K, "Interval valued fuzzy generalized semi-preclosed mappings", IOSR Journal of Mathematics, Vol.8, Issue 5(2013), 40 – 47.

[10] Lee, K.M., "Bipolar-valued fuzzy sets and their operations", Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, (2000), 307 –312.

[11] Levine.N, "Generalized closed sets in topology", Rend. Circ. Math. Palermo, 19 (1970),89 –96.

[12] Mondal.T.K., "Topology of interval valued fuzzy sets", Indian J. Pure Appl.Math. 30, No.1 (1999), 23 – 38.

[13] Murugan.V, U.Karuppiah & M.Marudai, Notes on multi fuzzy rw-closed, multi fuzzy rw-open sets in multi fuzzy topological spaces, Bulletin of Mathematics and Statistics Research, Vo.4, Issu. 1, 2016, 174-179.

[14] Sabu Sebastian, T.V.Ramakrishnan, Multi fuzzy sets, International Mathematical Forum, 5, no.50 (2010), 2471-2476.

[15] Saraf.R.K and Khanna.K., "Fuzzy generalized semipreclosed sets", Jour.Tripura. Math.Soc., 3 (2001), 59 – 68.

[16] Selvam.R, K.Arjunan & KR.Balasubramanian,"Bipolar interval valued multi fuzzy generalized semipreclosed sets", JASC Journal, Vol.6, ISS. 5(2019), 2688 –2697.

[17] Vinoth.S & K.Arjunan, "A study on interval valued intuitionistic fuzzy generalized semipreclosed sets", International Journal of Fuzzy Mathematics and Systems, Vol. 5, No. 1 (2015), 47 – 55.

[18] Vinoth.S & K.Arjunan, "Interval valued intuitionistic fuzzy generalized semipreclosed mappings", International Journal of Mathematical Archive, 6(8) (2015), 129 –135.

[19] Zadeh.L.A., "Fuzzy sets", Information and control, Vol.8 (1965), 338 –353.