BIPOLAR INTERVAL VALUED MULTI FUZZY GENERALIZED SEMIPRE CONTINUOUS MAPPINGS

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ABSTRACT: In this paper, bipolar interval valued multi fuzzy generalized semi-precontinuous mappings are defined and introduced. Using these definitions, some theorems are introduced.

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1. INTRODUCTION: The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [19] in the year 1965, the subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper C.L.Chang [4] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces many researchers like, and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] has introduced semipreclosed sets and Dontchev [5] has introduced generalized semipreclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by saraf and khanna [15]. Tapas kumar mondal and S.K.Samantha [12] have introduced the topology of interval valued fuzzy sets. Jeyabalan R and Arjunan [8, 9] have introduced interval valued fuzzy generalized semi pre continuous mapping. After that interval valued fuzzy generalized semi pre continuous mapping has been generalized into interval valued intuitionistic fuzzy generalized semi pre continuous mapping by S.Vinoth and K.Arjunan[17, 18]. The interval valued fuzzy set has been extended into the bipolar interval valued multi fuzzy topological spaces. R.Selvam et.al [16] have defined and introduced the bipolar interval valued multi fuzzy generalized semipreclosed sets. In this paper, we introduce bipolar interval valued multi fuzzy generalized semi-precontinuous mappings and some properties are investigated.

2. PRELIMINARIES:

Definition 2.1[19]. Let X be a non-empty set. A fuzzy subset A of X is a function $A : X \rightarrow [0, 1]$.

Definition 2.2[14]. A multi fuzzy subset A of a set X is defined as an object of the form $A = \{ (x, A_1(x), A_2(x), A_3(x), ..., A_n(x)) \mid x \in X \}$, where $A_i : X \rightarrow [0, 1]$ for all i and i = 1, 2, ..., n.

Definition 2.3[19]. Let X be any nonempty set. A mapping $A : X \rightarrow [0, 1]$ is called an interval valued fuzzy subset (briefly, IVFS) of X, where $D[0, 1]$ denotes the family of all closed subintervals of $[0, 1]$.

Definition 2.4[19]. A interval valued multi fuzzy subset A of a set X with degree n is defined as an object of the form $A = \{ (x, A_1(x), A_2(x), A_3(x), ..., A_n(x)) \mid x \in X \}$, where $A_i : X \rightarrow D[0, 1]$ for all i and i = 1, 2, ..., n.

Definition 2.5[10]. A bipolar valued fuzzy set A in X is defined as an object of the form $A = \{ (x, M(x), N(x)) \mid x \in X \}$, where $M : X \rightarrow [0, 1]$ and $N : X \rightarrow [-1, 0]$. The positive membership degree $M(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $N(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 2.6. $A = \{ (a, 0.7, -0.5), (b, 0.3, -0.8), (c, 0.2, -0.4) \}$ is a bipolar valued fuzzy subset of $X = \{ a, b, c \}$. 
Definition 2.7[10]. A bipolar valued multi fuzzy set \( A \) in \( X \) with degree \( n \) is defined as an object of the form \( A = \{ (x, M_i(x), M_i(x)) : x \in X \} \), where \( M_i : X \to [0, 1] \) and \( N_i : X \to [-1, 0] \) for all \( i \) and \( i = 1, 2, \ldots, n \). The positive membership degrees \( M_i(x) \) denotes the satisfaction degrees of an element \( x \) to the property corresponding to a bipolar valued multi fuzzy set \( A \) and the negative membership degrees \( N_i(x) \) denotes the satisfaction degrees of an element \( x \) to some implicit counter-property corresponding to a bipolar valued multi fuzzy set \( A \).

Example 2.8. \( A = \{ (k, 0.5, 0.4, 0.7, -0.2, -0.5, -0.8), (l, 0.3, 0.7, 0.3, -0.3, -0.4, -0.6), (m, 0.5, 0.8, 0.4, -0.5, -0.2, -0.9) \} \) is a bipolar valued multi fuzzy subset of \( X = \{k, l, m\} \) with degree 3.

Definition 2.9[16]. A bipolar interval valued fuzzy set \( A \) in \( X \) is defined as an object of the form \( A = \{ (x, M(x), N(x)) : x \in X \} \), where \( M : X \to D[0, 1] \) and \( N : X \to D[-1, 0] \). The positive membership interval degree \( M(x) \) denotes the satisfaction degree of an element \( x \) to the property corresponding to a bipolar interval valued fuzzy set \( A \) and the negative membership interval degree \( N(x) \) denotes the satisfaction degree of an element \( x \) to some implicit counter-property corresponding to a bipolar interval valued fuzzy set \( A \).

Example 2.10. \( A = \{ (k, [0.3, 0.9], [-0.5, -0.4]), (l, [0.2, 0.9], [-0.9, -0.5]), (m, [0.5, 0.8], [-0.8, -0.6]) \} \) is a bipolar interval valued fuzzy subset of \( X = \{a, b, c\} \).

Definition 2.11[16]. A bipolar interval valued multi fuzzy set \( A \) in \( X \) with degree \( n \) is defined as an object of the form \( A = \{ (x, M_i(x), M_i(x)) : x \in X \} \), where \( M_i : X \to D[0, 1] \) and \( N_i : X \to D[-1, 0] \) for all \( i \) and \( i = 1, 2, \ldots, n \). The positive membership degrees \( M_i(x) \) denotes the satisfaction degrees of an element \( x \) to the property corresponding to a bipolar interval valued multi fuzzy set \( A \) and the negative membership degrees \( N_i(x) \) denotes the satisfaction degrees of an element \( x \) to some implicit counter-property corresponding to a bipolar interval valued multi fuzzy set \( A \).

Example 2.12. \( A = \{ (k, [0.3, 0.7], [0.2, 0.6], [0.5, 0.9], [-0.3, -0.2], [-0.6, -0.3], [-0.8, -0.3]), (b, [0.6, 0.9], [0.1, 0.9], [0.5, 0.5], [-0.3, -0.2], [-0.5, -0.3], [-0.6, -0.4]), (m, [0.5, 0.8], [0.3, 0.6], [0.4, 0.9], [-0.5, -0.2], [-0.8, -0.5], [-0.9, -0.7]) \} \) is a bipolar interval valued multi fuzzy subset of \( X = \{a, b, c\} \) with degree 3.

Definition 2.13[16]. Let \( A = \{ M_i, N_i \} \) and \( B = \{ O_i, P_i \} \) be any two bipolar interval valued multi fuzzy subsets of a set \( X \) with degree \( n \). We define the following relations and operations:

(i) \( A \subseteq B \) if and only if \( M_i(x) \leq O_i(x) \) and \( N_i(x) \geq P_i(x) \) for all \( x \) in \( X \) and for all \( i \).

(ii) \( A \cap B = \{ x : M_i(x) = O_i(x) \) and \( N_i(x) = P_i(x) \) \) for all \( x \) in \( X \) and for all \( i \).

(iii) \( A \cap B = \{ x : x, \min\{M_i(x), O_i(x)\}, \max\{N_i(x), P_i(x)\} \in X \} \).

(iv) \( A \cup B = \{ x : \max\{M_i(x), O_i(x)\}, \min\{N_i(x), P_i(x)\} \in X \} \).

Remark 2.14. \( \mathcal{A}_0 = \{ x, [0, 0], [0, 0], \ldots, [0, 0] : x \in X \} \) and \( \mathcal{I} = \{ x, [1, 1], [1, 1], \ldots, [1, 1], [-1, -1], [-1, -1], \ldots, [-1, -1] : x \in X \} \).

Definition 2.15[16]. Let \( S \) be a set and \( \mathcal{F} \) be a family of bipolar interval valued multi fuzzy subsets of \( S \). The family \( \mathcal{F} \) is called a bipolar interval valued multi fuzzy topology (BIVMFT) on \( S \) if \( \mathcal{F} \) satisfies the following axioms:

(i) \( \mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \mathcal{F} \) if and only if \( \{ A_i : i \in I \} \subseteq \mathcal{F} \), then \( \bigcup_{i=1}^{\infty} A_i \in \mathcal{F} \).

(ii) If \( A_i, A_2, A_3, \ldots, A_n \in \mathcal{F} \), then \( \bigcap_{i=1}^{n} A_i \in \mathcal{F} \).

The pair \( (S, \mathcal{F}) \) is called a bipolar interval valued multi fuzzy topological space (BIVMFTS). The members of \( \mathcal{F} \) are called bipolar interval valued multi fuzzy open sets (BIVMFOS) in \( S \). An bipolar interval valued multi fuzzy subset \( A \) in \( S \) is said to be bipolar interval valued multi fuzzy closed set (BIVMFCS) in \( S \) if and only if \( \overline{A} \in \mathcal{F} \) is a BIVMFOS in \( S \).
Definition 2.16[16]. Let $\mathcal{S}$ be a BIVMFTS and $A$ be a BIVMFS in $\mathcal{S}$. Then the bipolar interval valued multi fuzzy interior and bipolar interval valued multi fuzzy closure are defined by $bivmfint(A) = \bigcup \{ H : H \text{ is a BIVMFS in } X \text{ and } H \subseteq A \}$, $bivmfcl(A) = \bigcap \{ K : K \text{ is a BIVMFS in } \mathcal{S} \text{ and } A \subseteq K \}$. For any BIVMFS $A$ in $(\mathcal{S}, \mathcal{G})$, we have $bivmfcl(A^c) = (bivmfint(A))^c$ and $bivmfint(A^c) = (bivmfcl(A))^c$.

Definition 2.17[16]. A BIVMFS $A$ of a BIVMFTS $(\mathcal{S}, \mathcal{G})$ is said to be a

(i) bipolar interval valued multi fuzzy regular closed set (BIVMFRCS for short) if $A = bivmfcl(bivmfint(A))$

(ii) bipolar interval valued multi fuzzy semiclosed set (BIVMFSCS for short) if $bivmfint(bivmfcl(A)) \subseteq A$

(iii) bipolar interval valued multi fuzzy preclosed set (BIVMFPCS for short) if $bivmfcl(bivmfint(A)) \subseteq A$

(iv) bipolar interval valued multi fuzzy $\alpha$ closed set (BIVMF$\alpha$CS for short) if $bivmfcl\left(bivmfint\left(bivmfcl(A)\right)\right) \subseteq A$

(v) bipolar interval valued multi fuzzy $\beta$ closed set (BIVMF$\beta$CS for short) if $bivmfint\left(bivmfcl\left(bivmfint(A)\right)\right) \subseteq A$.

Definition 2.18[16]. A BIVMFS $A$ of a BIVMFTS $(\mathcal{S}, \mathcal{G})$ is said to be a

(i) bipolar interval valued multi fuzzy generalized closed set (BIVMFGCS for short) if $bivmfcl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is a BIVMFOS

(ii) bipolar interval valued multi fuzzy regular generalized closed set (BIVMFRGCS for short) if $bivmfcl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is a BIVMFROS.

Definition 2.19[16]. A BIVMFS $A$ of a BIVMFTS $(\mathcal{S}, \mathcal{G})$ is said to be a

(i) bipolar interval valued multi fuzzy semipreclosed set (BIVMFSPCS for short) if there exists a BIVMFPCS $B$ such that $bivmfcl(B) \subseteq A \subseteq B$

(ii) bipolar interval valued multi fuzzy semipreopen set (BIVMFSPS for short) if there exists a BIVMFPOS $B$ such that $B \subseteq A \subseteq bivmfcl(B)$.

Definition 2.20[16]. Two BIVMFSs $A$ and $B$ are said to be not q-coincident if and only if $A \not\subseteq B^c$.

Definition 2.21[16]. Let $B$ be a BIVMFS in a BIVMFTS $(\mathcal{S}, \mathcal{G})$. Then the bipolar interval valued multi fuzzy semipre interior of $A$ ($bivmfspint(B)$ for short) and the bipolar interval valued multi fuzzy semipre closure of $A$ ($bivmfspcl(B)$ for short) are defined by $bivmfspint(B) = \bigcup \{ H : H \text{ is a BIVMFS in } \mathcal{S} \text{ and } H \subseteq B \}$, $bivmfspcl(B) = \bigcap \{ K : K \text{ is a BIVMFPCS in } \mathcal{S} \text{ and } B \subseteq K \}$. For any BIVMFS $B$ in $(\mathcal{S}, \mathcal{G})$, we have $bivmfspcl(B^c) = (bivmfspint(B))^c$ and $bivmfspint(B^c) = (bivmfspcl(B))^c$.

Definition 2.22[16]. A BIVMFS $B$ in BIVMFTS $(\mathcal{S}, \mathcal{G})$ is said to be a bipolar interval valued multi fuzzy generalized semipreclosed set (BIVMFGSPCS for short) if $bivmfspcl(B) \subseteq U$ whenever $B \subseteq U$ and $U$ is a BIVMFOS in $(\mathcal{S}, \mathcal{G})$.

Example 2.23. Let $\mathcal{S} = \{k, l\}$ and $\mathcal{G} = \{0, H \}$ is a BIVMFT on $\mathcal{S}$, where $H = \{(k, [0.5, 0.5], [0.6, 0.6], [0.4, 0.4], [-0.4, -0.4], [-0.5, -0.5], [-0.3, -0.3]) \cup \{ l, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [-0.3, -0.3], [-0.4, -0.4], [-0.2, -0.2] \} \}$. And the BIVMFS $B = \{(k, [0.4, 0.4], [0.5, 0.5], [0.3, 0.3], [-0.3, -0.3], [-0.4, -0.4], [-0.2, -0.2], \{l, [0.2, 0.2], [0.3, 0.3], [0.1, 0.1], [-0.1, -0.1], [-0.2, -0.2], [-0.05, -0.05] \} \}$ is a BIVMFGSPCS in $(\mathcal{S}, \mathcal{G})$. 

Definition 2.24[16]. The complement $B^c$ of a BIVMFGSPCS $B$ in a BIVMFTS $(S, \mathcal{F})$ is called a bipolar interval valued multi fuzzy generalized semi-preopen set (BIVMFGPOS) in $S$.

Definition 2.25[16]. A BIVMFTS $(S, \mathcal{F})$ is called a bipolar interval valued multi fuzzy semi-pre $T_{1/2}$ space (BIVMFSPT$_{1/2}$), if every BIVMFGSPCS is a BIVMFSPCS in $S$.

Definition 2.26. Let $(S, \mathcal{F})$ and $(T, \mathcal{G})$ be BIVMFTs. Then a map $h: S \to T$ is called a (i) bipolar interval valued multi fuzzy continuous (BIVMF continuous) mapping if $h^{-1}(B)$ is BIVMFOS in $S$ for all BIVMFOS $B$ in $T$.

(ii) a bipolar interval valued multi fuzzy closed mapping (BIVMFC mapping) if $h(B)$ is a BIVMFC in $T$ for each BIVMFC $B$ in $T$.

(iii) bipolar interval valued multi fuzzy semi-closed mapping (BIVMFS mapping) if $h(B)$ is a BIVMFS in $T$ for each BIVMFS $B$ in $S$.

(iv) bipolar interval valued multi fuzzy preclosed mapping (BIVMFPC mapping) if $h(B)$ is a BIVMFPCS in $T$ for each BIVMFCS $B$ in $S$.

(v) bipolar interval valued multi fuzzy semi-open mapping (BIVMFOS mapping) if $h(B)$ is a BIVMFOS in $T$ for each BIVMFOS $B$ in $S$.

(vi) bipolar interval valued multi fuzzy generalized semi-preopen mapping (BIVMFGSPC mapping) if $h(B)$ is a BIVMFGSPC in $T$ for each BIVMFCS $B$ in $S$.

(vii) bipolar interval valued multi fuzzy generalized semi-preclosed mapping (BIVMFGSPCS mapping) if $h(B)$ is a BIVMFGSPCS in $T$ for each BIVMFCS $B$ in $S$.

Theorem 2.27. For any BIVMFS $B$ in $(S, \mathcal{F})$ where $S$ is a BIVMFSPT$_{1/2}$ space, $B \in \text{BIVMFGSP}(S)$ if and only if for every BIVMF $p(\alpha, \beta) \in B$, there exists a BIVMFGPOS $C$ in $S$ such that $p(\alpha, \beta) \in C \subseteq B$.

Definition 2.28[14]. Let $\alpha \in D[0,1]$ and $\beta \in D[-1,0]$, where $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ and $\beta = (\beta_1, \beta_2, ..., \beta_n)$. A bipolar interval valued multi fuzzy point (BIVMFP), written as $P_{(\alpha, \beta)}$, is defined to be a BIVMF of $X$ is given by

$$P_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p \\ (0,0) & \text{otherwise.} \end{cases}$$

3. SOME PROPERTIES:

Based on the paper Selvam.R et al.[16], the following Theorems are stated and proved.

Definition 3.1. A mapping $j: (S, \mathcal{F}) \to (T, \mathcal{G})$ is called an bipolar interval valued multi fuzzy generalized semiprecontinuous (BIVMFGSP continuous for short) mapping if $j^{-1}(N)$ is a BIVMFGSPCS in $(S, \mathcal{F})$ for every BIVMFCS $N$ of $(T, \mathcal{G})$.

Example 3.2. Let $S = \{k, l\}$, $T = \{m, n\}$ and $H_1 = \{k, [0.4, 0.4], [0.5, 0.5], [0.6, 0.6], [-0.5, -0.5], [-0.6, -0.6], [-0.7, -0.7] \}$, $\langle l, [0.3, 0.3], [0.4, 0.4], [0.5, 0.5], [-0.4, -0.4], [-0.5, -0.5], [-0.6, -0.6] \rangle$ and $H_2 = \{m, [0.5, 0.5], [0.6, 0.6], [0.7, 0.7], [-0.5, -0.5], [-0.6, -0.6], [-0.7, -0.7] \}$, $\langle n, [0.6, 0.6], [0.7, 0.7], [0.8, 0.8], [-0.7, -0.7], [-0.8, -0.8], [-0.9, -0.9] \rangle$. Then $\mathcal{F} = \{H_1, H_2\}$ and $\mathcal{G} = \{\emptyset, H_1, H_2, T\}$ are BIVMFTs on $S$ and $T$ respectively. Define a mapping $j: (S, \mathcal{F}) \to (T, \mathcal{G})$ by $j(k) = m$ and $j(l) = n$. Then $j$ is a BIVMFGSP continuous mapping.

Theorem 3.3. Every BIVMF continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let $j: (S, \mathcal{F}) \to (T, \mathcal{G})$ be a BIVMF continuous mapping. Let $N$ be a BIVMFCS in $T$. Then $j^{-1}(N)$ is a BIVMFC in $S$. Since every BIVMFCS is a BIVMFGSPCS, $j^{-1}(N)$ is a BIVMFGSPCS in $S$. Hence $j$ is a BIVMFGSP continuous mapping.
Remark 3.4. The converse of the above theorem 3.3 need not be true.

Proof. Consider the following example: Let $S = \{ k, l \}$, $T = \{ m, n \}$ and $H_1 = \{ \langle k, \langle 0.3, 0.3 \rangle, \langle 0.4, 0.4 \rangle, \langle 0.5, 0.5 \rangle, \langle -0.4, -0.4 \rangle, \langle -0.5, -0.5 \rangle \rangle, \langle l, \langle 0.5, 0.5 \rangle, \langle 0.6, 0.6 \rangle, \langle 0.7, 0.7 \rangle \rangle \}$, $H_2 = \{ \langle m, \langle 0.4, 0.4 \rangle, \langle 0.5, 0.5 \rangle \rangle, \langle n, \langle 0.6, 0.6 \rangle, \langle 0.7, 0.7 \rangle \rangle \}$. Then $\exists \{ 0.5, H_1, 1 \} \subset S$ and $\forall \{ 0.5, H_2, 1 \} \subset T$ respectively. Define a mapping $j : (S, 9) \to (T, \psi)$ by $j(k) = m$ and $j(l) = n$. Then $j$ is a BIVMFGSP continuous mapping but not a BIVMFG continuous mapping.

Theorem 3.5. Every BIVMFG continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let $j : (S, 9) \to (T, \psi)$ be a BIVMFG continuous mapping. Let $N$ be a BIVMFCS in $T$. Then $j^*(N)$ is a BIVMFCS in $S$. Since every BIVMFCS is a BIVMFGSPCS, $j^*(N)$ is a BIVMFGSPCS in $S$. Hence $j$ is a BIVMFGSP continuous mapping.

Remark 3.6. The converse of the above theorem 3.5 need not be true.

Proof. Consider the following example: Let $S = \{ k, l \}$, $T = \{ m, n \}$ and $H_1 = \{ \langle k, \langle 0.2, 0.2 \rangle, \langle 0.3, 0.3 \rangle, \langle 0.4, 0.4 \rangle, \langle 0.5, 0.5 \rangle, \langle -0.4, -0.4 \rangle, \langle -0.5, -0.5 \rangle \rangle, \langle l, \langle 0.3, 0.3 \rangle, \langle 0.4, 0.4 \rangle, \langle 0.5, 0.5 \rangle, \langle -0.4, -0.4 \rangle \}$. Then $\exists \{ 0.5, H_1, 1 \} \subset S$ and $\forall \{ 0.5, H_2, 1 \} \subset T$ respectively. Define a mapping $j : (S, \sigma) \to (T, \psi)$ by $j(k) = m$ and $j(l) = n$. Then $j$ is a BIVMFG continuous mapping but not a BIVMFGSP continuous mapping. Since $H_2 = \{ \langle m, \langle 0.5, 0.5 \rangle \rangle, \langle n, \langle 0.6, 0.6 \rangle \rangle, \langle 0.7, 0.7 \rangle \}$. Then $H_2 = \{ \langle m, \langle 0.5, 0.5 \rangle \rangle, \langle n, \langle 0.6, 0.6 \rangle \rangle, \langle 0.7, 0.7 \rangle \}$. But $bivmfc(l_j(H_2)) = H_2 \subset H_1$. Therefore $j^*(N)$ is not a BIVMFGCS in $S$.

Theorem 3.7. Every BIVMFS continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let $j : (S, 9) \to (T, \psi)$ be a BIVMFS continuous mapping. Let $N$ be a BIVMFCS in $T$. Then $j^*(N)$ is a BIVMFCS in $S$. Since every BIVMFCS is a BIVMFGSPCS, $j^*(N)$ is a BIVMFGSPCS in $S$. Hence $j$ is a BIVMFGSP continuous mapping.

Remark 3.8. The converse of the above theorem 3.7 need not be true.

Proof. Consider the following example: Let $S = \{ k, l \}$, $T = \{ m, n \}$ and $H_1 = \{ \langle k, \langle 0.4, 0.4 \rangle, \langle 0.5, 0.5 \rangle \rangle, \langle l, \langle 0.5, 0.5 \rangle \rangle \}$, $H_2 = \{ \langle m, \langle 0.5, 0.5 \rangle \rangle, \langle n, \langle 0.6, 0.6 \rangle \rangle, \langle 0.7, 0.7 \rangle \}$. Then $\exists \{ 0.5, H_1, 1 \} \subset S$ and $\forall \{ 0.5, H_2, 1 \} \subset T$ respectively. Define a mapping $j : (S, 9) \to (T, \psi)$ by $j(k) = m$ and $j(l) = n$. Then $j$ is a BIVMFGSP continuous mapping but not a BIVMFGCS continuous mapping. Since $H_2 = \{ \langle m, \langle 0.5, 0.5 \rangle \rangle, \langle n, \langle 0.6, 0.6 \rangle \rangle, \langle 0.7, 0.7 \rangle \}$. But $bivmfc(l_j(H_2)) = bivmfc(l_j(H_2)) = bivmfc(l_j(H_2))$ is not a BIVMFGCS in $S$.

Theorem 3.9. Every BIVMFP continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let $j : (S, 9) \to (T, \psi)$ be a BIVMFP continuous mapping. Let $N$ be a BIVMFCS in $T$. Then $j^*(N)$ is a BIVMFCS in $S$. Since every BIVMFCS is a BIVMFGSPCS, $j^*(N)$ is a BIVMFGSPCS in $S$. Hence $j$ is a BIVMFGSP continuous mapping.

Remark 3.10. The converse of the above theorem 3.9 need not be true.

Proof. Consider the following example: Let $S = \{ k, l \}$, $T = \{ m, n \}$ and $H_1 = \{ \langle k, \langle 0.4, 0.4 \rangle, \langle 0.5, 0.5 \rangle \rangle, \langle l, \langle 0.5, 0.5 \rangle \rangle \}$, $H_2 = \{ \langle m, \langle 0.5, 0.5 \rangle \rangle, \langle n, \langle 0.6, 0.6 \rangle \rangle, \langle 0.7, 0.7 \rangle \}$. Then $\exists \{ 0.5, H_1, 1 \} \subset S$ and $\forall \{ 0.5, H_2, 1 \} \subset T$ respectively. Define a mapping $j : (S, 9) \to (T, \psi)$ by $j(k) = m$ and $j(l) = n$. Then $j$ is a BIVMFGSP continuous mapping but not a BIVMFGCS continuous mapping. Since $H_2 = \{ \langle m, \langle 0.5, 0.5 \rangle \rangle, \langle n, \langle 0.6, 0.6 \rangle \rangle, \langle 0.7, 0.7 \rangle \}$. But $bivmfc(l_j(H_2)) = bivmfc(l_j(H_2)) = bivmfc(l_j(H_2))$ is not a BIVMFGCS in $S$. Therefore $j^*(N)$ is not a BIVMFGCS in $S$.
Theorem 3.11. Every BIVMFSP continuous mapping is a BIVMFGSP continuous mapping.

Proof. Let \( j : (S, \vartheta) \to (T, \psi) \) be a BIVMFSP continuous mapping. Let \( n \) be a BIVMFS in \( T \). Then \( j^{-1}(\vartheta) = \{ n, 0.5, 0.5 \} \) is a BIVMFCS in \( S \), because there exist no BIVMFPCS \( C \) in \( S \) such that \( \text{bivmfint}(C) \subset \vartheta \) and \( j(\vartheta) = n \). Then \( j \) is a BIVMFGSP continuous mapping. Since \( H_S = \{ (k, 0.5, 0.5), [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4] \} \). Then \( \vartheta = (0, H_S, 1_\vartheta) \) is a BIVMFCS in \( T \) and \( j(\vartheta) = n \). Then \( j \) is a BIVMFGSP continuous mapping but not a BIVMFSP continuous mapping. Since \( H_T = \{ (k, 0.5, 0.5), [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4] \} \). Then \( \vartheta = (0, H_T, 1_\psi) \) is a BIVMFCS in \( S \) and \( j(\vartheta) = n \). Then \( j \) is a BIVMFGSP continuous mapping but not a BIVMFSP continuous mapping. Since \( H_S = \{ (k, 0.5, 0.5), [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4] \} \). Then \( \vartheta = (0, H_S, 1_\vartheta) \) is a BIVMFCS in \( T \) and \( j(\vartheta) = n \). Then \( j \) is a BIVMFGSP continuous mapping but not a BIVMFSP continuous mapping. Since \( H_T = \{ (k, 0.5, 0.5), [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4], [0.3, 0.3], [0.4, 0.4] \} \). Then \( \vartheta = (0, H_T, 1_\psi) \) is a BIVMFCS in \( S \), because there exist no BIVMFPCS \( C \) in \( S \) such that \( \text{bivmfint}(C) \subset \vartheta \) and \( j(\vartheta) = n \). Then \( j \) is a BIVMFGSP continuous mapping.
Theorem 3.17. Let \( j : (S, \Omega) \rightarrow (T, \Psi) \) be a mapping, where \( j^{-1}(N) \) is a BIVMFRCs in \( S \) for every BIVMFCS \( N \) in \( T \). Then \( j \) is a BIVMFGSP continuous mapping.

Proof. Assume that \( j : (S, \Omega) \rightarrow (T, \Psi) \) is a mapping. Let \( B \) be a BIVMFCS in \( T \). Then \( j^{-1}(N) \) is a BIVMFRCs in \( S \), by hypothesis. Since every BIVMFRCs is a BIVMFGSPCS, \( j^{-1}(N) \) is a BIVMFGSPCS in \( S \). Hence \( j \) is a BIVMFGSP continuous mapping.

Remark 3.18. The converse of the above theorem 3.17 need not be true.

Proof. Consider the following example: Let \( S = \{ k, l \} \), \( T = \{ m, n \} \) and \( H_1 = \{ (k, [0.4, 0.4]), (l, [0.5, 0.5]), (m, [0.6, 0.6]), (n, [0.7, 0.7]) \} \). Then \( j(H_1) = \{ (k, [0.6, 0.6]), (l, [0.7, 0.7]) \} \) is a BIVMFG continuous mapping but not a mapping as defined in Theorem 2.17, since \( H_2 = \{ (k, [0.5, 0.5]), (l, [0.6, 0.6]) \} \) and \( j(H_2) = \{ (k, [0.5, 0.5]), (l, [0.6, 0.6]) \} \) is not a BIVMFRCs in \( S \), because \( \text{bivmfcl}(\text{bivmfint}(j^{-1}(H_2^c))) \neq \text{bivmfcl}(H_1) \).

Theorem 3.19. If \( j : (S, \Omega) \rightarrow (T, \Psi) \) is a BIVMFGSP continuous mapping, then for each BIVMFP \( p_{(\alpha, \beta)} \) of \( S \) and each \( B \in \Psi \) such that \( j(p_{(\alpha, \beta)}) \in B \), there exists a BIVMFGSPOS \( C \) of \( S \) such that \( p_{(\alpha, \beta)} \in C \) and \( j(C) \subseteq B \).

Proof. Let \( p_{(\alpha, \beta)} \) be a BIVMFP of \( S \) and \( B \in \Psi \) such that \( j(p_{(\alpha, \beta)}) \in B \). Then hypothesis, \( C \) is a BIVMFGSPOS in \( S \) such that \( p_{(\alpha, \beta)} \in C \) and \( j(C) = j^{-1}(B) \subseteq B \).

Theorem 3.20. If \( j : (S, \Omega) \rightarrow (T, \Psi) \) is a BIVMFSP continuous mapping. Then \( j \) is a BIVMFSPT \( \frac{1}{2} \) space.

Proof. Let \( N \) be a BIVMFCS in \( T \). Then \( j^{-1}(N) \) is a BIVMFGSPCS in \( S \), by hypothesis. Since \( S \) is a BIVMFSP\( \frac{1}{2} \) space, \( j^{-1}(N) \) is a BIVMFGSPCS in \( S \). Hence \( j \) is a BIVMFGSP continuous mapping.

Theorem 3.21. Let \( j : (S, \Omega) \rightarrow (T, \Psi) \) be a BIVMFSP continuous mapping and let \( \sigma : (T, \Psi) \rightarrow (Q, \Omega) \) be BIVMF continuous mapping where \( T \) is a BIVMF\( \frac{1}{2} \)-space. Then \( \sigma \circ j : (S, \Omega) \rightarrow (Q, \Omega) \) is a BIVMFGSP continuous mapping.

Proof. Let \( N \) be a BIVMFCS in \( T \). Then \( j^{-1}(N) \) is a BIVMFGSPOS in \( S \), by hypothesis. Since \( T \) is a BIVMF\( \frac{1}{2} \) space, \( j^{-1}(N) \) is a BIVMFGSPOS in \( S \). Therefore \( j^{-1}(n) \) is a BIVMFGSPOS in \( S \), by hypothesis. Hence \( j^{-1}(n) \) is a BIVMFGSP continuous mapping.

Theorem 3.22. Let \( j : (S, \Omega) \rightarrow (T, \Psi) \) be a BIVMFSP continuous mapping and let \( \sigma : (T, \Psi) \rightarrow (Q, \Omega) \) be a BIVMF continuous mapping, then \( \sigma \circ j : (S, \Omega) \rightarrow (Q, \Omega) \) is a BIVMFGSP continuous mapping.

Proof. Let \( N \) be a BIVMFCS in \( T \). Then \( j^{-1}(N) \) is a BIVMFGSPOS in \( S \), by hypothesis. Since \( T \) is a BIVMF\( \frac{1}{2} \) space, \( j^{-1}(N) \) is a BIVMFGSPOS in \( S \). Hence \( j^{-1}(N) \) is a BIVMFGSP continuous mapping.

Theorem 3.23. Let \( j : (S, \Omega) \rightarrow (T, \Psi) \) be a mapping from a BIVMF\( S \) into a BIVMF\( T \). Then the following conditions are equivalent if \( S \) and \( T \) are BIVMF\( \frac{1}{2} \) spaces.

(i) \( j \) is a BIVMFSP continuous mapping,

(ii) \( j^{-1}(C) \) is a BIVMFSP in \( S \) for each BIVFOS \( C \) in \( T \),

(iii) for every BIVMFP \( p_{(\alpha, \beta)} \) in \( S \) and for every BIVFOS \( C \) in \( T \) such that \( j(p_{(\alpha, \beta)}) \in C \), there exists a BIVMFSP in \( S \) such that \( p_{(\alpha, \beta)} \in C \) and \( j(C) \subseteq C \).

Proof. (i) \( \Leftrightarrow \) (ii) is an obvious, since \( j^{-1}(B^c) = (j^{-1}(B))^c \).
(ii) ⇒ (iii) Let C any BIVMFOS in T and let \( p_{(\alpha, \beta)} \in E^X \). Given \( j(p_{(\alpha, \beta)}) \in C \). By hypothesis \( j^{-1}(C) \) is a BIVMFGSPOS in S. Take \( B = j^{-1}(C) \). Now \( p_{(\alpha, \beta)} \in j^{-1}((p_{(\alpha, \beta)})) \). Therefore \( j^{-1}(j(p_{(\alpha, \beta)})) \in j^{-1}(C) = B \). This implies \( (p_{(\alpha, \beta)}) \in B \) and \( j(B) = j^{-1}(C) \subseteq C \).

(iii) ⇒ (i) Let \( B \) be a BIVMFCS in T. Then its complement, say \( C = B' \), is a BIVMFOS in T. Let \( p_{(\alpha, \beta)} \in E^X \) and \( j((p_{(\alpha, \beta)})) \in C \). Then there exists a BIVMFGSPOS, say \( D \) in S such that \( p_{(\alpha, \beta)} \in D \) and \( j(D) \subseteq C \). Now \( D \subseteq j^{-1}(j(D)) \subseteq j^{-1}(C) \). Thus \( p_{(\alpha, \beta)} \in j^{-1}(C) \). Therefore \( j^{-1}(C) \) is a BIVMFGSPOS in S, by Theorem 2.27. That is \( j^{-1}(B) \) is a BIVMFGSPOS in S and hence \( j^{-1}(B) \) is a BIVMFGSPCS in S. Thus \( j \) is a BIVMFGSP continuous mapping.

**Theorem 3.24.** Let \( j : (S, \gamma) \to (T, \psi) \) be a mapping from a BIVMFT S into a BIVMFT T. Then the following conditions are equivalent if S and T are BIVMFSP\(_{1/2} \) spaces.

(i) \( j \) is a BIVMFGSP continuous mapping,

(ii) for each BIVMFP \( p_{(\alpha, \beta)} \) in S and for every BIVMFN \( B \) of \( j(p_{(\alpha, \beta)}) \), there exists a BIVMFGSPS C in S such that \( p_{(\alpha, \beta)} \in C \subseteq j^{-1}(B) \),

(iii) for each BIVMFP \( p_{(\alpha, \beta)} \) in S and for every BIVMFN \( B \) of \( j(p_{(\alpha, \beta)}) \), there exists a BIVMFGSPOS C in S such that \( p_{(\alpha, \beta)} \in C \subseteq j^{-1}(B) \).

**Proof.** (i) ⇒ (ii) Let \( p_{(\alpha, \beta)} \in E^X \) and let \( B \) be a BIVMFN of \( j(p_{(\alpha, \beta)}) \). Then there exists a BIVMFGSPOS \( D \) in T such that \( j(p_{(\alpha, \beta)}) \in D \subseteq B \). Since \( j \) is a BIVMFGSP continuous mapping, \( j^{-1}(D) = C \) (say), is a BIVMFGSPOS in S and \( p_{(\alpha, \beta)} \in C \subseteq j^{-1}(B) \).

(ii) ⇒ (iii) Let \( p_{(\alpha, \beta)} \in E^X \) and let \( B \) be a BIVMFN of \( j(p_{(\alpha, \beta)}) \). Then there exists a BIVMFGSPOS \( C \) in S such that \( p_{(\alpha, \beta)} \in C \subseteq j^{-1}(B) \), by hypothesis. Therefore \( p_{(\alpha, \beta)} \in C \subseteq j^{-1}(B) \).

(iii) ⇒ (i) Let \( C \) be any BIVMFOS in T and let \( p_{(\alpha, \beta)} \in j^{-1}(C) \). Then \( j(p_{(\alpha, \beta)}) \in C \). Therefore \( C \) is a BIVMFOS of \( j(p_{(\alpha, \beta)}) \). Since \( C \) is BIVMFOS, by hypothesis there exists a BIVMFGSPOS \( B \) in S such that \( p_{(\alpha, \beta)} \in B \subseteq j^{-1}(B) \subseteq j^{-1}(C) \). Therefore \( j^{-1}(C) \) is a BIVMFGSPOS in S, by Theorem 2.27. Hence \( j \) is a BIVMFGSP continuous mapping.

**Theorem 3.25.** Let \( j : (S, \gamma) \to (T, \psi) \) be a mapping from a BIVMFT S into a BIVMFT T. Then the following conditions are equivalent if S is a BIVMFSP\(_{1/2} \) space.

(i) \( j \) is a BIVMFGSP continuous mapping,

(ii) if \( C \) is a BIVMFS in T then \( j^{-1}(C) \) is a BIVMFGSPOS in S,

(iii) \( j^{-1}(\ bivmfint(C)) \subseteq bivmfcl(bivmfcl(j^{-1}(C))) \) for every BIVMFS C in T.

**Proof.** (i) ⇔ (ii) is obviously true by Theorem 3.23.

(ii) ⇒ (iii) Let \( C \) be any BIVMFS in T. Then \( \text{bivmfint}(C) \) is a BIVMFS in T. Then \( j^{-1}( \text{bivmfint}(C) ) \) is a BIVMFGSPOS in S. Since S is a BIVMFS\(_{1/2} \) space, \( j^{-1}( \text{bivmfint}(C) ) \) is a BIVMFGSPOS in S. Therefore \( j^{-1}( \text{bivmfint}(C)) \subseteq \text{bivmfcl}(\text{bivmfcl}(j^{-1}(\text{bivmfint}(C)))) \). Since \( j^{-1}(C) \) is a BIVMF\(_{1/2} \) space, \( j^{-1}(C) \subseteq \text{bivmfcl}(\text{bivmfcl}(j^{-1}(C))) \).

(iii) ⇒ (i) Let \( C \) be a BIVMFOS in T. By hypothesis \( j^{-1}(C) = j^{-1}(\text{bivmfint}(C)) \subseteq \text{bivmfcl}(\text{bivmfcl}(j^{-1}(C)))) \). This implies \( j^{-1}(C) \) is a BIVMFGSPOS in S. Therefore, \( j \) is a BIVMFGSP continuous mapping, by Theorem 2.23.

**Theorem 3.26.** Let \( j : (S, \gamma) \to (T, \psi) \) be a mapping from a BIVMFT S into a BIVMFT T. Then the following conditions are equivalent if S and T are BIVMFSP\(_{1/2} \) space.

(i) \( j \) is a BIVMFGSP continuous mapping,

(ii) \( \text{bivmfint}( \text{bivmfcl}(j^{-1}(C))) \subseteq j^{-1}(\text{bivmfspcl}(C)) \) for each BIVMFS C in T,

(iii) \( j^{-1}(\text{bivmfspcl}(C)) \subseteq \text{bivmfcl}(\text{bivmfcl}(j^{-1}(C))) \) for each BIVMFS C of T,

(iv) \( j^{-1}(\text{bivmfcl}(\text{bivmfcl}(B))) \subseteq \text{bivmfcl}(j(B)) \) for each BIVMFS B of S.
Proof. (i) ⇒ (ii) Let C be a BIVMFCS in T. Then \( j^{-1}(C) \) is a BIVMFSPCS in S. Since S is a BIVMFSPT\(_{1/2}\) space, \( j^{-1}(C) \) is a BIVMFSPCS. Therefore \( \text{bivmfint}(\text{bivmfcl}( j^{-1}(C) )) \subseteq j^{-1}(C) = j^{-1}(\text{bivmfscl}(C)). \)

(ii) ⇒ (iii) It can be easily proved by taking complement in (ii).

(iii) ⇒ (iv) Let \( B \) be a BIVMFOS in T. Taking \( C = j(B) \) we have \( B \subseteq j^{-1}(C). \)

Here \( \text{bivmfint}(j(B)) = \text{bivmfint}(C) \) is a BIVMFOS in T. Then (iii) implies \( j^{-1}(\text{bivmfscl}(\text{bivmfcl}(C) )) \subseteq \text{bivmfsp}(\text{bivmfcl}(\text{bivmfcl}(j^{-1}(C) ))). \) Now we have \( \text{bivmfcl}(\text{bivmfcl}(B^{\uparrow})) \subseteq \text{bivmfcl}(j \circ \text{bivmfscl}(\text{bivmfcl}(C) )). \) This implies \( \text{bivmfcl}(j \circ \text{bivmfscl}(\text{bivmfcl}(C) ) \subseteq j^{-1}(\text{bivmfscl}(B)). \)

(iv) ⇒ (i) Let \( C \) be any BIVMFCS in T. Then \( j^{-1}(C) \) is a BIVMFS in S. By hypothesis \( \text{bivmfcl}(\text{bivmfcl}(\text{bivmfcl}(j^{-1}(C) )) \subseteq \text{bivmfcl}(j \circ \text{bivmfscl}(\text{bivmfcl}(B))). \) Now \( \text{bivmfcl}(\text{bivmfcl}(j^{-1}(C) )) \subseteq \text{bivmfcl}(j \circ \text{bivmfscl}(\text{bivmfcl}(B))). \)

Theorem 3.27. A mapping \( j : (S, \varnothing) \to (T, \psi) \) is a BIVMFGSP continuous mapping if \( \text{bivmfcl}(\text{bivmfcl}(\text{bivmfcl}(j^{-1}(B)))) \subseteq j^{-1}(\text{bivmfcl}(B)) \) for every BIVMFS \( B \) in T.

Proof. Let \( B \) be a BIVMFOS in T. Then \( B^{\uparrow} \) is a BIVMFCS in T. Therefore \( \text{bivmfcl}(B^{\uparrow}) \) is a BIVMFS in S. By hypothesis, \( \text{bivmfcl}(\text{bivmfcl}(B^{\uparrow})) \subseteq j^{-1}(\text{bivmfscl}(B^{\uparrow}) = j^{-1}(B^{\uparrow}). \) Now \( \text{bivmfcl}(\text{bivmfcl}(j^{-1}(B^{\uparrow}))) \subseteq j^{-1}(\text{bivmfscl}(B^{\uparrow}) \subseteq j^{-1}(B) \subseteq \text{bivmfcl}(\text{bivmfcl}(j^{-1}(B))). \) Hence \( j^{-1}(B) \) is a BIVMF\_fos in S and hence it is a BIVMFSPCS in S. Therefore \( j \) is a BIVMFGSP continuous mapping.

Theorem 3.28. Let \( j : (S, \varnothing) \to (T, \psi) \) be a mapping from a BIVMF S into a BIVMFT T. Then the following conditions are equivalent if S is a BIVMFSPT\(_{1/2}\) space.

(i) \( j \) is a BIVMF\_g spor\_c continuous mapping,

(ii) \( j^{-1}(C) \) is a BIVMFGSPCS in S for every BIVMFCS \( C \) in T,

(iii) \( \text{bivmfint}(\text{bivmfcl}(j^{-1}(B))) \subseteq j^{-1}(\text{bivmfcl}(B)) \) for every BIVMFS \( B \) in T.

Proof. (i) ⇔ (ii) is obvious from the Definition 3.1. (ii) ⇒ (iii) Let \( B \) be a BIVMFCS in T. Then \( \text{bivmfcl}(B) \) is an BIVMFCS in T. By hypothesis, \( j^{-1}(\text{bivmfcl}(B)) \) is a BIVMFS in S. Since S is a BIVMFSPT\(_{1/2}\) space, \( j^{-1}(\text{bivmfcl}(B)) \) is an BIVMFSPCS in (S, \( \varnothing \)) therefore we have \( \text{bivmfcl}(\text{bivmfcl}(j^{-1}(\text{bivmfcl}(B))) \subseteq j^{-1}(\text{bivmfcl}(B)). \) Now \( \text{bivmfcl}(\text{bivmfcl}(\text{bivmfcl}(j^{-1}(B)))) \subseteq j^{-1}(\text{bivmfcl}(B)). \) (iii) ⇒ (i) Let \( B \) be an BIVMFCS in T. By hypothesis \( \text{bivmfcl}(\text{bivmfcl}(\text{bivmfcl}(j^{-1}(B)))) \subseteq j^{-1}(\text{bivmfcl}(B)) = j^{-1}(B). \) Hence \( j^{-1}(B) \) is a BIVMFGSPCS in S and hence it is a BIVMF\_gos in S. Therefore \( j \) is a BIVMFGSP continuous mapping.

4. CONCLUSION:

We conclude that, every BIVMF continuous mapping, BIVMF\alpha continuous mapping, BIVMF\beta continuous mapping, BIVMFP continuous mapping, BIVMFS continuous mapping are an BIVMF\_g spor\_c continuous mapping. Also some equivalent condition Theorems are proved in this paper. Using this concept, we can develop some new theorems and properties.

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