

FORCED DAMPED VIBRATION ANALYSIS OF OPTIMIZED CANTILEVER BEAM USING DIFFERENT METHODS

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Abstract - The Paper focuses on the study of the vibration analysis of different materials that can be used for manufacturing of the engine casing i.e. structural steels (namely Mild Steel and Stainless Steel) and aluminum or its alloys. The objective of the paper is to identify a suitable material for the manufacturing of Engine casing considering the parameters such as Material of the beam to be used, weight of the beam, Damping Co-efficient, Spring stiffness and the natural frequency of the material using the Taguchi Method (Regression Analysis) the effect of several variables which are involved in the experimental setup of Forced damped vibrations such as Material of the beam to be used, weight of the beam, Damping Co-efficient, Spring stiffness and the natural frequency of the material etc. The natural frequency of the optimized material is then evaluated and verified using theoretical, experimental methods (FFT) and software (Ansys).

Key Words: Cantilever Beam, Forced Vibration, Regression Analysis, FFT.

1. INTRODUCTION

Horizontal cantilever beam is a beam, which carries transverse load at its end point. Due to exciting load the vibration occurs in the beam. We know that, when the shaft or disc moves approximately perpendicular to the axis of shaft, then the vibrations are called transverse vibrations. If the body vibrates under influence of external force, then the body is under forced vibration. In case of forced vibrations, an external periodic disturbing force keeps the system vibrating. The vibrations of air compressors, internal combustion engines, machine tools and various other machinery are all examples of forced vibrations.

In some cases, the excitation is non deterministic or random. This will mean that the excitation magnitude at any instance is unknown. In such situations, sufficient data is required statistically determine the excitation characteristics. Some of the examples of random vibration are the excitations produced by winds, seismic activities and road surfaces.

If the value or magnitude of the excitation is known at any given time, the excitation is called deterministic and resulting vibration is called as deterministic vibration. When the external force is SHM, the corresponding vibrations are said to be with harmonic excitation. Many mechanical systems are subjected to forced vibrations. Such systems normally do have some form of damping in the system.

Forced vibration can be explained as when an alternating force or motion is applied to a mechanical system. Examples of this type of vibration include a washing machine imbalance, vibration during transportation (by engine, springs, road, etc), or the vibration of a building during an earthquake. In case of forced vibration the vibrating frequency is the frequency of the force applied, with its magnitude dependent on the actual mechanical system. The Engine casing particularly is subjected to forced damped vibration and becomes essential to study the effect of damped vibrations using different types of material with different densities, springs having different spring stiffness and oils with different damping co-efficient.

1.1 Problem Statement

For many years Cast Iron has been used for the manufacturing of engine casing, it is due to its low cost and it is due to its strength and low cost and its wear resistance. But as the engine become more complicated engineers found new materials to reduce its weight as well as to increase strength and wear resistance such as structural steel & aluminum alloy.

For Engine application engine is to be operated at high speed with min amplitude of vibration. Hence it becomes very essential that the proper material is selected for Engine application and its behavior is studied under forced damped vibration (which is the typical working conditions of the engine).

1.2 Methodology

1. Beam Material Selection
2. Taguchi Method of Orthogonal Arrays using Minitab Software
3. Analytical Method
4. Experimental Analysis
5. Analysis using FFT Analyzer
6. Analysis using Ansys Software
7. Comparison of Results

1.3 Objective

1. To do the regression analysis considering three different density beam material, three oils with different damping coefficient and three spring stiffness.
2. Selection of the optimum parameters such as material, damping coefficient, and spring stiffness.
3. Development of the experimental setup for forced damped vibration of Cantilever beam.
4. Vibration analysis of the beam considering optimum material, damping coefficient and spring stiffness.
5. Obtaining results by theoretical and numerical method with the help of FFT Analyzer Forced vibration of cantilever beam.
6. To obtain frequency response of the cantilever beam under the excitation force and damped conditions.
7. To validate the results using theoretical method, FFT Analyzer and Ansys.

Runs: 9

Columns of L9(3^4) Array

1 2 3

Response Table for Signal to Noise Ratios

Smaller is better

Level	M	C	K
1	-13.80	-14.03	-14.26
2	-13.98	-14.12	-14.01
3	-14.45	-14.09	-13.97
Delta	0.65	0.09	0.29
Rank	1	3	2

Response Table for Means

Level	M	C	K
1	4.900	5.030	5.163
2	5.003	5.083	5.023
3	5.280	5.070	4.997
Delta	0.380	0.053	0.167
Rank	1	3	2

2. TAGUCHI METHOD OF ORTHOGONAL ARRAYS

The goal of experimental work is to find out the best material from Mild Steel, Stainless Steel (SS306) and Aluminum alloy (3003) in manufacturing of and engine casing which usually faces forced damped vibrations. This is done by analyzing the parameters that have direct effect such as Damping Co-efficient of the Oil used for damping, Spring Stiffness and the natural frequency of the material used which influencing optimum selection of the material.

Parameters

1. Mass of the beam- e.g. m1, m2, m3
2. Damping Co-efficient - e.g.C1, C2, C3
3. Spring Stiffness – K1, K2, K3

Table -1: Process Parameters with their values at corresponding levels

Process Parameters	Level 1	Level 2	Level 3
Mass of the beam(m) in Kg	2.07	2.11	0.71
Damping Co-efficient (C) in N/mm	0.69	0.78	0.86
Spring Stiffness (K) in N/m	146 X 10 ²	147.15X 10 ²	148 X 10 ²

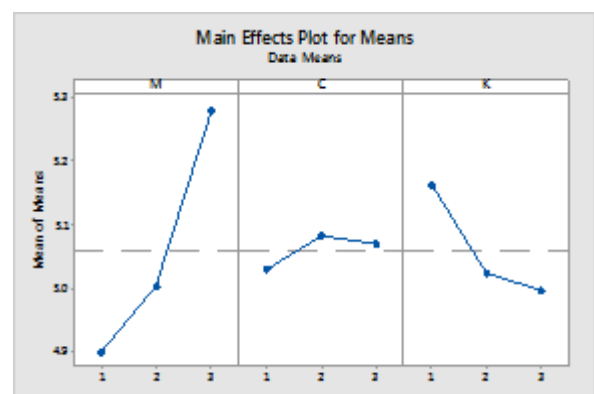


Chart.1 Main Effect Plot for SN Means

2.1 Taguchi orthogonal array design by MINITAB software:

L9(3^3)

Factors: 3

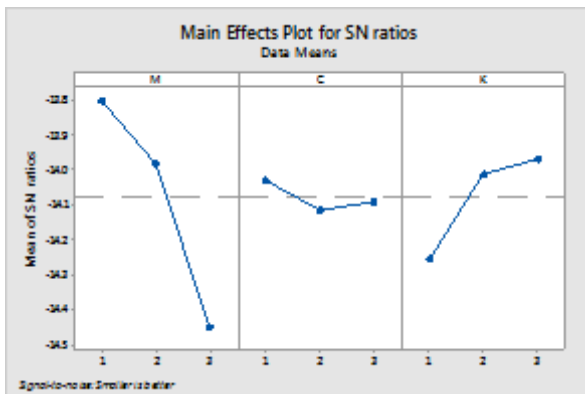


Chart.2 Main Effect Plot for SN ratios

Regression Equation

$$W_n = 4.808 + 0.1900 M + 0.0200 C - 0.0833 K$$

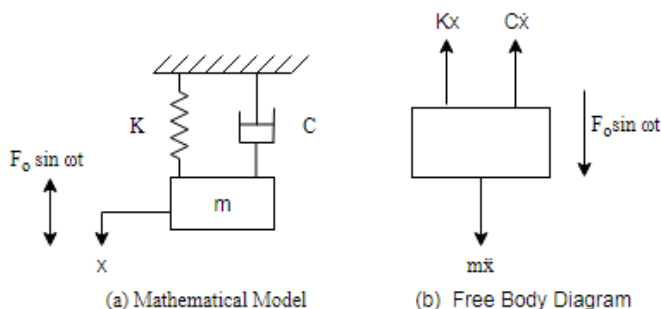
From signal to noise ratio (which is minimum) it's cleared that M3 material means Aluminum and C2 and K1 are the optimum parameter to be used in application.

3. ANALYTICAL METHOD:

3.1 Mathematical Model of Forced Damped Vibrating System

The System comprises of the following elements,

- a. Mass (M)
- b. Spring of Stiffness (K)
- c. Damper having damping coefficient (C)
- d. An external periodic vibrating force having magnitude F_0 and frequency ω



Forced Damped System

Fig.1 Mathematical model of Spring-Mass-Damper [7]

The free body diagram of the mass is shown in the Fig 1. The equation of motion can be worked out using the FBD method.

From Newton's Law of Motion,

$$m\ddot{x} + C\dot{x} + Kx = F_0 \sin \omega t \quad (1)$$

Inertia Force + Damping Force + Stiffness Force = External Force

This is linear, non-homogenous, second order differential equation. The solution of this equation consists of complementary function and particular integral.

$$X = X_c + X_p \quad (2)$$

Complimentary Solution,

$$X_c = Me^{-\omega n \xi t} \sin(\omega d t + \phi)$$

Where, damped frequency

$$\omega d = \omega n \sqrt{1 - \xi^2}$$

$$\text{Particular Solution, } X_p = X \sin(\omega t - \phi) \quad (3)$$

3.2 Analytical Analysis:

Below diagram shows Mathematical model of Cantilever beam with spring-mass-damper

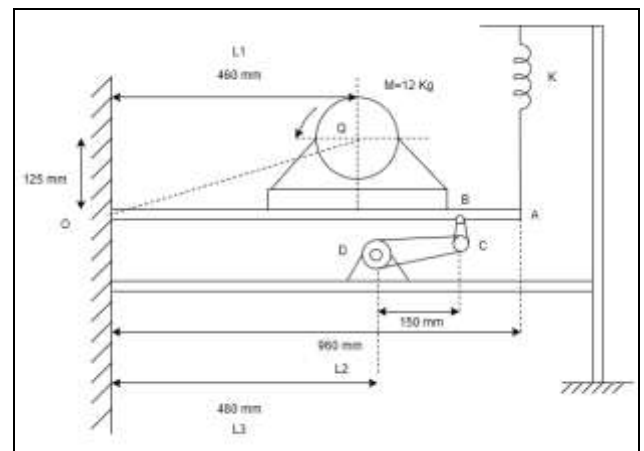


Fig.2 Model for Cantilever beam with spring-mass-damper

$$I_A = ML_1 + \frac{mL_2^2}{3}$$

$$I = 2.74 \text{ Kg m}^2$$

$$K = 147.15 \times 10^2 \text{ N/m}$$

$$\xi = 0.5$$

$$l(OQ) = \sqrt{460^2 + 125^2}$$

$$= 476.68 \text{ mm}$$

$$= 0.476 \text{ mm}$$

Consider the angular deflection of Stand and the damper as shown in the Fig .3. For Analysis, it is assumed that the angular deflections are small.

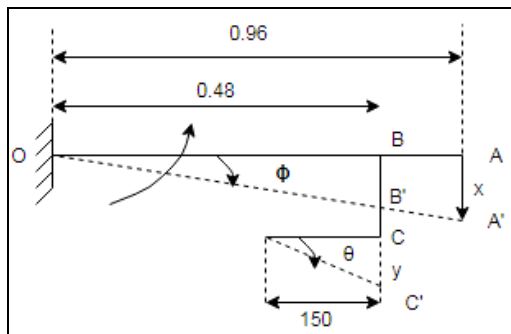


Fig . 3 Force analysis of cantilever beam

If the end of A deflects by A' by a distance x,

$$x = 0.96 \phi$$

$$\text{Or } \phi = 1.04 x$$

$$\phi' = 1.04 x'$$

$$\phi'' = 1.04 x'' \quad (4)$$

For this deflection, the points B and C move by a distance y to B' and C' respectively. The angular deflection theta of the damper will be the same as that of the link CD.

$$\text{Also, } y = 0.5 x = 0.15 \theta$$

$$\theta = 3.33 x$$

$$\theta' = 3.33 x' \quad (5)$$

The damping torque in the rotational damper about the damper axis would be $C_T \theta'$. This torque can be expressed in the form of force F as:

$$\text{Force} \times \text{Distance} = \text{Torque}$$

$$F \times 0.15 = C_T \theta' = 3.33 C_T x'$$

$$F = 22.2 C_T x' \quad (6)$$

This force acts at point Consider the Freebody diagram of the arrangement as shown in the fig 4

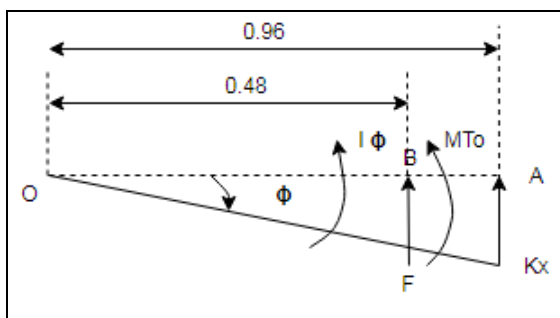


Fig.4. Free Body Diagram

From Newton's Law of Motion,

$$I\phi' + F \times 0.48 + K X \times 0.96 = m_T \ddot{\phi}$$

$$5.98 x'' + 10.65 C_T x' + 309.13 \times 10^2 x = m_e \omega^2 \sin \omega t \quad (7)$$

Hence,

$$M_1 = 5.98$$

$$C_1 = 10.65 C_T$$

$$K_1 = 309.13 \times 10^3$$

The natural frequency will be,

$$\omega_n = \sqrt{\frac{KL^2}{I_A}}$$

$$= 33.71 \text{ rad/sec} \quad (8)$$

$$= 5.36 \text{ Hz}$$

4. EXPERIMENTAL METHOD:

Below picture shows experimental setup of Cantilever beam with spring-mass-damper.



Fig.5 Experimental Setup of Cantilever beam with spring-mass-damper

The arrangement for this experimental set up is as shown in the picture. It consists of a rectangular beam supported at one end by pin pivoted in ball bearings which are located in a fixed housing. The other end of beam is supported by a spring of the known stiffness bolted to the bracket which is attached to the upper frame. This bracket allows fine adjustment of the spring, thus raising and dropping the end of the beam. The DC Motor rigidly fastens to the beam with

added masses placed on the platform attached. Two out of disks on the output shaft of the belt -driven unit provide the exciting force. The exciting frequency can be adjusted by the means of the speed control unit.

The Chart recorder fits to the left-hand vertical member of the frame and provides the means of obtaining a trace for the vibration. The recorder unit consists of slowly rotating drum driven by synchronous motor, operated from auxiliary supply on the speed control unit. A role of the recorder paper is next to the drum and is wound round the drum so that the paper is driven at constant speed. A felt -tilted pen fits to the free end of the beam; so that the pen just traces graph on the paper. By switching the motor on, we can obtain a trace to show the oscillatory motion of beam. Damping can be introduced with the help of dashpot assembly near the pivoted end of the beam.

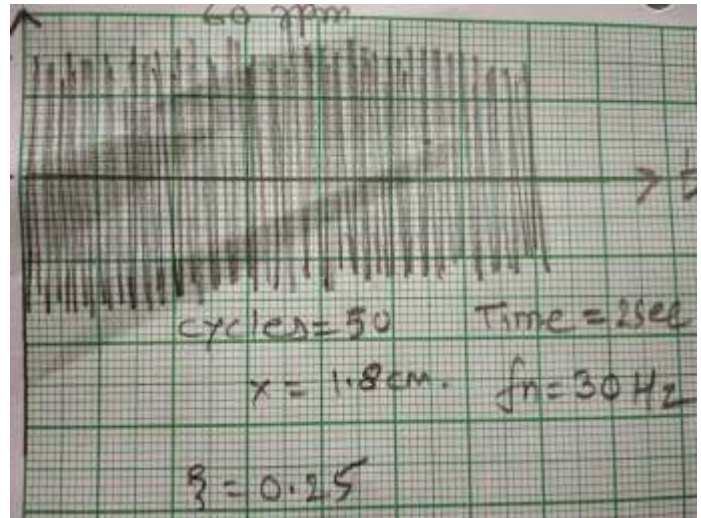


Chart.4 Time Graph, f3= 30 Hz

5. Analysis using FFT Analyzer

Frequency analysis based on the Fast-Fourier Transform (FFT) Algorithm is the tool of choice for measurement and diagnostic of vibration. The FFT Analyzer is recently developed pc based virtual instrument. It uses impulse execution and either frequency domain analysis or time - domain Analysis to entrant the model parameter from the response measurement in real time. Following impulse execution of the specimen, the measured analog response signal may be digitalized and analyzed using the domain techniques or transformed for analysis in the frequency domain using FFT Analyzer. The peaks in the frequency response spectrum are the location of natural frequency.

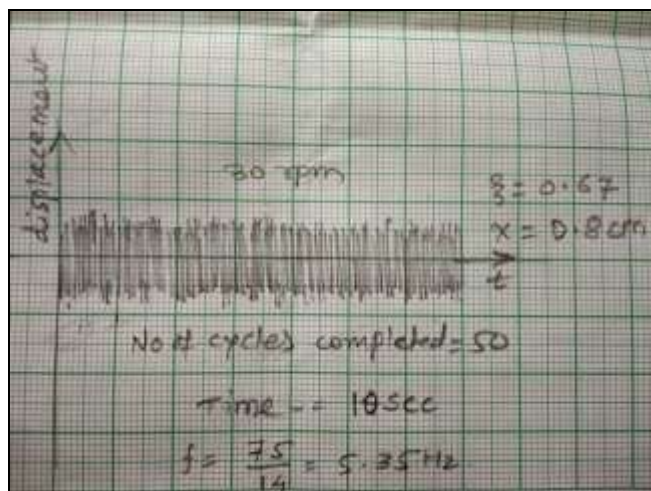


Chart.2 Time Graph, f1 = 5.35 Hz

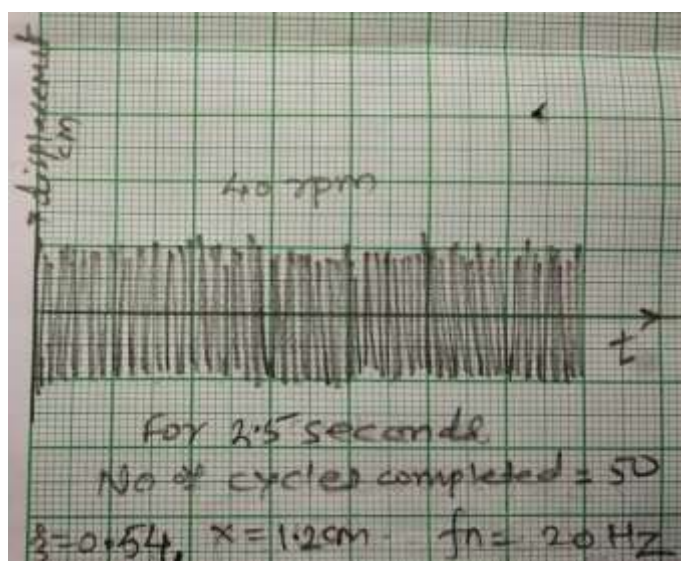


Chart.3 Time Graph, f2= 20 Hz



Fig.6 -Experimental Set up using FFT

FFT Graphs:

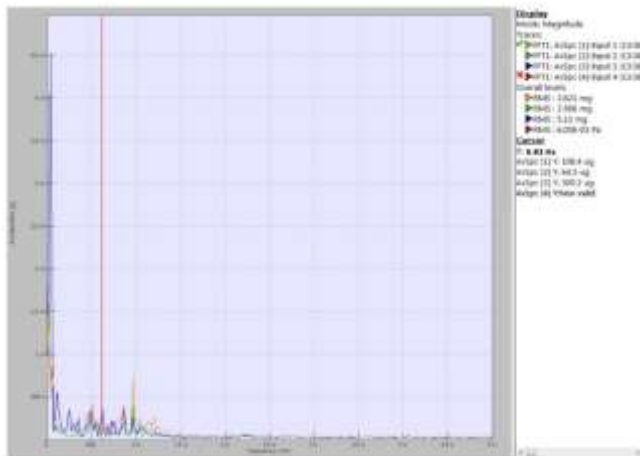


Chart.5 FFT reading observed for 30 rpm at frequency 5.83 Hz

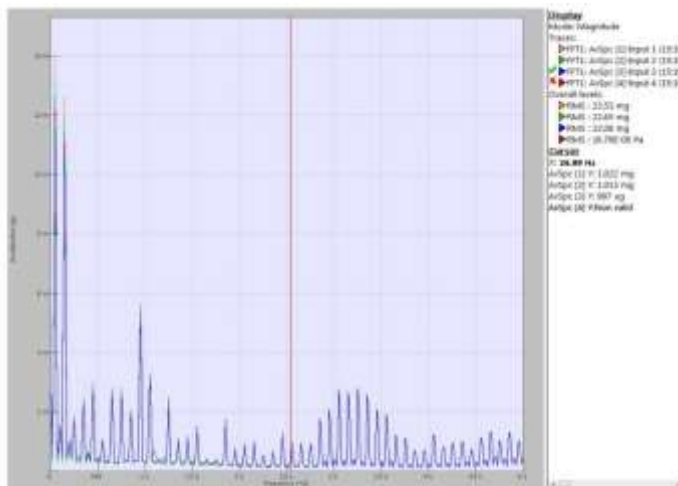


Chart.6 FFT reading observed for 40 rpm at frequency 26.89 Hz



Chart.6 FFT reading observed for 60 rpm at frequency 33.2 Hz

6. Analysis using Ansys Software (FEA):

The problem involves a structure or structural component subjected to certain loads. The conversion of the physical problem to a mathematical model requires certain assumptions that collectively lead to differential Eq.s governing the mathematical model. Since the finite element solution technique is a numerical procedure, it is required to access the solution correctness. If the accuracy criteria are not met, the numerical solution must be repeated with revised solution parameters until a sufficient precision is reached. Division of whole into parts, which allows illustration of geometrically complex domains as collection of simple domains that, enables a systematic derivation of the approximation functions. Derivation of approximation functions over each element the approximation functions are algebraic polynomials that are resultant of interpolation theory. Group of elements, which are based on continuity of the solution and equilibrium of internal fluxes.

Table -2: Experimental Parameters for Al Beam

Parameters	Symbol	Value
Material	AL	-
Total Effective Length	L	0.96 m
Width	B	0.023 m
Thickness	T	0.015 m
Moment of Inertia	I	2.74 Kg m ²
Young's Modulus	E	0.69 x10 ¹¹ N/m ²
Mass Density	ρ	2700 kg/m ³

A. Boundary conditions

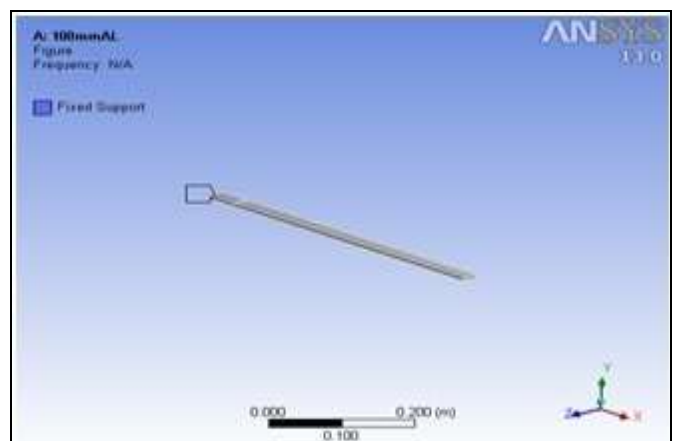


Fig.7- Beam Element (Plane 82)

The beam considered here has a fixed support at one end and spring supported at the other end of beam. For the fixed end all DOF are fixed as shown in Fig 7. And at the simply supported end the displacement in Y direction is taken as free.

Table 3 Natural frequencies at different mode

Mode	Numerical Frequency in Hz (for Al Beam)
1	5.10
2	25.54
3	31.95

B. Vibration Analysis of Beam

The first step in the vibration analysis of the beam is to find its natural frequencies. In ANSYS, Eigen natural frequencies are found by modal analysis. A minimum of first three mode shapes and natural frequencies are obtained. The mode shape and obtained lowest frequency for the beam are shown in in Fig.8

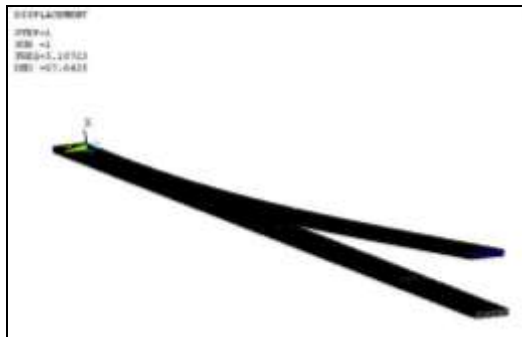


Fig. 8(a) Mode Shape1 with first node observed at frequency 5.10 Hz

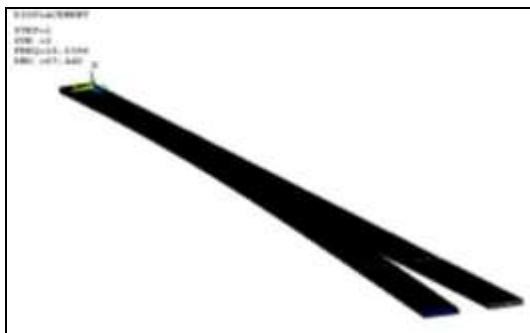


Fig. 8(b) Mode shape Mode Shape2 with first node observed at frequency 25.54 Hz

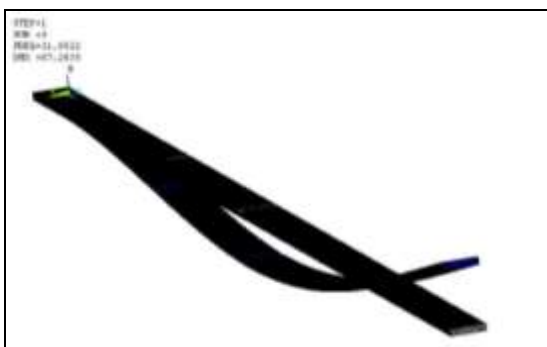


Fig. 8(c) Mode Shape3 with first node observed at frequency 31.95Hz

8. RESULT AND DISCUSSION

Comparison table of the frequencies obtained by Analytical, Experimental, by FFT Analyzer and by Ansys have been tabulated in the below table,

Table -4: Result Comparison

Sr No	Speed (n)	Fn(Hz) Ana	Fn(Hz) Exp	Fn(Hz) FFT	Fn (Hz) Ansys	Max Deviation (%)
1	30	5.36	5.35	5.83	5.1	8%
2	40		20	26.89	25.54	26%
3	60		30	33.2	31.95	10%

Following observations can be made:

- 1.8% is the Maximum deviation occurs between analytical and other methods at 30 RPM
- 2.26% is the Maximum deviation occurs between analytical and other methods at 40 RPM
- 3.26% is the Maximum deviation occurs between analytical and other methods at 60 RPM

9. CONCLUSIONS

1. From the present study it is revealed that Aluminum alloy material is optimum using regression analysis.
2. This material is further chosen for manufacturing of cantilever beam and its vibration analysis is carried for the case of forced damped condition.
3. From the analytical vibration analysis, frequency response curve shows that as the frequency ratio increases then the amplitude decreases with increase in damping ratio.
4. Similarly, from the experimental Time Graph it can be observed that the as the damping increased the amplitude decreases.

Vibration analysis of beam is studied under forced damped vibration is studied by analytical, experimental, using FFT Analyzer and Ansys.

It is observed that at different values of damping factor under damped case with increase in the damping ratio amplitude of the vibration goes on decreasing with increase in the frequency ratio for Aluminum Alloy which is the most suitable material for the engine casing because in practical application engine is to be operated at high speed with minimum amplitude of vibration and low natural frequency.

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