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W-R₀ SPACE IN MINIMAL g-CLOSED SETS OF TYPE1

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ABSTRACT - In this article we introduce and study Weakly minimal closes sets of Type1- R_0 spaces, Weakly minimal g-closed sets of Type1- R_0 spaces, Weakly minimal closed sets of Type1-i- R_0 spaces, Weakly minimal closed sets of Type1-d- R_0 spaces, Weakly minimal closed sets of Type1b- R_0 spaces, and also discuss the inter-relationships among separation properties along with several counter examples.

Key Wards: minimal open sets, minimal g-open sets, minimal open sets of Type1, minimal g-open set of Type1.

1. INTRODUCTION.

L.Nachbin[6] Topology and order, D.Van Nostrand Inc., Princeton, New Jersy studied increasing [resp. decreasing, balanced] open sets in 1965. K. Bhagya Lakshmi, J. Venkateswara Rao[13] studied W-R₀ Type spaces in topological ordered spaces in 2014. G.Venkareswarlu, V.Amarendra Babu, and M.K.R.S Veera kumar [11] introduced and studied minimal open sets of Type1 sets, minimal g-open sets of Type1 sets in 2016. G.Venkateswarlu, V.Amarendra Babu, K. Bhagya Lakshmi and V.B.V.N. Prasad [14] studied W-C₀ Spaces in 2019.

In this article we introduce new separation axioms of type Weakly- minimal closed sets $Type1-R_0$ spaces, Weakly minimal g-closed sets of $Type1-R_0$ spaces, Weakly minimal g- closed sets of $Type1-i-R_0$ spaces, Weakly minimal g- closed sets of $Type1-d-R_0$ spaces, Weakly minimal g- closed sets of $Type1-d-R_0$ spaces, and discuss the inter-relationships among separation properties along with several counter examples.

2. PRELIMINARIES.

DEFINITION 2.1[11]: In a topological space (X, T), an open sub set U of X is called a minimal open sets of Type! If \exists at least one non-empty closed set F such that $F \subseteq U$ or U = Φ .

DEFINITION 2.2[11]: In a topological space (X, T), an open sub set U of X is called a minimal g- open sets of Type! If \exists at least one non-empty g- closed set F such that $F \subseteq U$ or $U = \Phi$.

3. MAIN RESULT:

Now we state and prove our first main result. Before that we first introduce the following definitions and notations.

Note:

The collections of all increasing Weakly-minimal closed sets of Type1, increasing Weakly-minimal g-closed sets Type1 is denoted by W-i- $m_i^{cl}(Z, T1)$, W-i- m_i - $g^{cl}(Z, T1)$. [resp. decreasing and balanced is denoted by W-d- $m_i^{cl}(Z, T1)$, W-d- m_i - $g^{cl}(Z, T1)$, W-b- $m_i^{cl}(Z, T1)$, W-b- $m_i^{cl}(Z, T1)$]. Topological ordered space is denoted by TOS, Weakly-Minimal closed sets of Type 1, Weakly-Minimal g-closed sets of Type 1, set is denoted by W- $m_i^{cl}(Z, T1)$, W- m_i - $g^{cl}(Z, T1)$ and α -closed, β -closed, Ψ -closed is denoted by α^{cl} , β^{cl} , Ψ^{cl} .

WE INTRODUCE THE FOLLOWING DEFINITIONS:

DEFINITION 3.1: In a topological space (Z, T), a non empty closed subset F of Z is called a minimal closed sets of Type1 if \exists at least one nom-empty open set U such that $F \subseteq U$ or U=Z.

DEFINITION 3.2: In a topological space (Z, T), a non empty g- closed subset F of Z is called a minimal g- closed sets of Type1 if \exists at least one nom-empty g- open set U such that F \subseteq U or U=Z.

DEFINITION 3.3: A space (Z, T) is called a minimal closed sets of Type1 R_0 -space if $m_i^{cl}{x}$ contained in G where G is Minimal closed sets of Type1(brifly $m_i^{cl}{x}$) and $x \in G \in T$

DEFINITION 3.4: A space (Z, T) is called a minimal g-closed sets of Type1 R₀-space if m_i -g^{cl}{x} contained in G where G is Minimal g-closed sets of Type1(brifly m_i -g^{cl}{x}) and xeGeT

DEFINITION 3.5: A space (Z, T) is called a minimal gclosed sets of Type1 α -R₀-space if for x \in G \in m_i-g-O(Z, T) α m_i-g^{cl}{x} contained in G where G is Minimal g-closed sets of Type1(brifly m_i-g^{cl}{x}) and x \in G \in T

DEFINITION 3.6: A space (Z, T) is called a minimal gclosed sets of Type1 Ψ -R₀-space if for x \in G \in m_i-g-O(Z, T) Ψ m_i-g^{cl}{x} contained in G where G is Minimal g-closed sets of Type1(brifly m_i-g^{cl}{x}) and x \in G \in T **DEFINITION 3.7**: A space (Z, T) Called Weakly minimal closed sets of Type1-- R_0 if the intersection of micl{x} is non-empty set $\forall x \in Z$

DEFINITION 3.8: A space (Z, T) Called Weakly minimal gclosed sets of Type1--R₀ if the intersection of $mi-g^{cl}{x}$ is non-empty set $\forall x \in Z$

DEFINITION 3.9: A space (Z, T) Called Weakly minimal closed sets of Type1-i-R₀ if the intersection of i- $m_i^{cl}{x}$ ia non-empty set $\forall x \in Z$

DEFINITION 3.10: A space (Z, T) Called Weakly minimal g-closed sets of Type1-i-R₀ if the intersection of $i-m_i-g^{cl}{x}$ is non-empty set $\forall x \in Z$

DEFINITION 3.11: A space (Z, T) Called Weakly minimal closed sets of Type1-d--R₀ if the intersection of $d-m_i^{cl}{x}$ non-empty set $\forall x \in Z$

DEFINITION 3.12: A space (Z, T) Called Weakly minimal g-closed sets of Type1-d--R₀ if the intersection of d-m_i- g^{cl} {x} is non-empty set $\forall x \in Z$

DEFINITION 3.13: A space (Z, T) Called Weakly minimal closed sets of Type1-b--R₀ if the intersection of $b-m_i{}^{cl}{x}$ non-empty set $\forall x \in Z$

DEFINITION 3.14: A space (Z, T) Called Weakly minimal g-closed sets of Type1-b-R₀ if the intersection of $b-m_i-g^{cl}{x}$ is non-empty $\forall x \in Z$

THEOREM 3.15: In a TOS (Z, T, \leq), every W- m_i -g^{cl}(Z, T1) – R_0 space is a W- α^{R0} space but not converse.

 $\alpha^{cl} \text{ set Then } \alpha^{cl}\{x\} \text{ contained in } m_i\text{-}g^{cl}\{x\} \; \forall \; x {\in} Z.$

That implies the intersection of $\alpha^{cl}\{x\}$ contained in m_i - $g^{cl}\{x\}$. But the intersection of m_i - $g^{cl}\{x\}$ empty set $x \in \mathbb{Z}$

we get the intersection of $\alpha^{cl}\{x\}$ is empty set $x \varepsilon Z.$ Hence (Z, T) is W- α^{R0} space.

EXAMPLE 3.16: Let $Z = \{\zeta_1, \delta_2, \Omega_3\}$ and $T = \{\Phi, Z, \{\zeta_1\}\}$.

 $\begin{array}{l} m_i \text{-gc}^{cl}(Z,T1) \ \text{are}\ \Phi, Z \\ \alpha^{cl}\ \text{sets}\ \text{are} \ \ \Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\} \\ m_i \text{-gc}^{cl}\{\zeta_1\} \ \text{is}\ Z \\ m_i \text{-gc}^{cl}\{\Omega_3\} \ \text{is}\ Z \\ \text{The intersection of}\ m_i \text{-gc}^{cl}\{x\} \ \text{is}\ Z \ \forall \ x \in Z \\ \alpha^{cl}\{\zeta_1\} \ \text{is}\ Z \\ \alpha^{cl}\{\delta_2\} \ \text{is}\ \{\delta_2\} \\ \alpha^{cl}\{\Omega_3\} \ \text{is}\ \{\Omega_3\} \\ \text{The intersection of}\ \alpha^{cl}\{x\} \ \text{is}\ \text{empty}\ \text{set not equal to}\ Z \ \forall \ x \in Z \end{array}$

THEOREM 3.17: In a TOS (Z, T, \leq), every W-i- m_i -g^{cl}(Z, T1) – R_0 space is a W-i α^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-i- m_i -gcl(Z, T1) – R_0 space. Then the intersection of i- m_i -gcl{x} is empty $\forall x \in Z$ by fact, everyi- m_i -gcl(Z, T1) is a i- m_i cl(Z, T1) and then every i- m_i cl(Z, T1) is $i\alpha^{cl}$ set Then $i\alpha^{cl}{x}$ contained in i- m_i -gcl{x} $\forall x \in Z$. That implies the intersection of $i\alpha^{cl}{x}$ contained ini- m_i gcl{x}. But the intersection of $i-m_i$ -gcl{x} empty set $\forall x \in Z$.

we get the intersection of $i\alpha^{cl}{x}$ is empty set $\forall x \in Z$ Hence (Z, T) is W- $i\alpha^{R0}$ space.

EXAMPLE 3.18: Let $Z=\{\zeta_1, \delta_2, \Omega_3\}$ and $T=\{\Phi, Z, \{\zeta_1\}, \{\zeta_1, \Omega_3\}\}$. $\leq_4 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2)\}, (\Omega_3, \zeta_1), (\Omega_3, \delta_2)\}.$ $m_i \cdot gc^{l}(Z, T1)$ are $\Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\}$ $i \cdot m_i \cdot gc^{l}(Z, T1)$ are Φ, Z α^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}.$ $i\alpha^{cl}$ sets are $\Phi, Z, \{\delta_2\}.$ $i \cdot m_i \cdot gc^{l}\{\zeta_1\}$ is Z $i \cdot m_i \cdot gc^{l}\{\Omega_3\}$ is ZThe intersection of $i \cdot m_i \cdot gc^{l}\{x\}$ is $Z \forall x \in Z$ $i\alpha^{cl}\{\zeta_2\}$ is $\{\delta_2\}.$ $i\alpha^{cl}\{\zeta_1\}$ is ZThe intersection of $i\alpha^{cl}\{x\}$ is $\{\delta_2\}$ not equal to $Z \forall x \in Z$

THEOREM 3.19: In a TOS (Z, T, \leq), every W-d- m_i -g^{cl}(Z, T1) – R_0 space is a W-d α^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-d- m_i - $g^{cl}(Z, T1) - R_0$ space. Then the intersection of d- m_i - $g^{cl}\{x\}$ is empty $\forall x \in Z$ by fact, every d- m_i - $g^{cl}(Z, T1)$ is a d- m_i - $d^{cl}(Z, T1)$ and then every d- m_i - $d^{cl}(Z, T1)$ is $d\alpha^{cl}$ set Then $d\alpha^{cl}\{x\}$ contained in d- m_i - $g^{cl}\{x\} \forall x \in Z$. That implies the intersection of $d\alpha^{cl}\{x\}$ contained in d- m_i $g^{cl}\{x\}$. But the intersection of $d-m_i$ - $g^{cl}\{x\}$ empty set $\forall x \in Z$. we get the intersection of $d\alpha^{cl}\{x\}$ is empty set $\forall x \in Z$. (Z, T) is W- $d\alpha^{R0}$ space.

EXAMPLE 3.20: Let $Z=\{\zeta_1, \delta_2, \Omega_3\}$ and $T=\{\Phi, Z, \{\zeta_1\}, \{\zeta_1, \Omega_3\}\}$. $\leq_2 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2)\}, (\Omega_3, \delta_2)\}$. $m_i \cdot g^{cl}(Z, T1)$ are Φ, Z $d \cdot m_i \cdot g^{cl}(Z, T1)$ are Φ, Z α^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}$. $d\alpha^{cl}$ sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}$. $d^{-m_i} \cdot g^{cl}\{\zeta_1\}$ is Z $d \cdot m_i \cdot g^{cl}\{\zeta_2\}$ is Z $d \cdot m_i \cdot g^{cl}\{\Omega_3\}$ is ZThe intersection of $d \cdot m_i \cdot g^{cl}\{x\}$ is $Z \forall x \in Z$ $d\alpha^{cl}\{\delta_2\}$ is Z $d\alpha^{cl}\{\delta_2\}$ is Z $d\alpha^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$ The intersection of $d\alpha^{cl}\{x\}$ is $\{\Omega_3\}$ not equal to $Z \forall x \in Z$



THEOREM 3.21: In a TOS (Z, T, \leq), every W-b- m_i -g^{cl}(Z, T1) - R_0 space is a W-b α^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-b- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of b- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every b- m_i -g^{cl}(Z, T1) is a b- m_i -cl(Z, T1) and then every $b-m_i^{cl}(Z, T1)$ is $b\alpha^{cl}$ set Then $b\alpha^{cl}{x}$ contained in $b-m_i-g^{cl}{x} \forall x \in \mathbb{Z}$. That implies the intersection of $b\alpha^{cl}{x}$ contained in b- m_{i} $g^{cl}{x}$. But the intersection of b-m_i- $g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $b\alpha^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-b α^{R0} space.

EXAMPLE 3.22: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { ζ_1 }, $\{\delta_2\}, \{\zeta_1, \delta_2\}, \{\delta_2, \Omega_3\} \}.$ $\leq_9 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3) \} \}.$ m_i -g^{cl}(Z, T1) are Φ , Z, { ζ_1 }, { Ω_3 }, { δ_2 , Ω_3 } b-mi-g^{cl}(Z, T1) are Φ , Z α^{cl} sets are Φ , Z, { ζ_1 }, { Ω_3 }, { δ_2 , Ω_3 }, { ζ_1 , Ω_3 } $b\alpha^{cl}$ sets are Φ , Z, { ζ_1 , Ω_3 }. b-m_i-g^{cl}{ ζ_1 } is Z $b-m_i-g^{cl}\{\delta_2\}$ is Z b-m_i-g^{cl}{ Ω_3 } is Z The intersection of b-m_i-g^{cl}{x} is Z \forall x \in Z $b\alpha^{cl}{\zeta_1}$ is $\{\zeta_1, \Omega_3\}$ $b\alpha^{cl}{\delta_2}$ is Z $b\alpha^{cl}\{\Omega_3\}$ is $\{\zeta_1, \Omega_3\}$ The intersection of $b\alpha^{cl}{x}$ is ${\zeta_1, \Omega_3}$ not equal to $Z \forall x \in Z$

THEOREM 3.23: In a TOS (Z, T, \leq), every W-i- m_i -g^{cl}(Z, T1) – R_0 space is a W-b α^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-i- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of i- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every i- m_i -g^{cl}(Z, T1) is a b- m_i ^{cl}(Z, T1) and then every $b-m_i^{cl}(Z, T1)$ is $b\alpha^{cl}$ set Then $b\alpha^{cl}{x}$ contained in i-m_i-g^{cl}{x} $\forall x \in \mathbb{Z}$. That implies the intersection of $b\alpha^{cl}{x}$ contained in i- m_{i} $g^{cl}{x}$. But the intersection of $i-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $b\alpha^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-b α^{R0} space.

EXAMPLE 3.24: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\} \}$. $\leq_9 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3)\}\}.$ m_i -g^{cl}(Z, T1) are Φ , Z i-mi-g^{cl}(Z, T1) are Φ , Z α^{cl} sets are Φ , Z, { δ_2 }, { Ω_3 }, { δ_2 , Ω_3 }. b α^{cl} sets are Φ , Z, { δ_2 }. i-m_i-g^{cl}{ ζ_1 } is Z $i\text{-}m_i\text{-}g^{cl}\{\delta_2\} \text{ is } Z$ i-m_i-g^{cl}{ Ω_3 } is Z The intersection of $i-m_i-g^{cl}$ {Zx is Z $\forall x \in \mathbb{Z}$. $b\alpha^{cl}{\zeta_1}$ is Z $b\alpha^{cl}{\delta_2}$ is ${\delta_2}$ $b\alpha^{cl}\{\Omega_3\}$ is Z The intersection of $b\alpha^{cl}{x}$ is ${\delta_2}$ not equal to $Z \forall x \in Z$.

THEOREM 3.25: In a TOS (Z, T, \leq), every W-i- m_i-g^{cl}(Z, T1) - R_0 space is a W-d α^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-i- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of i- m_i -g^{cl}{x} is empty $\forall x \in \mathbb{Z}$ by fact, every i- m_i -g^{cl}(Z, T1) is a d- m_i ^{cl}(Z, T1) and then every $d-m_i^{cl}(Z, T1)$ is $d\alpha^{cl}$ set Then $d\alpha^{cl}{x}$ contained in i-m_i-g^{cl}{x} $\forall x \in Z$. That implies the intersection of $d\alpha^{cl}{x}$ contained in i- m_{i} $g^{cl}{x}$. But the intersection of $i-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $d\alpha^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-d α^{R0} space.

EXAMPLE3.26: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { ζ_1 }, { δ_2 }, $\{\zeta_1, \delta_2\}, \{\zeta_1, \Omega_3\}\}.$ $\leq_1 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3), (\delta_2, \Omega_3) \}$ m_i -g^{cl}(Z, T1) are Φ , Z, { δ_2 }, { Ω_3 }, { ζ_1 , Ω_3 } i-mi-g^{cl}(Z, T1) are Φ , Z, { Ω_3 } α^{cl} sets are Φ , Z, { δ_2 }, { Ω_3 }, { δ_2 , Ω_3 }, { ζ_1 , Ω_3 } d α^{cl} sets are Φ , Z i-m_i-g^{cl}{ ζ_1 } is Z i-m_i-g^{cl}{ δ_2 } is Z i- m_i - $g^{cl}{\Omega_3}$ is ${\Omega_3}$ The intersection of $i-m_i-g^{cl}\{x\}$ is $\{\Omega_3\} \forall x \in \mathbb{Z}$. $d\alpha^{c}$;{ ζ_1 } is Z $d\alpha^{cl}{\delta_2}$ is Z $d\alpha^{cl} \{\Omega_3\}$ is Z The intersection of $d\alpha^{cl}{x}$ is Z not equal to $\{\Omega_3\} \forall x \in \mathbb{Z}$.

THEOREM 3.27: In a TOS (Z, T, \leq), every W-b- m_i -g^{cl}(Z, T1) - R_0 space is a W-i α^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-b- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of b- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every b- m_i -g^{cl}(Z, T1) is a i- m_i ^{cl}(Z, T1) and then every $i-m_i^{cl}(Z, T1)$ is $i\alpha^{cl}$ set Then $i\alpha^{cl}{x}$ contained in b-m_i-g^{cl}{x} $\forall x \in \mathbb{Z}$. That implies the intersection of $i\alpha^{cl}{x}$ contained in b- m_{i} $g^{cl}{x}$. But the intersection of b-m_i- $g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $i\alpha^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-i α^{R0} space.

EXAMPLE 3.28: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { ζ_1 }, $\{\delta_2\}, \{\zeta_1, \delta_2\}, \{\delta_2, \Omega_3\} \}.$ $\leq_{1} = \{ (\zeta_{1}, \zeta_{1}), (\delta_{2}, \delta_{2}), (\Omega_{3}, \Omega_{3}), (\zeta_{1}, \delta_{2}) \}, (\zeta_{1}, \Omega_{3}), (\delta_{2}, \Omega_{3}) \}.$ m_i -g^{cl}(Z, T1) are Φ , Z, { ζ_1 }, { Ω_3 }, { δ_2 , Ω_3 } b-mi-g^{cl}(Z, T1) are Φ , Z α^{cl} sets are Φ , Z, { ζ_1 }, { Ω_3 }, { δ_2 , Ω_3 }, { ζ_1 , Ω_3 } $i\alpha^{cl}$ sets are Φ , Z, $\{\Omega_3\}$, $\{\delta_2, \Omega_3\}$ b-m_i-g^{cl}{ ζ_1 } is Z b-m_igcl{ δ_2 } is Z b-m_igcl{ Ω_3 } is Z The intersection of $b-m_i-g^{cl}\{x\}$ is $Z \forall x \in Z$. $i\alpha^{cl}\{\zeta_1\}$ is $\{\zeta_1\}$ $i\alpha^{cl}{\delta_2}$ is ${\delta_2, \Omega_3}$ $i\alpha^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$

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The intersection of $i\alpha^{cl}{x}$ is empty set not equal to $Z \forall$ x∈Z. **THEOREM 3.29**: In a TOS (Z, T, \leq), every W-b- m_i -g^{cl}(Z, T1) – R_0 space is a W-d α^{R0} space but not converse. Proof. Suppose (Z, T) be a W-b- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of b- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every b- m_i -g^{cl}(Z, T1) is a d- m_i ^{cl}(Z, T1) and then every $d-m_i^{cl}(Z, T1)$ is $d\alpha^{cl}$ set Then $d\alpha^{cl}{x}$ contained in b-m_i-g^{cl}{x} $\forall x \in \mathbb{Z}$. That implies the intersection of $d\alpha^{cl}{x}$ contained in b- m_i $g^{cl}{x}$. But the intersection of b-m_i- $g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $d\alpha^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-d α^{R0} space. **EXAMPLE 3.30**: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { ζ_1 }, $\{\delta_2\}, \{\zeta_1, \delta_2\}, \{\delta_2, \Omega_3\}\}.$ $\leq_2 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2)\}, (\Omega_3, \delta_2)\}$ m_i -g^{cl}(Z, T1) are Φ , Z, { δ_2 }, { Ω_3 }, { ζ_1 , Ω_3 } b-m_i-g^{cl}(Z, T1) are Φ ,Z α^{cl} sets are Φ , Z, { ζ_1 }, { Ω_3 }, { δ_2 , Ω_3 }, { ζ_1 , Ω_3 } $d\alpha^{cl}$ sets are Φ , Z, { ζ_1 , Ω_3 } $b-m_i-g^{cl}{\zeta_1}$ is Z $b-m_i-g^{cl}{\delta_2}$ is Z b-m_i-g^{cl}{ Ω_3 } is Z The intersection of b-m_i-g^{cl}{x} is $Z \forall x \in Z$. $d\alpha^{cl}{\zeta_1}$ is ${\zeta_1}$ $d\alpha^{cl}{\delta_2}$ is Z $d\alpha^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$ The intersection of $d\alpha^{cl}{x}$ is empty set not equal to Z ∀x∈Z. **THEOREM 3.31**: In a TOS (Z, T, \leq), every W-d- m_i -g^{cl}(Z, T1) - R_0 space is a W-i α^{R0} space but not converse. **Proof**: Suppose (Z, T) be a W-d- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of d- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every d- m_i -g^{cl}(Z, T1) is a i- m_i ^{cl}(Z, T1) and then every $i-m_i^{cl}(Z, T1)$ is $i\alpha^{cl}$ set Then $i\alpha^{cl}{x}$ contained in d-m_i-g^{cl}{x} $\forall x \in Z$. That implies the intersection of $i\alpha^{cl}{x}$ contained in d- m_i $g^{cl}{x}$. But the intersection of $d-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $i\alpha^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-i α^{R0} space. **EXAMPLE 3.32**: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\}, \{\zeta_1\}, \{\zeta_1\}, \{\zeta_2\}, \{\zeta_3\}, \{\zeta_4\}, \{\zeta_$ Ω_3 }. $\leq_3 = \{(\zeta_1,\zeta_1),\, (\delta_2,\delta_2),\, (\Omega_3,\Omega_3),\, (\zeta_1,\delta_2),\, (\zeta_1,\Omega_3)\,\}.$ m_i -g^{cl}(Z, T1) are Φ , Z d-mi-g^{ci}(Z, T1) are Φ ,Z α^{cl} sets are Φ , Z, { δ_2 }, { Ω_3 }, { δ_2 , Ω_3 } $i\alpha^{cl}$ sets are Φ , Z, { δ_2 }, { Ω_3 }, { δ_2 , Ω_3 }. d-m_i-g^{cl}{ ζ_1 } is Z $d-m_i-g^{cl}\{\delta_2\}$ is Z

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 $lpha^{cl}{\zeta_1}$ is Z $lpha^{cl}{\delta_2}$ is $\{\delta_2\}$

 $i\alpha^{l}\{\Omega_{3}\}$ is $\{\Omega_{3}\}$

The intersection of $i\alpha^{cl}{x}$ is empty set not equal to $Z \forall x \in Z$

THEOREM 3.33: In a TOS (Z, T, \leq), every W-d- m_i -g^{cl}(Z, T1) – R_0 space is a W-b α^{R0} space but not converse.

 $\begin{array}{l} \textbf{Proof: Suppose (Z, T) be a W-d-} m_i - g^{cl}(Z, T1) - R_0 \text{ space.} \\ \text{Then the intersection of } d-m_i - g^{cl}\{x\} \text{ is empty} \\ \forall x \in Z \text{ by fact, every } d-m_i - g^{cl}(Z, T1) \text{ is } a b-m_i ^{cl}(Z, T1) \text{ and} \\ \text{then every } b-m_i ^{cl}(Z, T1) \text{ is } \\ b\alpha^{cl} \text{ set Then } b\alpha^{cl}\{x\} \text{ contained in } d-m_i - g^{cl}\{x\} \forall x \in Z. \\ \text{That implies the intersection of } b\alpha^{cl}\{x\} \text{ contained in } d-m_i - g^{cl}\{x\} \\ \text{empty set } \forall x \in Z. \\ \text{we get the intersection of } b\alpha^{cl}\{x\} \text{ is empty set } \forall x \in Z. \\ \text{We get the intersection of } b\alpha^{cl}\{x\} \text{ is empty set } \forall x \in Z. \\ \text{We get the intersection of } b\alpha^{cl}\{x\} \text{ is empty set } \forall x \in Z. \\ \text{Hence } (Z, T) \text{ is } W - b\alpha^{R0} \text{ space.} \end{array}$

EXAMPLE 3.34: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\zeta_1, \delta_2\}, \{\zeta_1, \Omega_3\} \}$. $\leq_4 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\Omega_3, \zeta_1), (\Omega_3, \delta_2) \}$. $m_i \cdot g^{cl}(Z, T1)$ are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\zeta_1, \Omega_3\}$ $d \cdot m_i \cdot g^{cl}(Z, T1)$ are $\Phi, Z, \{\zeta_1, \Omega_3\}$ α^{ci} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}, \{\zeta_1, \Omega_3\}$ $b\alpha^{cl}$ sets are Φ, Z $d \cdot m_i \cdot g^{cl}(\zeta_1)$ is $\{\zeta_1, \Omega_3\}$ $d \cdot m_i \cdot g^{cl}\{\Omega_3\}$ is $\{\zeta_1, \Omega_3\}$ The intersection of $d \cdot m_i gcl\{x\}$ is $\{\zeta_1, \Omega_3\} \quad \forall x \in Z$. $b\alpha^{cl}\{\Omega_3\}$ is Z $b\alpha^{cl}\{\Omega_3\}$ is ZThe intersection of $b\alpha^{cl}\{x\}$ is Z not equal to $\zeta_1, \Omega_3\} \quad \forall x \in Z$.

THEOREM 3.35: In a TOS (Z, T, \leq), every W- m_i -g^{cl}(Z, T1) – R_0 space is a W- β^{R0} space but not converse.

 $\begin{array}{l} \textbf{Proof: Suppose (Z, T) be a W- m_i-g^{cl}(Z, T1) - R_0 \text{ space.} \\ Then the intersection of m_i-g^{cl}\{x\} is empty set \forall \\ x \in Z by fact, every m_i-g^{cl}(Z, T1) is a m_i^{cl}(Z, T1) and then \\ every m_i^{cl}(Z, T1) is \\ \beta^{cl} \text{ set Then } \beta^{cl}\{x\} \text{ contained in } m_i-g^{cl}\{x\} \forall x \in Z. \\ That implies the intersection of $\beta^{cl}\{x\}$ contained in $m_i-g^{cl}\{x\}$ but the intersection of $m_i-g^{cl}\{x\}$ empty set $x \in Z$ we get the intersection of $\beta^{cl}\{x\}$ is empty set $x \in Z$. \\ Hence (Z, T) is W- β^{R0} space. \\ \end{array}$

 $\begin{array}{l} \textbf{EXAMPLE 3.36:} \quad \text{Let } Z = \{ \ \zeta_1, \ \delta_2, \ \Omega_3 \ \} \text{ and } T = \{ \ \Phi, \ Z, \ \{\zeta_1\}, \\ \{\delta_2\}, \ \{\zeta_1, \ \delta_2\} \ \}. \\ m_i \cdot g^{cl}(Z, T1) \text{ are } \Phi, Z \\ \beta^{cl} \text{ sets are } \Phi, Z, \ \{\zeta_1\}, \ \{\delta_2\}, \ \{\Omega_3\}, \ \{\delta_2, \ \Omega_3\}, \ \{\zeta_1, \ \Omega_3\} \\ m_i \cdot g^{cl}(\zeta_1) \text{ is } Z \\ m_i \cdot g^{cl}(\zeta_1) \text{ is } Z \\ m_i \cdot g^{cl}(\Omega_3) \text{ is } Z \\ The intersection of m_i \cdot g^{cl}(x) \text{ is } Z \quad \forall x \in Z. \\ \beta^{cl}(\zeta_1) \text{ is } \{\zeta_1\} \\ \beta^{cl}(\delta_2) \text{ is } \{\delta_2\} \end{array}$

The intersection of d-m_i-g^{cl}{Z} is $Z \forall x \in Z$

d-m_i-g^{cl}{ Ω_3 } is Z

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 $\beta^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$

The intersection of $\beta^{cl}{x}$ is empty set not equal to $Z \forall x \in Z$.

THEOREM 3.37: In a TOS (Z, T, \leq), every W-i- m_i -g^{cl}(Z, T1) – R_0 space is a W-i β^{R0} space but not converse.

 $\begin{array}{l} \textbf{Proof: Suppose (Z, T) be a W-i- m_i-g^{cl}(Z, T1) - R_0 \text{ space.} \\ Then the intersection of i- m_i-g^{cl}\{x\} is empty \\ \forall x \in Z \ by fact, every i- m_i-g^{cl}(Z, T1) is a i-m_i^{cl}(Z, T1) and \\ then every i-m_i^{cl}(Z, T1) is \\ i\beta^{cl} \text{ set Then } i\beta^{cl}\{x\} \text{ contained in } i-m_i-g^{cl}\{x\} \forall x \in Z. \\ That implies the intersection of i\beta^{cl}\{x\} \text{ contained in } i-m_i-g^{cl}\{x\} \text{ But the intersection of } i\beta^{cl}\{x\} \text{ contained in } i-m_i-g^{cl}\{x\} \text{ mpty set } \forall x \in Z. \\ we get the intersection of i\beta^{cl}\{x\} \text{ is empty set } \forall x \in Z. \\ (Z, T) \text{ is } W-i\beta^{R0} \text{ space.} \end{array}$

EXAMPLE 3.38: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\}, \{\zeta_1, \Omega_3\} \}$. $\leq_4 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2) \}, (\Omega_3, \zeta_1), (\Omega_3, \delta_2) \}.$

$$\begin{split} & m_i \text{-gcl}(Z, T1) \text{ a re } \Phi, Z \\ & i \text{-mi-cl}g(Z, T1) \text{ are } \Phi, Z \\ & \beta^{cl} \text{ sets are } \Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}. \\ & I\beta^{cl} \text{ sets are } \Phi, Z, \{\delta_2\}. \\ & i \text{-m}_i \text{-gcl}\{\zeta_1\} \text{ is } Z \\ & i \text{-m}_i \text{-gcl}\{\delta_2\} \text{ is } Z \\ & i \text{-m}_i \text{-gcl}\{\Omega_3\} \text{ is } Z \\ & The intersection of i \text{-m}_i \text{-gcl}\{x\} \text{ is } Z \forall x \in Z. \\ & i\beta^{cl}\{\delta_2\} \text{ is } \{\delta_2\} \\ & i\beta^{cl}\{\Omega_3\} \text{ is } Z \\ & The intersection of i\beta^{cl}\{x\} \text{ is } \{\delta_2\} \text{ not equal to } Z \forall x \in Z. \end{split}$$

THEOREM 3.39: In a TOS (Z, T, \leq), every W-d- m_i -g^{cl}(Z, T1) – R_0 space is a W-d β^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-d- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of d- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every d- m_i -g^{cl}(Z, T1) is a d- m_i ^{cl}(Z, T1) and then every d-m_icl(Z, T1) is $d\beta^{cl}$ set Then $d\beta^{cl}{x}$ contained in $d-m_i-g^{cl}{x} \forall x \in \mathbb{Z}$. That implies the intersection of $d\beta^{cl}{x}$ contained in d- m_i $g^{cl}{x}$. But the intersection of $d-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $d\beta^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-d β^{R0} space. **EXAMPLE 3.40**: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\} \}$, $\leq_4 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2)\}, (\Omega_3, \zeta_1), (\Omega_3, \delta_2)\}.$ m_i -g^{cl}(Z, T1) are Φ , Z d-mi-g^{cl}(Z, T1) are Φ , Z β^{cl} sets are Φ , Z, $\{\delta_2\}$, $\{\Omega_3\}$, $\{\delta_2, \Omega_3\}$. $d\beta^{cl}$ sets are Φ , Z, { Ω_3 }. d-m_i-g^{cl}{ ζ_1 } is Z

 $\begin{array}{l} d\text{-}m_i\text{-}g^{cl}\{\delta_2\} \text{ is } Z\\ d\text{-}m_i\text{-}g^{cl}\{\Omega_3\} \text{ is } Z\\ \text{The intersection of } d\text{-}m_i\text{-}g^{cl}\{x\} \text{ is } Z \;\forall x {\in} Z.\\ d\beta^{cl}\{\zeta_1\} \text{ is } Z\\ d\beta^{cl}\{\delta_2\} \text{ is } \{\delta_2\}\\ d\beta^{cl}\{\Omega_3\} \text{ is } Z \end{array}$

 $\begin{array}{ll} & Then \ the \ intersection \ of \ b-\ m_i-g^{cl}\{x\} \ is \ empty \\ \forall \ x \in Z \ \ by \ fact, \ every \ b-\ m_i-g^{cl}(Z, \ T1) \ is \ a \ b-m_i^{cl}(Z, \ T1) \ and \\ & then \ every \ b-m_i^{cl}(Z, \ T1) \ is \end{array}$

bβ^{cl} set Then bβ^{cl}{x} contained in b-m_i-g^{cl}{x} $\forall x \in \mathbb{Z}$. That implies the intersection of bβ^{cl}{x} contained in b-m_i-g^{cl}{x}. But the intersection of b-m_i-g^{cl}{x} empty set $\forall x \in \mathbb{Z}$. we get the intersection of bβ^{cl}{x} is empty set $\forall x \in \mathbb{Z}$. (Z, T) is W-bβ^{R0} space.

The intersection of $d\beta^{ck}{x}$ is ${\delta_2}$ not equal to $Z \forall x \in Z$.

– R_0 space is a W-b β^{R0} space but not converse.

THEOREM 3.41: In a TOS (Z, T, \leq), every W-b- m_i -g^{cl}(Z, T1)

Proof. Suppose (Z, T) be a W-b- m_i -gcl(Z, T1) – R_0 space.

EXAMPLE 3.42: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { ζ_1 }, { δ_2 }, { ζ_1 , δ_2 , { ζ_1 , δ_2 , Ω_3 } }. $\leq_9 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3) \}$. m_i-g^{cl}(Z, T1) are Φ , Z, { δ_2 }, { Ω_3 }, { ζ_1 , Ω_3 } b-mi-g^{cl}(Z, T1) are Φ , Z, { δ_2 } β^{cl} sets are Φ , Z, { δ_2 }, { Ω_3 }, { δ_2 , Ω_3 }, { ζ_1 , Ω_3 } b β^{cl} sets are Φ , Z, { δ_2 }, { Ω_3 }, { δ_2 , Ω_3 }, { ζ_1 , Ω_3 } b β^{cl} sets are Φ , Z, { δ_2 }, { ζ_1 , Ω_3 }. b-m_i-g^{cl}{ Ω_3 } = Z The intersection of b-m_i-g^{cl}{x} is { δ_2 } $\forall x \in Z$. b $\beta^{cl}{\zeta_1}$ is { ζ_1 , Ω_3 } b $\beta^{cl}{\delta_2}$ is { δ_2 } b $\alpha^{cl}{\Omega_3}$ is { ζ_1 , Ω_3 } The intersection of b- β cl{x} is Φ not equal to{ δ_2 } $\forall x \in Z$.

THEOREM 3.43: In a TOS (Z, T, \leq), every W-i- m_i -gcl(Z, T1) – R_0 space is a W-b β^{R0} space but not converse.

 $\begin{array}{l} \textbf{Proof: Suppose (Z, T) be a W-i- m_i-gcl(Z, T1) - R_0 space.} \\ Then the intersection of i- m_i-gcl{x} is empty \\ \forall x \in Z \ by fact, every i- m_i-gcl(Z, T1) is a b-m_icl(Z, T1) and then every b-m_icl(Z, T1) is \\ b\beta^{cl} set Then b\beta^{cl}{x} contained in i-m_i-gcl{x} \forall x \in Z. \\ That implies the intersection of b\beta^{cl}{x} contained in i- m_i-gcl{x}. \\ But the intersection of i-m_i-gcl{x} empty set \forall x \in Z. \\ we get the intersection of b\beta^{cl}{x} is empty set \forall x \in Z. \\ (Z, T) is W-b\beta^{R0} space. \\ \end{array}$

EXAMPLE 3.44: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\} \}$.

 $\leq_{5} = \{(\zeta_{1}, \zeta_{1}), (\delta_{2}, \delta_{2}), (\Omega_{3}, \Omega_{3}), (\zeta_{1}, \Omega_{3}), (\delta_{2}, \Omega_{3}) \}$ $m_{i} \cdot g^{cl}(Z, T1) \text{ are } \Phi, Z, \{\zeta_{1}\}, \{\delta_{2}, \Omega_{3}\}$ $i \cdot m_{i} \cdot g^{cl}(Z, T1) \text{ are } \Phi, Z, \{\zeta_{1}\}, \{\delta_{2}\}, \{\Omega_{3}\}, \{\zeta_{1}, \delta_{2}\}, \{\delta_{2}, \Omega_{3}\}, \{\zeta_{1}, \Omega_{3}\}$ $\beta^{cl} \text{ sets are } \Phi, Z, \{\zeta_{1}\}, \{\delta_{2}\}, \{\Omega_{3}\}, \{\zeta_{1}, \delta_{2}\}, \{\delta_{2}, \Omega_{3}\}, \{\zeta_{1}, \Omega_{3}\}$ $b\beta^{cl} \text{ sets are } \Phi, Z$ $i \cdot m_{i} \cdot g^{cl}(\zeta_{1}) \text{ is } Z$ $i \cdot m_{i} \cdot g^{cl}(\Omega_{3}) \text{ is } \{\delta_{2}, \Omega_{3}\}$ $The intersection of i \cdot m_{i} \cdot g^{cl}\{Z\} \text{ is } \{\delta_{2}, \Omega_{3}\}$ $\forall x \in Z.$ $b\beta^{cl}\{\zeta_{1}\} \text{ is } Z$ $b\beta^{cl}\{\Omega_{3}\} \text{ is } Z$ $The intersection of b\beta^{cl}\{x\} \text{ is } Z \text{ not equal to } \{\delta_{2}, \Omega_{3}\}$ $\forall x \in Z.$



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THEOREM 3.45: In a TOS (Z, T, \leq), every W-i- m_i-g^{cl}(Z, T1)

– R_0 space is a W-d β^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-i- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of i- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every i- m_i -g^{cl}(Z, T1) is a d- m_i ^{cl}(Z, T1) and then every $d-m_i^{cl}(Z, T1)$ is $d\beta^{cl}$ set Then $d\beta^{cl}{x}$ contained in i-m_i-g^{cl}{x} $\forall x \in \mathbb{Z}$. That implies the intersection of $d\beta^{cl}{x}$ contained in i- m_{i} $g^{cl}{x}$. But the intersection of $i-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $d\beta^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-d β^{R0} space. **EXAMPLE 3.46**: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { ζ_1 }, { ζ_1 , δ_2 , { ζ_1 , Ω_3 } }. $\leq_1 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3), (\delta_2, \Omega_3)\}$ m_i -g^{cl}(Z, T1) are Φ , Z, { δ_2 }, { Ω_3 } i-mi-g^{cl}(Z, T1) are Φ , Z, { Ω_3 } β^{cl} sets are Φ , Z, $\{\delta_2\}$, $\{\Omega_3\}$, $\{\delta_2, \Omega_3\}$ $d\beta^{cl}$ sets are Φ , Z i-m_i-g^{cl}{ ζ_1 } is Z i-m_i-g^{cl}{ δ_2 } is Z i-m_i-g^{cl}{ Ω_3 } is { Ω_3 } The intersection of $i-m_i-g^{cl}\{x\}$ is $\{\Omega_3\} \forall x \in \mathbb{Z}$. $d\beta^{cl}{\zeta_1}$ is Z

 $d\beta^{cl}{\delta_2}$ is Z

 $d\beta^{cl}\{\Omega_3\}$ isZ

The intersection of $d\beta^{cl}{x}$ is Z not equal to $\{\Omega_3\} \forall x \in \mathbb{Z}$.

THEOREM 3.47: In a TOS (Z, T, \leq), every W-b- m_i -g^{cl}(Z, T1) – R_0 space is a W-i β^{R0} space but not converse.

 $\begin{array}{l} \textbf{Proof: Suppose (Z, T) be a W-b- $m_i-g^{cl}(Z, T1) - R_0$ space.} \\ Then the intersection of b- $m_i-g^{cl}{x}$ is empty} \\ \forall $x \in Z$ by fact, every b- $m_i-g^{cl}(Z, T1)$ is a $i-m_i^{cl}(Z, T1)$ and then every $i-m_i^{cl}(Z, T1)$ is $i\beta^{cl}$ set Then $i\beta^{cl}{x}$ contained in $b-m_i-g^{cl}{x}$ $\forall $x \in Z$.} \\ That implies the intersection of $i\beta^{cl}{x}$ contained in $b-m_i-g^{cl}{x}$ } \end{array}$

empty set $\forall x \in \mathbb{Z}$.

EXAMPLE 3.48: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \} \}$. $\leq_1 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\delta_2, \zeta_1) \}, (\zeta_1, \Omega_3), (\delta_2, \Omega_3) \}$. $m_i \cdot g^{cl}(Z, T1) \text{ are } \Phi, Z, \{ \zeta_1 \}, \{ \Omega_3 \}, \{ \delta_2, \Omega_3 \}$ $b \cdot m_i \cdot g^{cl}(Z, T1) \text{ are } \Phi, Z$ β^{cl} sets are $\Phi, Z, \{ \zeta_1 \}, \{ \delta_2 \}, \{ \Omega_3 \}, \{ \zeta_1, \delta_2 \}, \{ \zeta_1, \Omega_3 \}$ $i\beta^{cl}$ sets are $\Phi, Z, \{ \Omega_3 \}, \{ \zeta_1, \Omega_3 \}$ $b \cdot m_i \cdot g^{cl}\{ \zeta_1 \} \text{ is } Z$ $b \cdot m_i \cdot g^{cl}\{ \delta_2 \} \text{ is } Z$ $b \cdot m_i \cdot g^{cl}\{ \Omega_3 \} \text{ is } Z$ The intersection of $b \cdot m_i \cdot g^{cl}\{ Z \}$ is $Z \forall x \in Z$. $i\beta^{cl}\{ \Omega_3 \}$ is $\{ \Omega_3 \}$ The intersection of $i \cdot \beta^{cl}\{ Z \}$ is $\{ \Omega_3 \}$ not equal to $Z \forall x \in Z$. **THEOREM 3.49**: In a TOS (Z, T, \leq), every W-b- m_i -g^{cl}(Z, T1) – R_0 space is a W-d β^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-b- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of b- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every b- m_i -g^{cl}(Z, T1) is a d- m_i ^{cl}(Z, T1) and then every $d-m_i^{cl}(Z, T1)$ is $d\beta^{cl}$ set Then $d\beta^{cl}{x}$ contained in b-m_i-g^{cl}{x} $\forall x \in Z$. That implies the intersection of $d\beta^{cl}{x}$ contained in b- m_i $g^{cl}{x}$. But the intersection of $b-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $d\beta^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-d β^{R0} space. **EXAMPLE 3.50**: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\}, \{\zeta_1\}, \{\zeta_1\}, \{\zeta_2\}, \{\zeta_3\}, \{\zeta_4\}, \{\zeta_$ δ_2 }. $\leq_{10} = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\Omega_3, \zeta_1) \}, (\delta_2, \Omega_3), (\delta_2, \zeta_1) \}.$ m_i -g^{cl}(Z, T1) are Φ , Z b-mi-g^{cl}(Z, T1) are Φ ,Z β^{cl} sets are Φ , Z, { δ_2 }, { Ω_3 }, { δ_2 , Ω_3 } } $d\beta^{cl}$ sets are Φ , Z, { ζ_1 , Ω_3 } b-m_i-g^{cl}{ ζ_1 } is Z b-m_i-g^{cl}{ δ_2 } is Z b-m_i-g^{cl}{ Ω_3 } is Z The intersection of $b-m_i-g^{cl}\{x\}$ is $Z \forall x \in Z$. $d\beta^{cl}{\zeta_1}$ is Z $d\beta^{cl}{\delta_2}$ is ${\delta_2}$ $d\beta^{cl}\{\Omega_3\}$ are $\{\delta_2, \Omega_3\}$ The intersection of d- $\beta^{cl}{x}$ is ${\delta_2}$ not equal to Z $\forall x \in Z$.

 $\begin{array}{l} \textbf{THEOREM 3.51: In a TOS (Z, T, \leq), every W-d-m_i-g^{cl}(Z, T1) \\ - R_0 \text{ space is a W-i}\beta^{R0} \text{ space but not converse.} \\ \end{array} \\ \begin{array}{l} \textbf{Proof: Suppose (Z, T) be a W-d-m_i-g^{cl}(Z, T1) - R_0 \text{ space.} \\ Then the intersection of d-m_i-g^{cl}\{x\} \text{ is empty} \\ \forall x \in Z \text{ by fact, every d-} m_i-g^{cl}(Z, T1) \text{ is } a i-m_i^{cl}(Z, T1) \text{ and} \\ then every i-m_i^{cl}(Z, T1) \text{ is } \\ i\beta^{cl} \text{ set Then } i\alpha^{cl}\{x\} \text{ contained in } d-m_i-g^{cl}\{x\} \forall x \in Z. \\ That implies the intersection of i\beta^{cl}\{x\} \text{ contained in } d-m_i-g^{cl}\{x\} \\ empty \text{ set } \forall x \in Z. \end{array} \\ \end{array}$

we get the intersection of $i\beta^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W- $i\beta^{R0}$ space.

EXAMPLE 3.52: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\Omega_3\}, \{\delta_2, \Omega_3\} \}, \leq_7 = \{(\zeta_1, \zeta_1), (\delta_2, \Omega_3), (\Omega_3, \Omega_3), (\delta_2, \zeta_1) \}\}$. $m_i \cdot g^{cl}(Z, T1)$ are Φ, Z $d \cdot m_i \cdot g^{cl}(Z, T1)$ are Φ, Z β^{cl} sets are $\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\zeta_1, \delta_2\}$ $i\beta^{cl}$ sets are $\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\zeta_1, \delta_2\}$ $d \cdot m_i \cdot g^{cl}\{\zeta_1\}$ is Z $d \cdot m_i \cdot g^{cl}\{\delta_2\}$ is Z $d \cdot m_i \cdot g^{cl}\{\Omega_3\}$ is ZThe intersection of $d \cdot m_i \cdot g^{cl}\{x\}$ is $Z \forall x \in Z$. $i\beta^{cl}\{\zeta_1\}$ is $\{\zeta_1\}$ $i\beta^{cl}\{\Omega_3\}$ is ZThe intersection of $i\beta^{cl}\{x\}$ is $\{\zeta_1\}$ not equal to $Z \forall x \in Z$.



THEOREM 3.53: In a TOS (Z, T, \leq), every W-d- m_i -g^{cl}(Z, T1) – R_0 space is a W-b β^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-d- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of d- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every d- m_i -g^{cl}(Z, T1) is a b- m_i ^{cl}(Z, T1) and then every $b-m_i^{cl}(Z, T1)$ is $b\beta^{cl}$ set Then $b\beta^{cl}{x}$ contained in $d-m_i-g^{cl}{x} \forall x \in Z$. That implies the intersection of $b\beta^{cl}{x}$ contained in d- m_i $g^{cl}{x}$. But the intersection of $d-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $b\beta^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-b β^{R0} space. **EXAMPLE 3.54**: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\}, \{\zeta_$ $\{\delta_2\}, \{\zeta_1, \delta_2\}, \{\delta_2, \Omega_3\}\},\$ $\leq_3 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3) \}.$ $m_i\text{-}g^{cl}(Z,\,T1)$ are $\Phi,\,Z,\,\{\zeta_1\},\,\{\Omega_3\},\,\{\delta_2,\,\Omega_3\}$ d-migcl(Z, T1) are Φ ,Z, { ζ_1 } β^{cl} sets are Φ , Z, { ζ_1 }, { Ω_3 }, { δ_2 , Ω_3 }, { ζ_1 , Ω_3 } $b\beta^{cl}$ sets are Φ , Z d-m_i-g^{cl}{ ζ_1 } is { ζ_1 } $d-m_i-g^{cl}{\delta_2}$ is Z d-m_i-g^{cl}{ Ω_3 } is Z The intersection of d-m_i-g^{cl}{x} is { ζ_1 } $\forall x \in \mathbb{Z}$. $b\beta^{cl}{\zeta_1}$ is Z $b\beta^{cl}{\delta_2}$ is Z $b\beta^{cl}\{\Omega_3\}$ is Z

The intersection of $b\beta^{cl}{x}$ is Z not equal to ${\zeta_1} \forall x \in Z$.

THEOREM 3.55: In a TOS (Z, T, \leq), every W- m_i -g^{cl}(Z, T1) – R_0 space is a W- Ψ^{R0} space but not converse.

 $\begin{array}{l} \textbf{Proof: Suppose (Z, T) be a W- m_i \text{-}g^{cl}(Z, T1) - R_0 \text{ space.} \\ Then the intersection of m_i \text{-}g^{cl}\{x\} \text{ is empty set } \forall \\ x \in Z & by fact, every m_i \text{-}g^{cl}(Z, T1) \text{ is } a m_i^{cl}(Z, T1) \text{ and then} \\ every m_i^{cl}(Z, T1) \text{ is } \\ \Psi^{cl} \text{ set Then } \beta^{cl}\{x\} \text{ contained in } m_i \text{-}g^{cl}\{x\} \forall x \in Z. \\ That implies the intersection of <math>\Psi^{cl}\{x\} \text{ contained in } m_i \text{-}g^{cl}\{x\} \text{ is empty set } \forall x \in Z. \\ we get the intersection of <math>\Psi^{cl}\{x\} \text{ is empty set } \forall x \in Z. \\ \text{we get the intersection of } \Psi^{cl}\{x\} \text{ is empty set } \forall x \in Z. \\ \text{Hence } (Z, T) \text{ is } W \text{-} \Psi^{R0} \text{ space.} \\ \end{array}$

EXAMPLE 3.56: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { δ_2 }, { Ω_3], { δ_2 , Ω_3 } } m_i-g^{cl}(Z, T1) are Φ , Z Ψ^{cl} sets are Φ , Z, { ζ_1 }, { δ_2 }, { Ω_3 }, { ζ_1 , δ_2 } { ζ_1 , Ω_3 } m_i-g^{cl}{ ζ_1 } is Z m_i-g^{cl}{ Ω_3 } is Z The intersection of m_i-g^{cl}{x} is Z $\forall x \in Z$. $\Psi^{cl}{\zeta_1}$ is { ζ_1 } $\Psi^{cl}{\Omega_3}$ is { Ω_3 } The intersection of $\Psi^{cl}{x}$ ia empty set but not equal to Z \forall x $\in Z$. **THEOREM 3.57**: In a TOS (Z, T, \leq), every W-i- m_i -g^{cl}(Z, T1) – R_0 space is a W-i Ψ^{R0} space but not converse.

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Proof: Suppose (Z, T) be a W-i- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of i- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every i- m_i -g^{cl}(Z, T1) is a i- m_i ^{cl}(Z, T1) and then every i- m_i ^{cl}(Z, T1) is $i\Psi^{cl}$ set Then $i\Psi^{cl}$ {x} contained in i- m_i -g^{cl}{x} $\forall x \in Z$. That implies the intersection of $i\Psi^{cl}$ {x} contained in i- m_i g^{cl}{x}. But the intersection of i- m_i -g^{cl}{x} empty set $\forall x \in Z$. we get the intersection of $i\Psi^{cl}$ {x} is empty set $\forall x \in Z$. We get the intersection of $i\Psi^{cl}$ {x} is empty set $\forall x \in Z$. EXAMPLE 3.58: Let Z = { $\zeta_1, \delta_2, \Omega_3$ } and T= { $\Phi, Z, \{\delta_2\}$.

 $\begin{cases} \Omega_3 \}, \{ \delta_2, \Omega_3 \} \end{cases}$ $\leq_7 = \{ (\zeta_1, \zeta_1), (\delta_2, \Omega_3), (\Omega_3, \Omega_3), (\delta_2, \zeta_1) \} \}. \\ m_i \cdot g^{cl}(Z, T1) \text{ are } \Phi, Z \\ i \cdot mi \cdot g^{cl}(Z, T1) \text{ are } \Phi, Z \\ \Psi^{cl} \text{ sets are } \Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\Omega_3\}, \{\zeta_1, \delta_2\}, \{\zeta_1, \Omega_3\}. \\ i \Psi^{cl} \text{ sets are } \Phi, Z, \{\zeta_1\}, \{\Omega_3\}, \{\zeta_1, \delta_2\}, \{\zeta_1, \Omega_3\}. \\ i \cdot m_i \cdot g^{cl}(\zeta_1) \text{ is } Z \\ i \cdot m_i \cdot g^{cl}(\delta_2) \text{ is } Z \\ i \cdot m_i \cdot g^{cl}(\Omega_3) \text{ is } Z \\ The intersection of i \cdot m_i \cdot g^{cl}(x) \text{ is } Z \forall x \in Z. \\ i \Psi^{cl}\{\delta_2\} \text{ is } \{\zeta_1, \delta_2\}. \\ i \Psi^{cl}\{\Omega_3\} \text{ is } \{\Omega_3\} \\ The intersection of i \cdot \Psi^{cl}(x) \text{ is empty but not equal to } Z \\ \forall x \in Z. \end{cases}$

THEOREM 3.59: In a TOS (Z, T, \leq), every W-d- m_i -g^{cl}(Z, T1) – R_0 space is a W-d\Psi^{R0} space but not converse.

 $\begin{array}{l} \textbf{Proof: Suppose (Z, T) be a W-d-} m_i - g^{cl}(Z, T1) - R_0 \ \text{space.} \\ Then the intersection of d-} m_i - g^{cl}\{x\} \ \text{is empty} \\ \forall \ x \in Z \ by \ fact, \ every \ d-} m_i - g^{cl}(Z, T1) \ \text{is } a \ d-} m_i^{cl}(Z, T1) \ \text{and} \\ \text{then every } d-m_i^{cl}(Z, T1) \ \text{is } \\ d\Psi^{cl} \ \text{set Then } d\Psi^{cl}\{x\} \ \text{contained in } d-m_i - g^{cl}\{x\} \ \forall \ x \in Z. \\ That \ implies \ the \ intersection \ of \ d\Psi^{cl}\{x\} \ \text{contained in } d-m_i - g^{cl}\{x\} \ \forall \ x \in Z. \\ \text{That implies the intersection of } d\Psi^{cl}\{x\} \ \text{contained in } d-m_i - g^{cl}\{x\} \ \text{empty set } \forall x \in Z. \\ \text{we get the intersection of } d\Psi^{cl}\{x\} \ \text{is empty set } \forall x \in Z. \\ \text{Hence } (Z, T) \ \text{is } W - d\Psi^{R0} \ \text{space.} \end{array}$

EXAMPLE 3.60: Let $Z=\{\zeta_1, \delta_2, \Omega_3\}$ and $T=\{\Phi, Z, \{\zeta_1\}, \{\zeta_1, \Omega_3\}\}$. $\leq_6 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\delta_2, \zeta_1)\}, (\zeta_1, \Omega_3), \{\delta_2, \Omega_3\}\}.$ $m_i \cdot g^{cl}(Z, T1)$ are Φ, Z Ψ^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}.$ $d\Psi^{cl}$ sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}.$ $d\Psi^{cl}$ sets are $\Phi, Z, \{\delta_2\}.$ $d \cdot m_i \cdot g^{cl}\{\zeta_1\}$ is Z $d \cdot m_i \cdot g^{cl}\{\Omega_3\}$ is ZThe intersection of $d \cdot m_i \cdot g^{cl}\{x\}$ is $Z \forall x \in Z.$ $d\Psi^{cl}\{\delta_2\}$ is $\{\delta_2\}.$ $d\Psi^{cl}\{\Omega_3\}$ is ZThe intersection of $d\alpha^{cl}\{x\}$ is $\{\delta_2\}$ not equal to $Z \forall x \in Z.$



THEOREM 3.61: In a TOS (Z, T, \leq), every W-b- m_i -g^{cl}(Z, T1) – R_0 space is a W-b Ψ^{R_0} space but not converse.

Proof: Suppose (Z, T) be a W-b- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of b- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every b- m_i -g^{cl}(Z, T1) is a b- m_i -cl(Z, T1) and then every b- m_i -cl(Z, T1) is

 $b\Psi^{cl}$ set Then $b\Psi^{cl}\{x\}$ contained in $b-m_i-g^{cl}\{x\} \forall x \in \mathbb{Z}$. That implies the intersection of $b\Psi^{cl}\{x\}$ contained in $b-m_i-g^{cl}\{x\}$. But the intersection of $b-m_i-g^{cl}\{x\}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $b\Psi^{cl}\{x\}$ is empty set $\forall x \in \mathbb{Z}$. Hence (Z, T) is $W-b\Psi^{R0}$ space.

EXAMPLE 3.62: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\}, \{\zeta_1, \Omega_3\} \}$. $S = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \Omega_3) \} \}$. $m_i \cdot g^{cl}(Z, T1)$ are Φ, Z Ψ^{cl} sets are $\Phi, Z, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}$ $b\Psi^{cl}$ sets are $\Phi, Z, \{\delta_2\}$. $b - m_i \cdot g^{cl}(\zeta_1)$ is Z $b - m_i \cdot g^{cl}\{\Omega_3\}$ is Z The intersection of $b - m_i \cdot g^{cl}\{x\}$ is Z $\forall x \in Z$. $b\Psi^{cl}\{\delta_2\}$ is $\{\delta_2\}$. $b\Psi^{cl}\{\delta_2\}$ is $\{\delta_2\}$. $b\Psi^{cl}\{\Omega_3\}$ is Z The intersection of $b - \Psi^{cl}\{x\}$ is $\{\delta_2\}$ is not equal to Z $\forall x \in Z$.

THEOREM 3.63: In a TOS (Z, T, \leq), every W-i- m_i -g^{cl}(Z, T1) – R_0 space is a W-b Ψ^{R_0} space but not converse.

Proof: Suppose (Z, T) be a W-i- m_i -g^{cl}(Z, T1) – R₀ space. Then the intersection of i- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every i- m_i -g^{cl}(Z, T1) is a b- m_i -g^{cl}(Z, T1) and then every b- m_i -g^{cl}(Z, T1) is b Ψ -cl set Then b Ψ -cl{x} contained in i- m_i -g^{cl}{x} $\forall x \in Z$. That implies the intersection of b Ψ -cl{x} contained in i- m_i g^{cl}{x}. But the intersection of i- m_i -g^{cl}{x} empty set $\forall x \in Z$. we get the intersection of b Ψ -cl{x} is empty set $\forall x \in Z$. Hence (Z, T) is W-b Ψ ^{R0} space.

EXAMPLE 3.64: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { ζ_1 }, { δ_2 }, { ζ_1 , δ_2], { ζ_1 , Ω_3 } }. $\leq_1 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3), (\delta_2, \Omega_3) \}$. $m_i \cdot g^{cl}(Z, T1)$ are Φ , Z, { δ_2 }, { Ω_3 }, { ζ_1 , Ω_3 } $i \cdot m \cdot g^{cl}(Z, T1)$ are Φ , Z, { Ω_3 } Ψ^{cl} sets are Φ , Z, { δ_2 }, { Ω_3 }, { δ_2 , Ω_3 }, { ζ_1 , Ω_3 } $b\Psi^{cl}$ sets are Φ , Z $i \cdot m_i \cdot g^{cl}{\zeta_1}$ is Z $i \cdot m_i \cdot g^{cl}{\Omega_3}$ is { Ω_3 } The intersection of $i \cdot m_i \cdot g^{cl}{x}$ is { Ω_3 } $\forall x \in Z$. $b\Psi^{cl}{\zeta_1}$ is Z $\Psi^{cl}{\zeta_1}$ is Z $\Psi^{cl}{\Omega_3}$ is Z The intersection of $b\Psi^{cl}{x}$ is Z but not equal to { Ω_3 } $\forall x \in Z$. **THEOREM 3.65**: In a TOS (Z, T, \leq), every W-i- m_i -g^{cl}(Z, T1) – R_0 space is a W-d\Psi^{R0} space but not converse. Proof. Suppose (Z, T) be a W-i- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of i- m_i -g^{cl}{x} is empty

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 $\forall x \in Z$ by fact, every i- m_i -g^{cl}(Z, T1) is a d- m_i ^{cl}(Z, T1) and then every d- m_i ^{cl}(Z, T1) is

 $d\Psi^{cl}$ set Then $d\Psi^{cl}{x}$ contained in $i-m_i-g^{cl}{x} \forall x \in Z$. That implies the intersection of $d\Psi^{cl}{x}$ contained in $i-m_i-g^{cl}{x}$. But the intersection of $i-m_i-g^{cl}{x}$ empty set $\forall x \in Z$. we get the intersection of $d\Psi^{cl}{x}$ is empty set $\forall x \in Z$. Hence (Z, T) is W- $d\Psi^{R0}$ space.

EXAMPLE 3.66: Let $Z=\{\zeta_1, \delta_2, \Omega_3\}$ and $T=\{\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\zeta_1, \delta_2\}\}$. $\leq_4 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\Omega_3, \zeta_1), (\Omega_3, \delta_2)\}$ $m_i \cdot g^{cl}(Z, T1)$ are Φ, Z Ψ^{cl} sets are $\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}, \{\zeta_1, \Omega_3\}$ $d\Psi^{cl}$ sets are $\Phi, Z, \{\zeta_1\}, \{\delta_2\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}, \{\zeta_1, \Omega_3\}$ $i \cdot m_i \cdot g^{cl}(\zeta_1)$ is Z $i \cdot m_i \cdot g^{cl}(\Omega_3)$ is Z The intersection of $i \cdot m_i \cdot g^{cl}\{x\}$ is Z $\forall x \in Z$. $d\Psi^{cl}\{\delta_2\}$ is Z $d\Psi^{cl}\{\delta_2\}$ is Z $d\Psi^{cl}\{\delta_2\}$ is Z $d\Psi^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$ The intersection of $d\Psi^{cl}\{x\}$ is $\{\Omega_3\}$ not equal to Z $\forall x \in Z$.

THEOREM 3.67: In a TOS (Z, T, \leq), every W-b- m_i -g^{cl}(Z, T1) – R_0 space is a W-i Ψ^{R0} space but not converse.

Proof: Suppose (Z, T) be a W-b- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of b- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every b- m_i -g^{cl}(Z, T1) is a i- m_i ^{cl}(Z, T1) and then every i- m_i ^{cl}(Z, T1) is $i\Psi^{cl}$ set Then $i\Psi^{cl}$ {x} contained in b- m_i -g^{cl}{x} $\forall x \in Z$. That implies the intersection of $i\Psi^{cl}$ {x} contained in b- m_i g^{cl}{x}. But the intersection of b- m_i -g^{cl}{x} empty set $\forall x \in Z$. we get the intersection of $i\Psi^{cl}$ {x} is empty set $\forall x \in Z$ Hence (Z, T) is W-i\Psi^{R0} space.

EXAMPLE 3.68: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\} \}$. $\leq_2 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2) \}, (\Omega_3, \delta_2) \}$. $m_i \cdot g^{cl}(Z, T1) \text{ are } \Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\}$ $b \cdot m_i \cdot g^{cl}(Z, T1) \text{ are } \Phi, Z$ Ψ^{cl} sets are $\Phi, Z, \{\zeta_1\}, \{\delta_2, \Omega_3\}$ $b \cdot m_i \cdot g^{cl}(\zeta_1) \text{ is } Z$ $b \cdot m_i \cdot g^{cl}(\delta_2) \text{ is } Z$ $b \cdot m_i \cdot g^{cl}(\Omega_3) \text{ is } Z$ The intersection of $b \cdot m_i \cdot g^{cl}(x)$ is $Z \forall x \in Z$. $i \Psi^{cl} \{\delta_2\} \text{ is } \{\delta_2, \Omega_3\}$ $i \Psi^{cl} \{\Omega_3\} \text{ is } \{\delta_2, \Omega_3\}$ The intersection of $i \Psi^{cl} \{x\}$ is $\{\delta_2, \Omega_3\}$ not equal to $Z \forall x \in Z$.

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THEOREM 3.69: In a TOS (Z, T, \leq), every W-b- m_i -g^{cl}(Z, T1) – R_0 space is a W-d Ψ^{R_0} space but not converse.

Proof: Suppose (Z, T) be a W-b- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of b- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every b- m_i -g^{cl}(Z, T1) is a d- m_i ^{cl}(Z, T1) and then every $d-m_i^{cl}(Z, T1)$ is $d\Psi^{cl}$ set Then $d\Psi^{cl}{x}$ contained in b-m_i-g^{cl}{x} $\forall x \in \mathbb{Z}$. That implies the intersection of $d\Psi^{cl}{x}$ contained in b- m_i $g^{cl}{x}$. But the intersection of $b-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $d\Psi^{cl}{x}$ is empty set $\forall x \in Z$ Hence (Z, T) is W-d Ψ^{R0} space. **EXAMPLE 3.70**: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { ζ_1 }, { δ_2 }, $\{\zeta_1, \delta_2\}, \{\delta_2, \Omega_3\}\}.$ $\leq_{5} = \{(\zeta_{1}, \zeta_{1}), (\delta_{2}, \delta_{2}), (\Omega_{3}, \Omega_{3}), (\zeta_{1}, \Omega_{3})\}, (\delta_{2}, \Omega_{3})\}$ m_i -g^{cl}(Z, T1) are Φ , Z, { ζ_1 }, { Ω_3 }, { δ_2 , Ω_3 } b-mi-g^{cl}(Z, T1) are Φ ,Z $Ψ^{cl}$ sets are Φ, Ζ, {ζ₁}, {Ω₃}, {δ₂, Ω₃}, {ζ₁, Ω₃}

$$\begin{split} &\Psi^{cl} \text{ sets are } \Phi, Z, \{\zeta_1\}, \{\Omega_3\}, \{\delta_2, \Omega_3\}, \{\zeta_1, \Omega_3\}, \{\zeta_1, \Omega_4\}, \{\zeta_1\} \text{ berm}_i\text{-}g^{cl}\{\zeta_1\} \text{ is } Z \text{ berm}_i\text{-}g^{cl}\{\delta_2\} \text{ is } Z \text{ berm}_i\text{-}g^{cl}\{\Omega_3\} \text{ is } Z \text{ The intersection of } \text{berm}_i\text{-}g^{cl}\{x\} \text{ is } Z \forall x \in Z. \\ &d\Psi^{cl}\{\zeta_1\} \text{ is } \{\zeta_1\} \end{split}$$

 $d\Psi^{cl}{\delta_2}$ is Z

 $d\Psi^{cl}\{\Omega_3\}$ is Z

The intersection of $\Psi^{cl}{x}$ is ${\zeta_1}$ not equal to $Z \forall x \in Z$.

THEOREM 3.71: In a TOS (Z, T, \leq), every W-d- m_i -g^{cl}(Z, T1) $-R_0$ space is a W-i Ψ^{R0} space but not converse. **Proof**: Suppose (Z, T) be a W-d- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of d- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every d- m_i -g^{cl}(Z, T1) is a i- m_i ^{cl}(Z, T1) and then every $i-m_i^{cl}(Z, T1)$ is $i\Psi^{cl}$ set Then $i\Psi^{cl}{x}$ contained in d-m_i-g^{cl}{x} $\forall x \in \mathbb{Z}$. That implies the intersection of $i\Psi^{cl}{x}$ contained in d- m_i $g^{cl}{x}$. But the intersection of $d-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $i\Psi^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-i Ψ^{R0} space. **EXAMPLE 3.72**: Let Z={ ζ_1 , δ_2 , Ω_3 } and T= { Φ , Z, { δ_2 }, $\{\Omega_3\}, \{\delta_2, \Omega_3\}\}$. $\leq_3 = \{(\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\zeta_1, \delta_2), (\zeta_1, \Omega_3) \}.$ m_i -g^{cl}(Z, T1) are Φ , Z d-mi-g^{cl}(Z, T1) are Φ , Z $Ψ^{cl}$ are Φ, Ζ, {ζ₁}, {δ₂}, {Ω₃}, {ζ₁, δ₂}, {ζ₁, Ω₃} $i\Psi^{cl}$ sets are Φ , Z, { δ_2 }, { Ω_3 } d-m_i-g^{cl}{ ζ_1 } is Z $d-m_i-g^{cl}\{\delta_2\}$ is Z d-m_i-g^{cl}{ Ω_3 } is Z The intersections of $d-m_i-g^{cl}\{x\}$ is $Z \forall x \in \mathbb{Z}$. $i\Psi^{cl}{\zeta_1}$ is Z $i\Psi^{cl}{\delta_2}$ is ${\delta_2}$ $i\Psi^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$ The intersection of $i\Psi^{cl}{x}$ is empty not equal to $Z \forall x \in Z$.

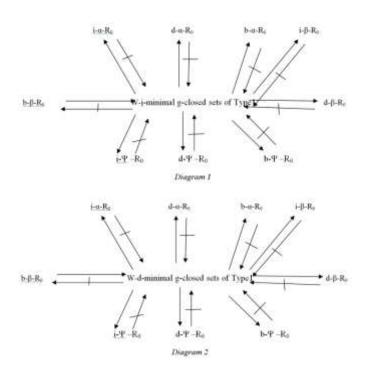
THEOREM 3.73: In a TOS (Z, T, \leq), every W-d- m_i -g^{cl}(Z, T1) – R_0 space is a W-b β^{R0} space but not converse.

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Proof: Suppose (Z, T) be a W-d- m_i -g^{cl}(Z, T1) – R_0 space. Then the intersection of d- m_i -g^{cl}{x} is empty $\forall x \in Z$ by fact, every d- m_i -g^{cl}(Z, T1) is a b- m_i ^{cl}(Z, T1) and then every $b-m_i^{cl}(Z, T1)$ is $b\Psi^{cl}$ set Then $b\Psi^{cl}{x}$ contained in $d-m_i-g^{cl}{x} \forall x \in \mathbb{Z}$. That implies the intersection of $b\Psi^{cl}{x}$ contained in d- m_i $g^{cl}{x}$. But the intersection of $d-m_i-g^{cl}{x}$ empty set $\forall x \in \mathbb{Z}$. we get the intersection of $b\Psi^{cl}{x}$ is empty set $\forall x \in \mathbb{Z}$ Hence (Z, T) is W-b Ψ^{R0} space. **EXAMPLE 3.74**: Let $Z = \{ \zeta_1, \delta_2, \Omega_3 \}$ and $T = \{ \Phi, Z, \{\zeta_1\}, \{\zeta_1\}, \{\zeta_1\}, \{\zeta_1\}, \{\zeta_2\}, \{\zeta_3\}, \{\zeta_4\}, \{\zeta_$ $\Omega_3 \} \}. \leq_7 = \{ (\zeta_1, \zeta_1), (\delta_2, \delta_2), (\Omega_3, \Omega_3), (\delta_2, \zeta_1) \}.$ m_i -g^{cl}(Z, T1) are Φ , Z, { ζ_1 }, { δ_2 , Ω_3 } d-mi-g^{cl}(Z, T1) are Φ ,Z, { δ_2 , Ω_3] Ψ^{cl} sets are Φ , Z, { δ_2 }, { Ω_3 }, { δ_2 , Ω_3 } bΨ^{cl} sets are Φ, Ζ, { Ω_3 } d-m_i-g^{cl}{ ζ_1 } is Z $b\Psi^{cl}{\zeta_1}$ is Z d-m_i-g^{cl}{ δ_2 } is { δ_2 , Ω_3 } $b\Psi^{cl}{\delta_2}$ is Z d-m_i-g^{cl}{ Ω_3 } is { δ_2 , Ω_3 } $b\Psi^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$ The intersection of d-m_i-g^{cl}{x} is { δ_2 , Ω_3 } The intersection of b Ψ^{cl} {x} is $[\Omega_3]$ not equal to $\{\delta_2, \Omega_3\}$ $b\Psi^{cl}{\zeta_1}$ is Z $b\Psi^{cl}{\delta_2}$ is Z $b\Psi^{cl}\{\Omega_3\}$ is $\{\Omega_3\}$

The intersection of $b\Psi^{cl}{x}$ is $[\Omega_3]$ not equal to $\{\delta_2, \Omega_3\}$

The following diagrams shows the above results.

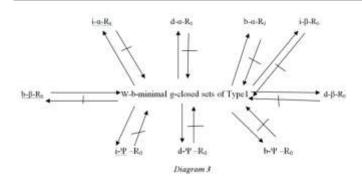




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