Free Vibration Analysis of Beams

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Abstract - Modal analysis is a technique to analyze the dynamic properties of the structures and its' components in the frequency domain. The theory allows finding the natural frequencies, mode shapes and the damping factor of a structure. The goal of this paper is to determine the natural frequencies and mode shapes of the beams of various cross-sections, material properties & support conditions with the help of mathematical models and to compare the results with ANSYS results. For mathematical analysis, Euler-Bernoulli's beam theory is used. The results obtained by two methods, are satisfactory and hold a good correlation.

Key Words: Natural frequency, mode shapes, ANSYS.

1. INTRODUCTION

In the past two decades modal analysis has become a major technology in the quest for determining, improving and optimizing dynamic characteristics of engineering structures. Not only has it been recognized in mechanical and aeronautical engineering but modal analysis has profound applications for civil and building structures, transportations etc.

The free vibrations takes place when a system oscillates under the action of forces integral in system due to internal deflection, under the absence of externally applied forces. The system will vibrate at one or more of its natural frequencies which are the properties of the system dynamics, established by its system and mass distribution. If these natural frequencies match the frequencies due to external loads (such as – live load, wind load, earth-quake load), the structure may continue to resonate and experience structural damage. If we can predict the frequencies due to the external loads, we can set the natural frequencies of the beams by distributing the mass and stiffness and thereby find the most optimum section and material.

2. OBJECTIVE AND SCOPE OF WORK

In this paper, we have found out the frequency of three types of beam cross sections (rectangular ,circular and I-section) for two different types of materials namely steel and aluminum with three different support conditions , viz.-fixed-fixed, cantilever and simply supported. Based on the natural frequency with an initial displacement we have tried to do a comparative study of behavior of different sections under different support conditions with Euler Bernoulli's beam theory and ANSYS 19.2 software. The study has been done mainly to predict the most efficient cross section and its materials for optimum design of structural systems.

3. LITERATURE SURVEY

A study on Dynamic dispersion curves for non-homogeneous, anisotropic beams with cross section of arbitrary geometry was done by Volvoi V.V et al. In this paper a code was developed to calculate the dispersion as well as the corresponding mode shapes. The code was based on finite element discretion over the cross sectional domain.[1]

With advances in researches a lot of progress was made in analysing the vibrations of beams. In one of the research work by Gavali A.L, he studied vibration analysis of beam by numerical discretisation scheme such as finite element method, the differential quadrature method (DQM) etc.[2]

A study of Free Vibration Analysis of circular plates with holes and cut outs was done by Thakare S.B et al. In this study an experimental method to determine the modal characteristics of a plate with multiple holes and slots are used which is verified by the finite element analysis (FEA) and ANSYS. Also the relationship between parameter variations and vibration modes is investigated. These results can be used as guidance for the modal analysis and damage detection of a circular plate with hole.[3]

A research was done by Raj R. and Sinha P.K in which they studied the modelling simulation and analysis of cantilever beams of different materials by finite element method analysis and MATLAB programming.[4]

Later on certain specific study was made on cantilever beam by Chaphalkar S.P et al, where modal analysis of cantilever beam structure using finite element analysis and experimental analysis was studied.[5]

A research on modal analysis of beam structures was done by Kumar P. et al where they studied three types of beams namely cantilever, simply supported and fixed beam and obtained their mode shapes and natural frequencies.[6]

A research on Free vibration analysis of eccentric and concentric isotropic stiffened plate using ANSYS was done by Siddiqui H.R and Shivhare V. In that paper free vibrations analysis of eccentric and concentric stiffened isotropic plates with central stiffener and double stiffener has been studied and effect of various parameters such as boundary condition aspect ratio of non – dimensional frequency parameters of plates are investigated. [7]

A study of free vibration of fixed free beam with theoretical and numerical approach was done by Chopade J.P. et al. Their paper mainly focuses on the theoretical analysis of transverse vibration of fixed free beam and investigates the mode shape frequency.[8]

Further research was carried one of which includes Numerical Investigation of Natural Frequencies for Clamped Longitudinal Composite Plates by Firas T. Al- Maliky in which two finite element model was performed. The fibre and matrix represented as two different materials in the first plate, while the second showed a composite plate. The numerical equations that performed from this study used to investigate the natural frequencies for longitudinal clamped composite plates.[9]

A study on modal analysis of central crack stainless steel plate using ANSYS was done by Maliky F. T. Al.- et al. The purpose of their research was to detect the cracks in stainless steel plates. The modal analysis of free vibration on central crack plate was done. The finite element analysis was performed in ANSYS 18.2 program workbench. The final mesh was generated in square plate with total number of nodes and elements. The prediction in cracks of stainless steel plates was done by studying the various in dynamic response of structures which provided a benefit method to investigate central cracks in stainless steel plates.[10]

4. THEORY

Using Euler-Bernoulli Beam theory, we can know that, the n th natural frequency can be calculated by –

$$\omega_n = \frac{1}{2\pi}(\beta l)^2\sqrt{\frac{EI}{\rho A l^4}} \qquad (1)$$

Where,

A= area of the cross section of the beam

l= length of the beam

ρ= density of the material

EI = equivalent bending stiffness and is the constant relative to the vibration bound condition

Using the formula, we can derive the fundamental mode shape frequencies of the beam specimens of different materials and support conditions.

Natural frequencies for first three modes of vibration for different support conditions can be written as –

(1). Pinned-pinned

$$\omega_1 = \pi^2\sqrt{\frac{EI}{\rho A l^4}} \qquad (2)$$

$$\omega_2 = 4\pi^2\sqrt{\frac{EI}{\rho A l^4}} \qquad (3)$$

$$\omega_3 = 9\pi^2\sqrt{\frac{EI}{\rho A l^4}} \qquad (4)$$

(2). Cantilever

$$\omega_1 = 3.156 \times \sqrt{\frac{EI}{\rho A l^4}} \quad (5)$$

$$\omega_2 = 22.03 \times \sqrt{\frac{EI}{\rho A l^4}} \quad (6)$$

$$\omega_3 = 61.69 \times \sqrt{\frac{EI}{\rho A l^4}} \quad (7)$$

5. PROBLEM DEFINITION

The geometrical properties, cross-section and the material considered for the beam section is presented in the Table 1 below

Table- 1: Beam specification

Cross section	Material	Support condition	Beam dimension (mm)
Rectangular	Steel	Cantilever	Length = 5000 Width = 100 Depth = 2, 4, 6, 8, 10
		Simply supported	Length = 5000 Width = 100 Depth = 2, 4, 6, 8, 10
	Aluminum	Cantilever	Length = 5000 Width = 100 Depth = 2, 4, 6, 8, 10
		Simply supported	Length = 5000 Width = 100 Depth = 2, 4, 6, 8, 10
Circular	Steel	Cantilever	Length = 5000 Diameter = 2, 4, 6, 8, 10
		Simply supported	Length = 5000 Diameter = 2, 4, 6, 8, 10
	Aluminum	Cantilever	Length = 5000 Diameter = 2, 4, 6, 8, 10
		Simply supported	Length = 5000 Diameter = 2, 4, 6, 8, 10

The table alongside shows the two different cross- section of beams (rectangular , circular) along with the two different materials (steel , aluminum) that has been taken into consideration in the present work. Based on these beam specifications with various support conditions (cantilever and simply supported) and various dimensions for the same a comparison has been drawn to find out the most efficient and optimum section of the beam. The various frequency ranges for each of these materials have been studied analytically and then compared with results obtained by ANSYS simulation to find out the optimum material and cross sectional properties.

The analytical and software simulation results are presented in the subsequent tables and figures below. During the analysis, material properties are taken as –

Young's modulus of steel, E_{steel}= 2×10^5 N/mm^2
Density of steel, ρ_{steel}= 7850 kg/m^3
Young's modulus of aluminum, $E_{aluminum}$= 69×10^3 N/mm^2
Density of aluminum, $\rho_{aluminum}$= 2700 kg/m^3

Table -2: Natural frequency in first three modes for support condition- cantilever, material – Steel and cross section-Circular

Beam diameter (mm)	Modes of frequency					
	Analytical value			ANSYS value		
	ω1	ω2	ω3	ω1	ω2	ω3
10	5.56	34.83	97.53	5.67	35.52	99.45
8	4.44	27.86	78.02	4.53	28.41	79.56
6	3.33	20.99	58.52	3.40	21.31	59.67
4	2.22	13.93	39.01	2.26	14.20	39.78
2	1.11	6.97	19.51	1.33	7.10	19.89

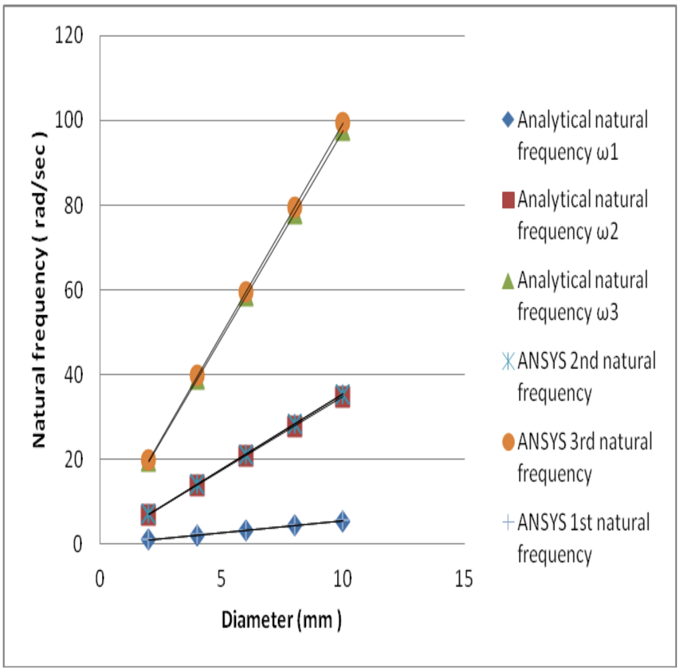


Fig- 1: Natural frequency vs. diameter plot of cantilever beam of material steel

Table-3: Natural frequency in first three modes for support condition- cantilever, material – Steel and cross section-rectangular

Beam Depth (mm)	Modes of frequency					
	Analytical value			ANSYS value		
	ω1	ω2	ω3	ω1	ω2	ω3
10	3.20	20.10	56.31	3.26	20.44	57.23
8	2.56	16.09	45.05	2.61	16.35	45.72
6	1.93	12.06	33.78	1.96	12.26	34.34
4	1.28	8.04	23.53	1.31	8.18	23.89
2	0.64	4.02	11.26	0.65	4.08	11.44

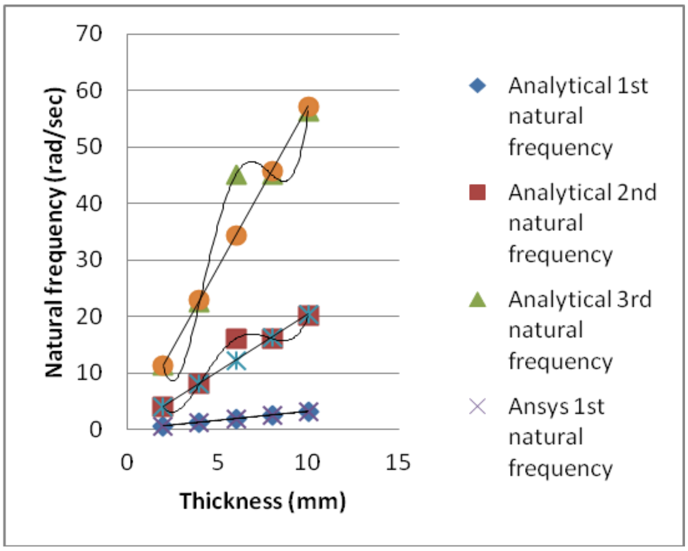


Fig- 2: Natural frequency vs. thickness plot of cantilever beam of material steel

Table-4: Natural frequency in first three modes for support condition- cantilever, material – Aluminum and cross section- Circular

Beam diameter (mm)	Modes of frequency					
	Analytical value			ANSYS value		
	ω1	ω2	ω3	ω1	ω2	ω3
10	5.52	34.57	96.82	5.65	35.43	99.20
8	4.46	27.94	78.24	4.53	28.34	79.36
6	3.34	20.93	58.61	3.39	21.26	59.27
4	2.22	14.12	39.55	2.26	14.17	39.69
2	1.11	6.98	19.54	1.1	7.08	19.84

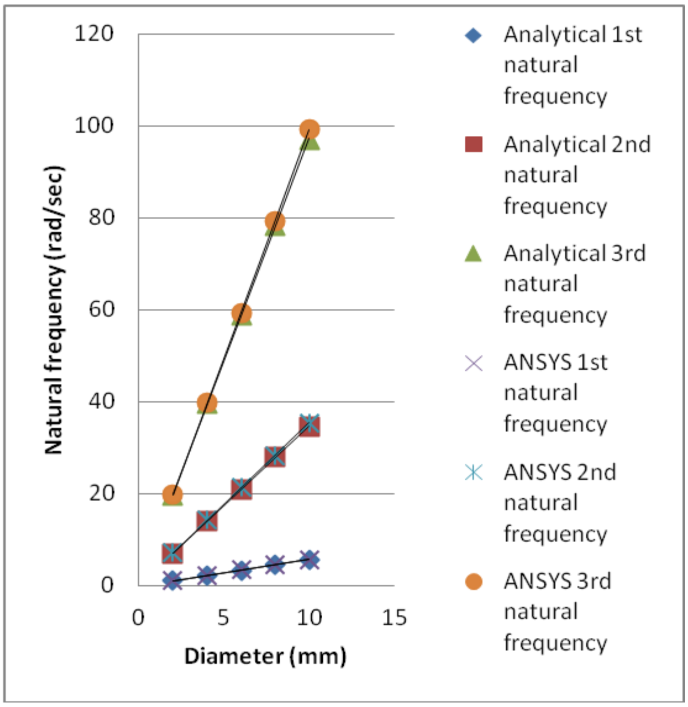


Fig- 3: Natural frequency vs. diameter plot of cantilever beam of material aluminum

Table-5: Natural frequency in first three modes for support condition- cantilever, material – Aluminum and cross section-rectangular

Beam Depth (mm)	Modes of frequency					
	Analytical value			ANSYS value		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
10	3.21	20.13	56.39	3.27	20.47	57.31
8	2.57	16.11	45.12	2.61	16.37	45.85
6	1.92	12.08	33.84	1.95	12.28	34.39
4	1.28	8.05	22.57	1.31	8.19	22.92
2	0.64	4.03	11.28	0.65	4.09	11.46

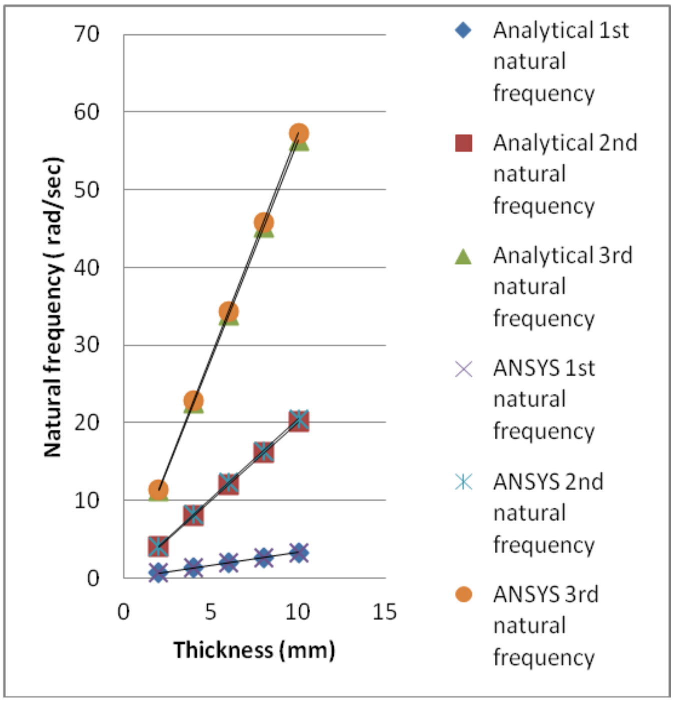


Fig- 4: Natural frequency vs. thickness plot of cantilever beam of material aluminum

Table-6: Natural frequency in first three modes for support condition- simply supported, material – steel and cross section- circular

Beam diameter (mm)	Modes of frequency					
	Analytical value			ANSYS value		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
10	15.60	32.41	140.42	15.91	63.34	143.1
8	12.48	44.93	112.34	12.72	50.91	114.5
6	9.37	37.47	84.30	9.54	38.18	85.91
4	6.26	25.03	56.32	6.36	25.46	57.27
2	3.12	12.48	28.08	3.18	12.72	28.64

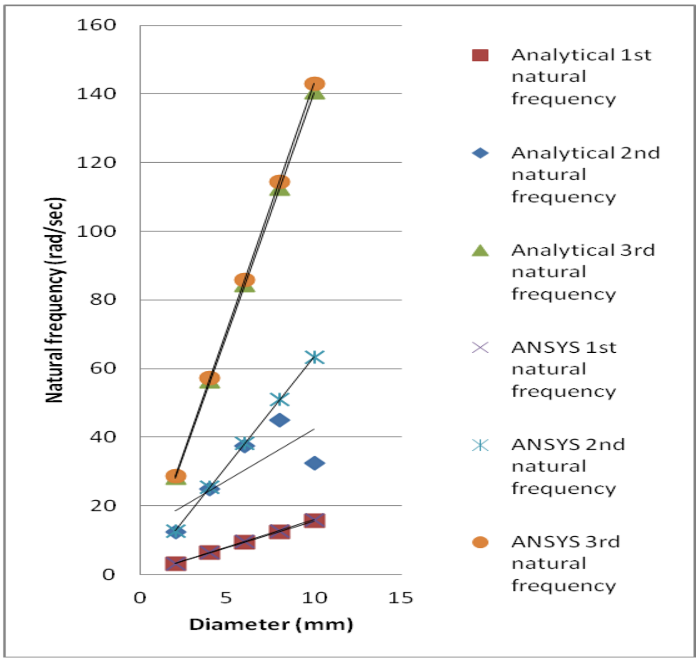


Fig-5: Natural frequency vs. diameter plot of simply supported beam of material steel

Table-7: Natural frequency in first three modes for support condition- simply supported, material – steel and cross section- rectangular

Beam Depth (mm)	Modes of frequency					
	Analytical value			ANSYS value		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
10	18.02	72.07	162.15	18.38	73.54	165.4
8	14.41	57.66	129.73	14.07	58.83	132.6
6	10.82	43.24	97.29	11.03	44.12	99.27
4	7.21	28.83	64.87	7.35	29.41	66.18
2	3.6	14.42	32.44	3.68	14.70	33.09

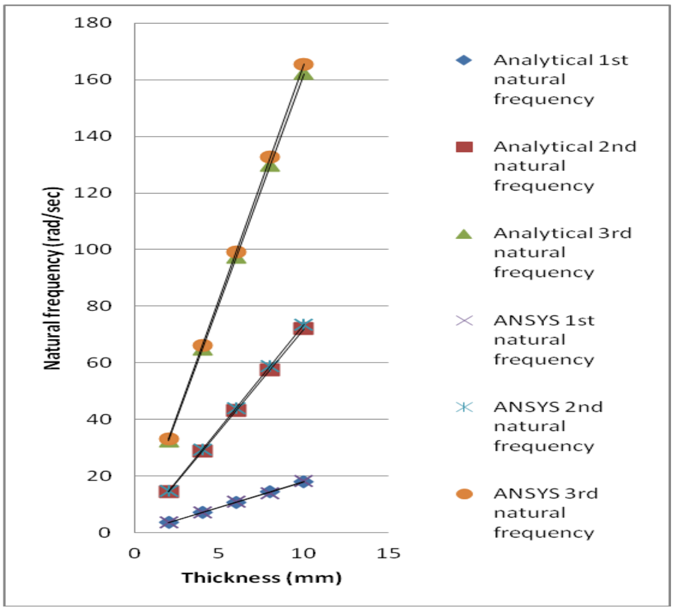


Fig- 6: Natural frequency vs. thickness plot of simply supported beam of material steel

Table-8: Natural frequency in first three modes for support condition- simply supported, material – aluminium and cross section- circular

Beam diameter (mm)	Modes of frequency					
	Analytical value			ANSYS value		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
10	15.5	61.96	139.41	16.1	64.25	144.55
8	12.5	50.07	112.65	12.9	51.40	115.65
6	9.4	37.5	84.38	9.6	38.55	86.74
4	6.2	25	56.25	6.4	25.70	57.82
2	3.1	12.5	28.13	3.2	12.85	28.91

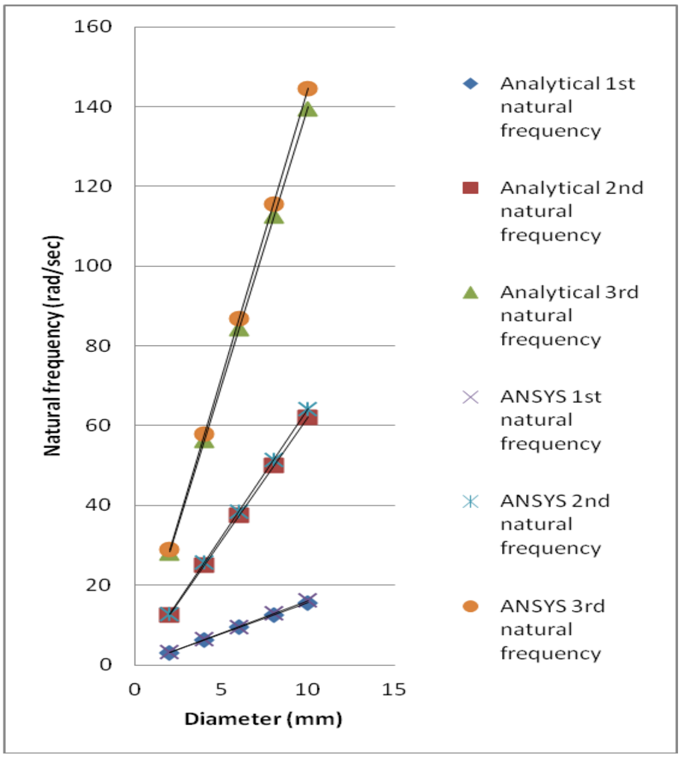


Fig-7: Natural frequency vs. diameter plot of simply supported beam of material aluminum

Table-9: Natural frequency in first three modes for support condition- simply supported, material – aluminium and cross section- rectangular

Beam Depth (mm)	Modes of frequency					
	Analytical value			ANSYS value		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
10	15.4	61.96	139.4	15.93	63.73	143.4
8	12.5	50.07	112.7	12.75	50.9	114.7
6	9.4	37.5	84.39	9.56	38.25	86.05
4	6.3	25	56.24	6.37	25.49	57.36
2	3.1	12.5	28.13	3.19	12.74	28.68

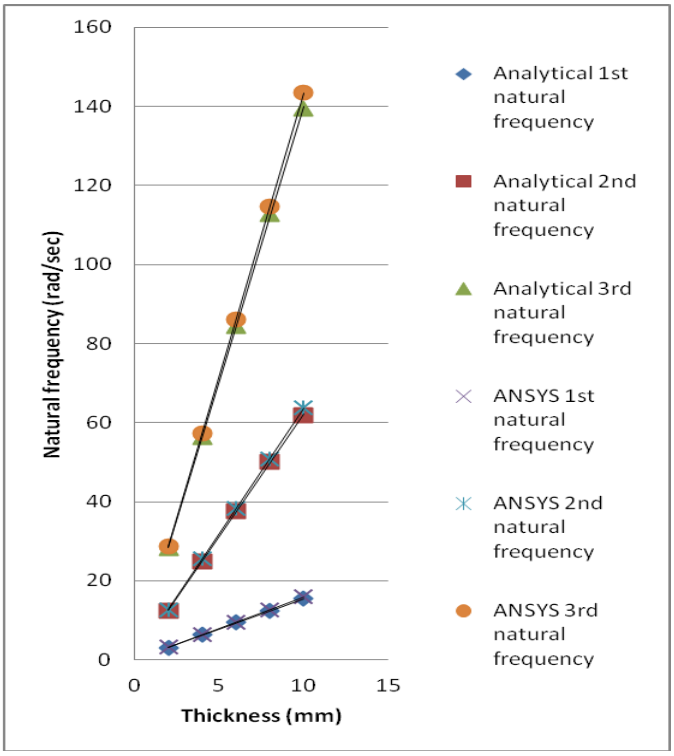
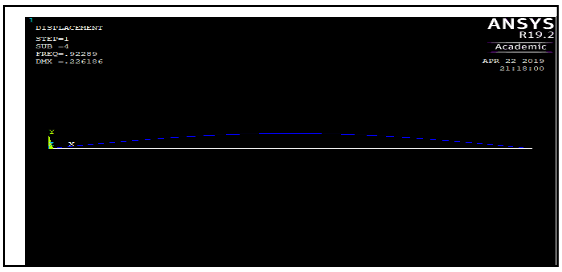


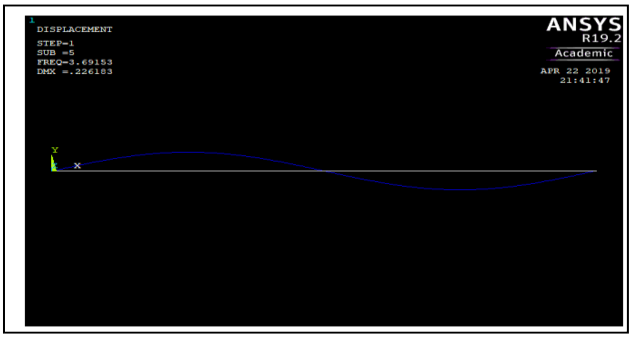
Fig 8: Natural frequency vs. thickness plot of simply supported beam of material aluminum

6. MODAL ANALYSIS ON ANSYS

The results obtained by modal analysis in ANSYS are shown in the Fig-9 and Fig-10 for simply supported and cantilever support conditions



(a)



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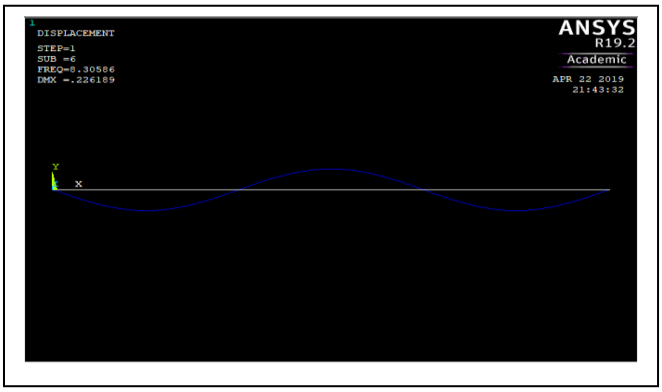
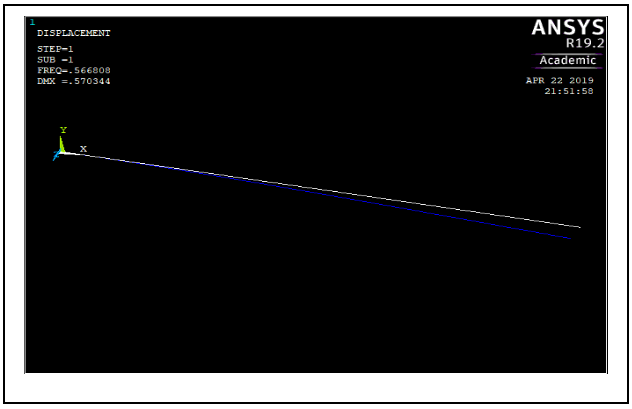
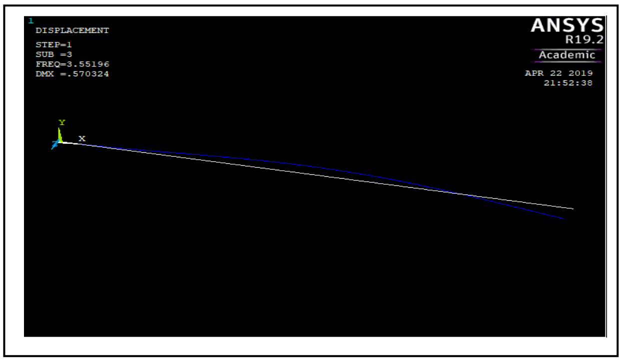


(C)

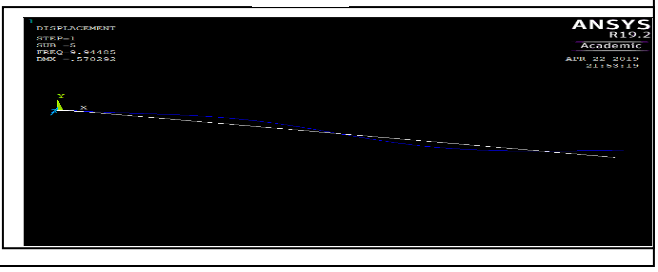
Fig 9: The first three mode shapes of simply supported steel beam with rectangular cross-section by ANSYS simulation



(a)



(b)



(c)

Fig 10: The first three mode shapes of cantilever steel beam with rectangular cross-section by ANSYS simulation

7. RESULTS AND DISCUSSION

To find the most economical material, a comparative study has been performed for the first three modes of natural frequencies for the chosen materials for particular section & support conditions as presented in Table 2 to Table 9 and fig- 1 to fig-8. The analytical results are cross checked against the software output. For a given support condition and cross sectional geometry natural frequency for different materials is plotted to compare in which case we get less response. This is depicted in figure 11 to figure 14

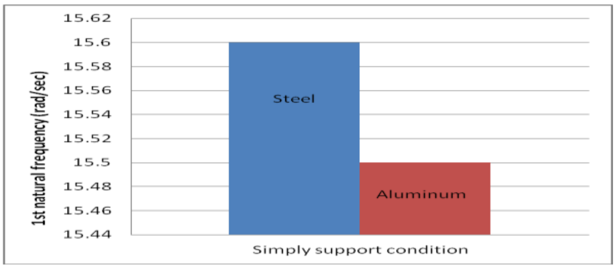


Fig-11: Simply supported beam with circular cross- section

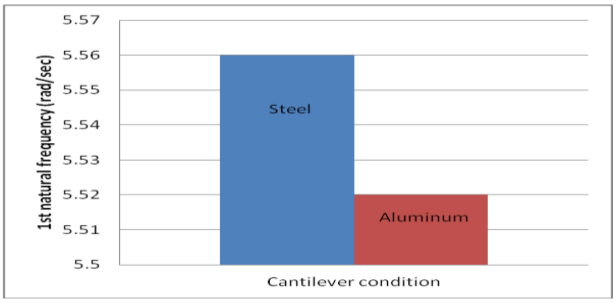


Fig-12: Cantilever beam with circular cross- section

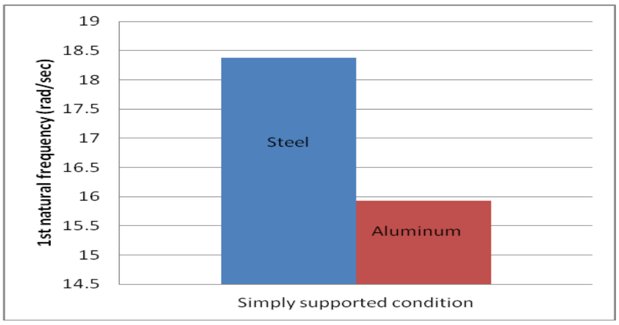


Fig-13: Simply supported beam with rectangular cross- section

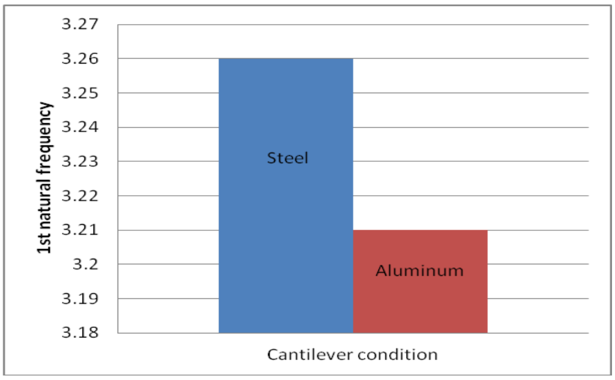


Fig-14: Cantilever beam with rectangular cross- section

From the above graphs it is clear that, frequency values are greater in case of material steel. Now, greater frequency means lesser time period of oscillation and greater stability of the overall structure. So, it can be said that steel is the most efficient material in this study when compared to aluminum.

8. CONCLUSIONS

An attempt has been made to find out the section which would give us minimum time period and maximum frequency, as compared through several graphs to do the comparative study.

By finding the frequency range for different materials with different support conditions we also can get the resonance of the structure. So we can easily justify the range of frequency for resonance of the structure and thus may avoid it.

By doing the comparative study we came to a final conclusion that the most effective section is the circular section among the two.

The most efficient & economical material is steel.

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