Detection of Angle-of-Arrival of the Laser Beam using a Cylindrical Lens and Gaussian Beam Curve Fitting

Anand Kumar Rai[1], Saurabh Bansod[2], Satya Prakash[3], Ajay Rathee[4*]

[1]M. Tech in Electronics Design Technology, National Institute of Electronics & Information technology Aurangabad, Maharashtra

[2]Scientist 'B', National Institute of Electronics & Information technology Aurangabad, Maharashtra

[3]Scientist 'F', Laser Science and Technology Centre (LASTEC), Defence Research& Development Organization (DRDO)

[4]Scientist 'E', Laser Science and Technology Centre (LASTEC), Defence Research& Development Organization (DRDO)

---***---

Abstract – *This paper presents a simple approach for finding the direction of incoming laser radiation on combat vehicles. This finds application in warning the crew of combat vehicles about incoming laser threats. The design is based on a linear array of photo-detectors and a Plano-convex cylindrical lens. The incoming laser radiation is focused by the lens on to linear array of photo-detectors and Gaussian curve fitting is used to find the position of center or peak of Gaussian, which in turn is calibrated to calculate the Angle of Arrival (A-o-A). Basic idea behind the design is the displacement of centroid of Gaussian beam with the change in angle-of-arrival. The system is able to measure the angle-of-arrival with the very high angular resolution, depending upon the number of sampling instants in the Gaussian fitted curve. The designing of the cylindrical lens and its simulation with laser beam is performed, at different angle-of-arrival using OSLO designer. The curve fitting and error analysis of the samples collected by the detector on being illuminated by the laser beam is performed using MATLAB curve fitting tool.*

Key Words: *Cylindrical lens; Gaussian curve fitting; angle-of-arrival (A-o-A); Laser Warning Receivers.*

1. INTRODUCTION

The detection of angle-of-arrival (A-o-A) is a technique used to detect the direction of the incident laser beam. The detection of A-o-A is useful in the functioning of the laser-based 3D scanners, and laser warning receivers.

1.1 Previous work

Researchers have proposed many schemes for detection of the angle-of-arrival of the incident beam of radiation, some of which employ the linear array of photo-detectors along with the coded mask or shadow mask techniques [1,2,3,4,5]. Later an improved method uses a cylindrical lens along with the top mask for focusing the laser radiation in the space between the cylindrical lens and the detector array which is concentric about the backside of the lens [1]. The top mask or coded mask with one slit or many parallel slits through it serves the purpose of encoding. The encoding schemes such as binary encoding using gray code is used to position slits on the top mask [2]. Each of these slots or rows of slots is positioned centrally directly above active area of photo-detectors [3]. The detector array is so arranged in rows that the radiation bars created by the slits of the mask are perpendicular to the detector row. The aperture of the slits is kept equal to the active area aperture of the used photo-detector [4]. When the radiation directed towards the device and falls on the detector plate, the radiation bars are created by slits above the few detectors, generating the electrical response for each detector. The generation of radiation bars depends on both the positions of slits in the encoded mask and the angle-of-arrival of radiation [3]. The electrical signal generated in photo-detectors is converted into digital by using reference signal and analog-to-digital converters which are then decoded to give the angle-of-arrival of the laser beam entering to the photo-detector. This scheme depends upon the use of coded mask and isolated channels, which is difficult to assemble. However, some proprietary solutions are available

Another kind of technology to serve the similar purpose makes use of the four-quadrant detector associated with a diaphragm formed by the central transparent zone of an opaque mask. These are kept in parallel with a transparent window and a circuit to measure deviation to process the detected Laser beam. This setup measures the angular deviation of a pinpoint radiation source. With two sensors of this kind, the locus of the radiating source can be ascertained [5].

1.2 Theoretical framework

The Laser beam is a coherent monochromatic electromagnetic radiation. Most of the Lasers follow the fundamental TEM_{00} Gaussian mode [6].

The mathematical expression for Gaussian function is:

$$f(x) = a e^{\frac{-(x-b)^2}{2c^2}} = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{\left(\frac{-1}{2}\right)\left(\frac{x-\mu}{\sigma}\right)^2} \qquad (1)$$

The fundamental transverse mode, TEM_{00} is of the lowest order and has a Gaussian intensity profile

$$I(r,z) = I_0 \left[\frac{\omega_0}{\omega(z)}\right]^2 e^{\frac{-2r^2}{\omega(z)^2}} \qquad (2)$$

The power contained in a Gaussian beam is

$$P = \int_0^\infty I(r,z) 2\pi r\, dr = \frac{1}{2} I_0 \pi \omega_0^2 \qquad (3)$$

In order to understand the intensity and power distribution, there is a need to know the beamwidth or beam diameter. The beamwidth is the diameter of the Laser beam at which its intensity is decremented to a $1/e^2$ value of its peak or axial value. FWHM (full width at half maximum) is the diameter of a Laser beam at which its intensity is decremented to 50% of its peak value [7]. The figure 1 below, describes both definitions graphically.
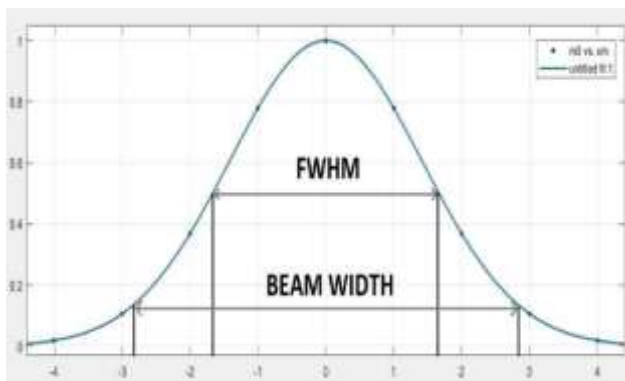

Fig - 1: Gaussian function curve showing FWHM and beamwidth at $1/e^2$ intensity.

Diffraction leads to the transverse spreading of light waves while propagation, due to which, it is impossible to acquire a perfectly collimated beam. Even if a TEM00 Laser beam wavefront is perfectly flat, the beam while propagation, quickly acquires curvature and spread [7] in accordance with

$$R(z) = z\left[1 + \left(\frac{\pi\omega_0^2}{\lambda z}\right)^2\right] \qquad (4)$$

And

$$w(z) = w_0 \left[1 + \left(\frac{\lambda z}{\pi\omega_0^2}\right)^2\right]^{1/2} \qquad (5)$$

Where z denotes the distance propagated by the wave from the plane where the wavefront is found to be flat, λ denotes the wavelength of Laser beam, $w0$ denotes the beamwidth radius i.e. radius at $1/e^2$ intensity contour at the plane where the flat wavefront achieved, $w\ (z)$ represents the radius of the $1/e^2$ contour at distance z, and $R\ (z)$ represents the radius of curvature of the wavefront at distance z. Plano-convex cylindrical lenses are the Aspheric lens and are curved only in one direction. Therefore, they focus or defocus light in the plane parallel to the Plano axis of the lens, into a line. It compresses the image along the power axis, which is perpendicular to the focal line, without altering it in a plane parallel to the Plano axis [8]. Their focusing behavior can be characterized by an effective focal length [9]. The same is represented in figure 2.
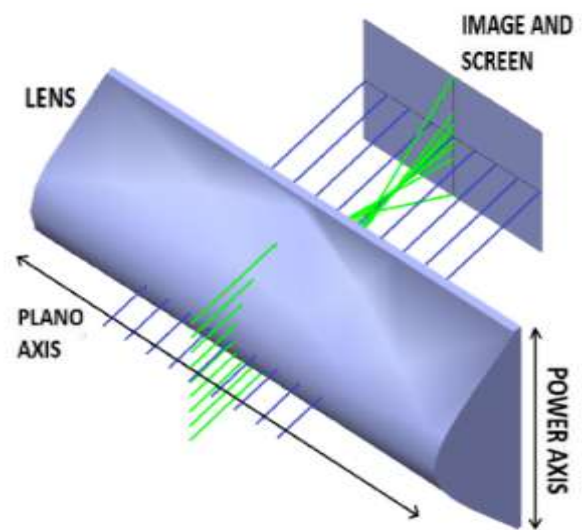

Fig - 2: Cylindrical lens showing focusing of light along Plano axis and Power axis.

The curve fitting tool is used to find a curve that best fits the data. Interpolation and smoothing are the two possible ways for curve fitting. There are several algorithms and method available that serves the purpose of curve fitting. Few of these methods are iterative method, nonlinear least square estimation, weighted least square estimation, and algorithms are Trust-Region algorithm, Caruana's algorithm, Guos algorithm, and Levenberg-Marquardt algorithm. Gaussian curve fitting fits the Gaussian function on to the given data points. Out of all methods, the Guos algorithm is the most appropriate method as it is least sensitive to the random noise [10].

1.3 Problem statement

To develop a simple and cost-effective method for the measurement of the angle-of-arrival of incident laser beam, with very high resolution by ascertaining the centroid of the beam using Gaussian curve fitting.

2. EXPERIMENTAL SETUP

The setup consists of three units, a Gaussian laser source, detection unit, and a processing unit. The laser source fires the laser beam towards the cylindrical lens at a specific angle. The beam suffers turbulence and also gets dispersed while propagating through the atmosphere. This effect is simulated in this work by adding white Gaussian noise to the signal. A cylindrical lens is used to focus the beam of laser, at the detector plane. The detector samples the wave and transfers it to the processing unit where Gaussian curve fitting, error analysis and calculation of angle-of-arrival is performed. The block diagram of the working experimental setup is explained in Figure 3 below.
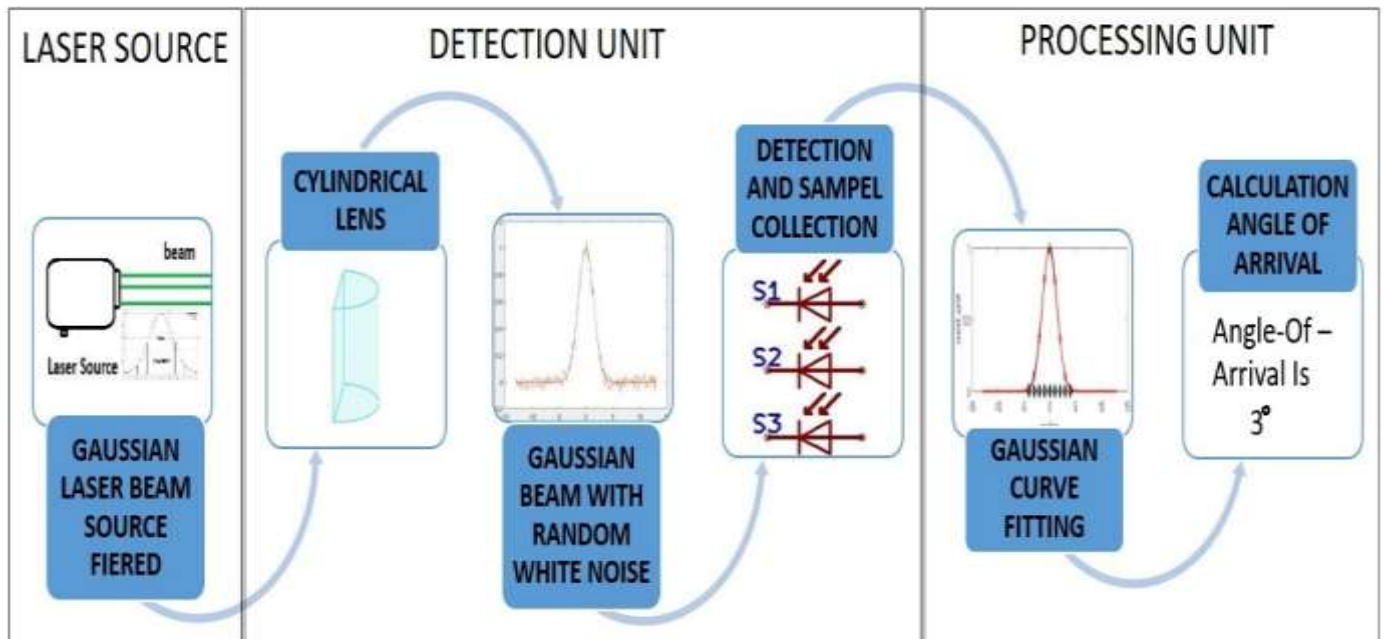

Fig - 3: Block diagram of the experimental setup for Gaussian curve fitting based angle-of-arrival detector.

3. METHODOLOGY

The Gaussian beam (wave) is generated using Gaussian function in MATLAB and different intensity of white Gaussian noise is added to the generated wave for achieving different signal to noise ratios as per experimental requirements. The Laser beam with a Gaussian profile is made to pass through a cylindrical lens. The beam, after propagating through the cylindrical lens remains Gaussian only at one axis i.e. at Plano axis. This beam falls on the linear detector array having nine photo-detectors each working as fixed sampling points or fixed sampling locations.

The central photo-detector is set to serve as a zero-point photo-detector which corresponds to the 0° (angle-of-arrival) of the incident beam at the photo-detector array. In this case, all nine photo-detectors are illuminated.

As the incident beam changes its angle-of-arrival (A-o-A) other than 0°, the spot at the detectors and the centroid of the spot shifts away from the central detector in the direction of propagation of the refracted beam namely;

positive angle-of-arrival (+A-o-A) and negative angle-of-arrival (-A-o-A), as the A-o-A increases the beam falls on a lesser number of detectors i.e. Fewer detectors are illuminated by beam spot. For both positive and negative A-o-A at a maximum value of A-o-A, a minimum of three photo-detectors is needed to be illuminated. This limitation comes into the picture because minimum three data points are required for single term Gaussian curve plotting using a nonlinear least square method using Trust-Region algorithm by Matlab curve plotting tool. As the beam falls on photo-detectors and the illuminated photo-detectors set the data value corresponding to the Gaussian power and intensity falling on the detectors and are fed to processing unit as a numerical value for curve fitting while data value corresponding to non-illuminated photo-detectors are set to not-a-number (NaN) state. NaN state is ignored automatically while plotting the curve and the detectors are treated as no signal or off. The Gaussian model that fits the intensity peaks of the received Laser beam and is given by the equation

$$y = \sum_{i=1}^{n} a_i e^{\left[\frac{-(x-b)^2}{c_i{}^2}\right]} \qquad (6)$$

Where a denotes the amplitude, b denotes the centroid of the Gaussian curve (location of the peak) while c is related to the FWHM beamwidth. Once the curve is plotted the system returns the value for three entities, namely a, b and c. Comparing them to mathematical expression for Gaussian function, we get

$$peak\,amplitude = a = \frac{1}{\sigma\sqrt{2\pi}} \qquad (7)$$

$$mean = b = \mu \qquad (8)$$

$$standard\,deviation = c = \sigma = w0 \qquad (9)$$

Using the values of a, b and c the errors related to noise and angle-of-arrival can be calculated. Using the locus of centroid related to the maximum and minimum A-o-A formula for calculation angle-of-arrival can be computed as the distance of the centroid increases proportionally to the increase in angle-of-arrival of the Laser beam.

4. SIMULATION AND RESULT

Using the Oslo designer, a rectangular Plano-convex cylindrical lens is designed with a radius of curvature 25mm, aperture radius 28mm, length 100mm, width 50mm, thickness 27mm. The 'BK7' material with a reflective index of 1.517 is selected for the lens. The effective focal length of the lens is 48.639mm.
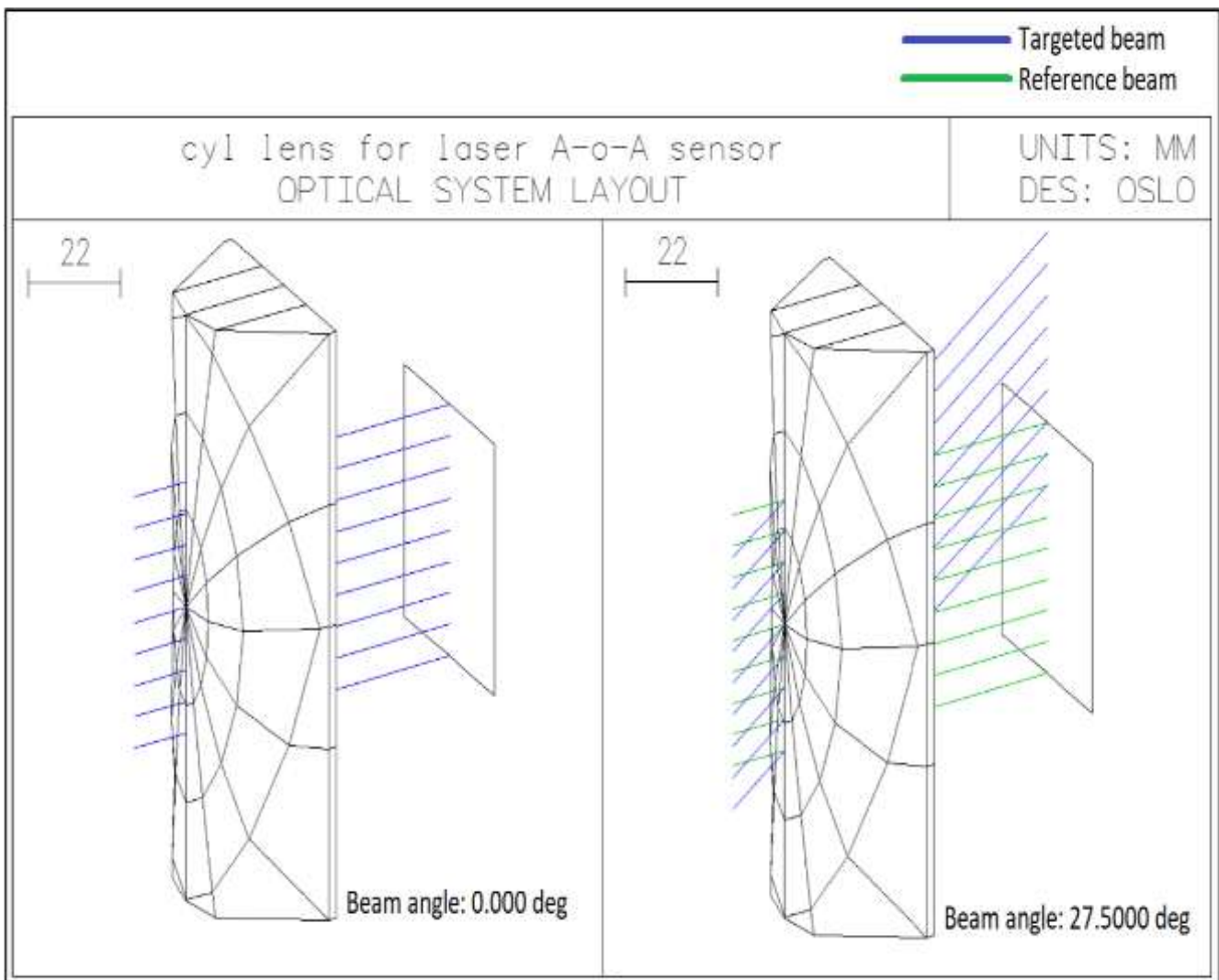
Fig - 4: Cylindrical lens showing the beam tilt and focusing of the beam at 0° angle-of-arrival (left) and at 27.5° angle-of-arrival (right) of the incident Laser beam

Described above in figure 4, the cylindrical lens projecting the image on the detector surface i.e. screen plane, at two different angle-of-arrival. The lens at left shows an image projection for 0°A-o-A while the one at right shows an image projection for +27.5°A-o-A.
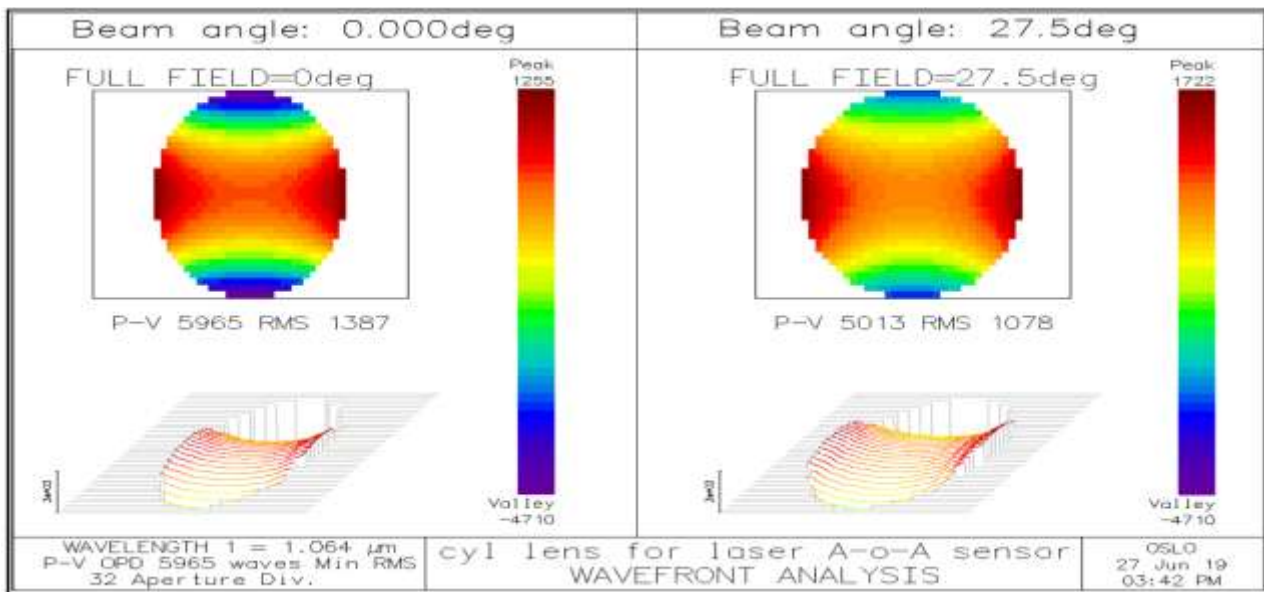
Fig - 5: Wavefront analysis of the focused beam showing Gaussian intensity distribution at the detector plane beam at 0° angle-of-arrival (left) and at 27.5° angle-of-arrival (right) of the incident Laser beam.

Figure 5 describes the wavefront analysis of the beam spot formed at detector surface i.e. screen plane by the cylindrical lens for two different angle-of-arrival. The plot at left shows wavefront and intensity distribution for 0° A-o-A while the plot at right shows wavefront and intensity distribution for +27.5° A-o-A. The diagram describes the intensity distribution follows the Gaussian profile along the planar axis of the cylindrical lens and is compressed along power axis i.e. the beam is no more Gaussian in a plane along the power axis of the lens.
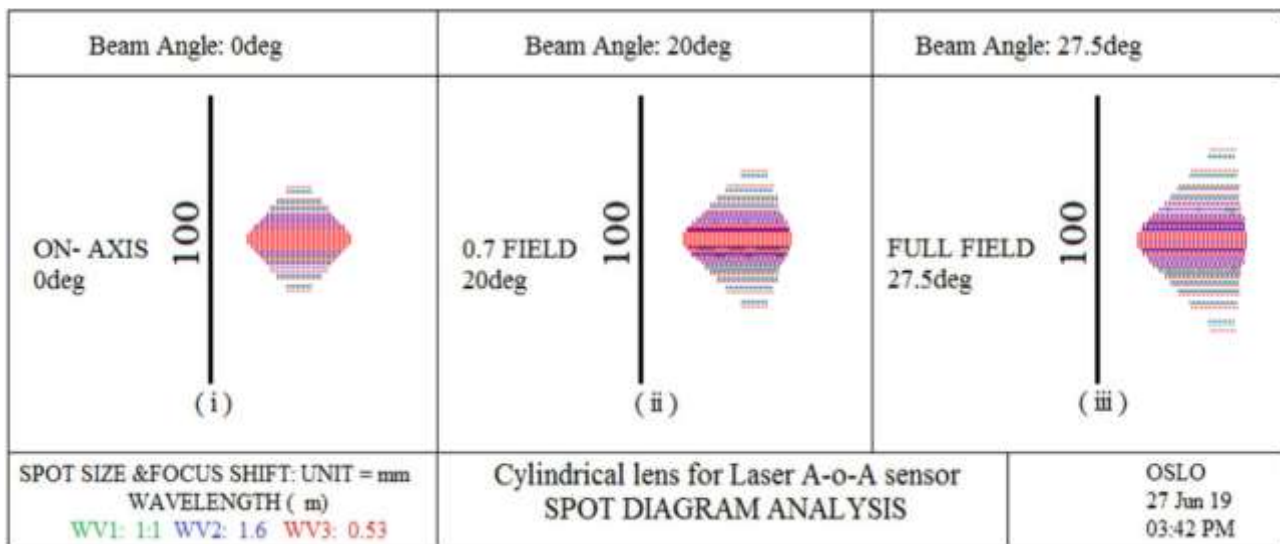

Fig - 6: Spot diagram showing size and shape of the spot formed at detector plane at 0 focal shift beam (i) at 0° angle-of-arrival, (ii) at 20° angle-of-arrival and (iii) at 27.5° angle-of-arrival of the incident Laser beam.

Figure 6 describes the spot diagram analysis of the beam spot formed at detector surface i.e. screen plane by the cylindrical lens for three different angle-of-arrival. The 6(i) shows a spot diagram for 0°A-o-A. The 6(ii) shows a spot diagram for 20°A-o-A while the 6(iii) shows a spot diagram for +27.5°A-o-A. The diagram describes the shape, size and the focal shift for the beam passing through the lens at some particular A-o-A.
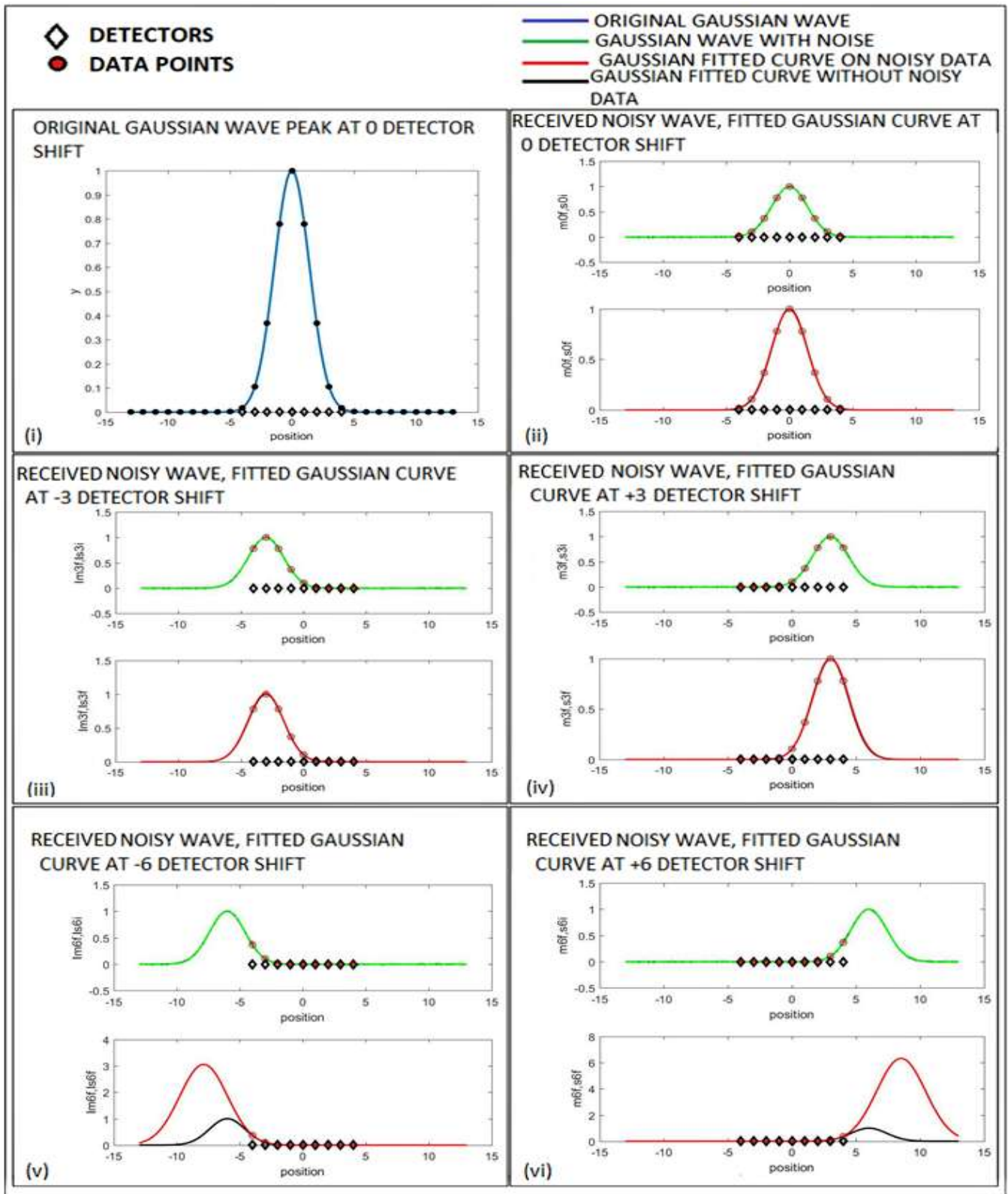
Fig - 7: Gaussian curve fitting of the detected signal and comparison with originally received noisy Gaussian signal.

Figure 7 graphs Gaussian curve of the noisy beam (green) that illuminate the detectors, and the plotted Gaussian curve (red) using samples received at the detectors. The figure 7(iii) describes the curve of wave at for -3 detector shift indicating that the image is shifted till 3 photo-detectors towards left (if the lens and detector are placed horizontally) or downward (if the lens and detector are placed vertically) and 3 detectors from right or upward

are not getting illuminated and their response is considered as NaN state. While the figure 7(iv) describes the curve of wave at +3 detector shift indicating that the image is shifted till 3 photo-detectors towards right (if the lens and detector are placed horizontally) or upward (if the lens and detector are placed vertically) and 3 detectors from left or downward are not getting illuminated and their response is considered as NaN state. The same applies for -6 to 0 and 0 to +6 detector shifts respectively. The distance corresponding to each detector can be calculated by multiplying the number of the detector with 5.36mm
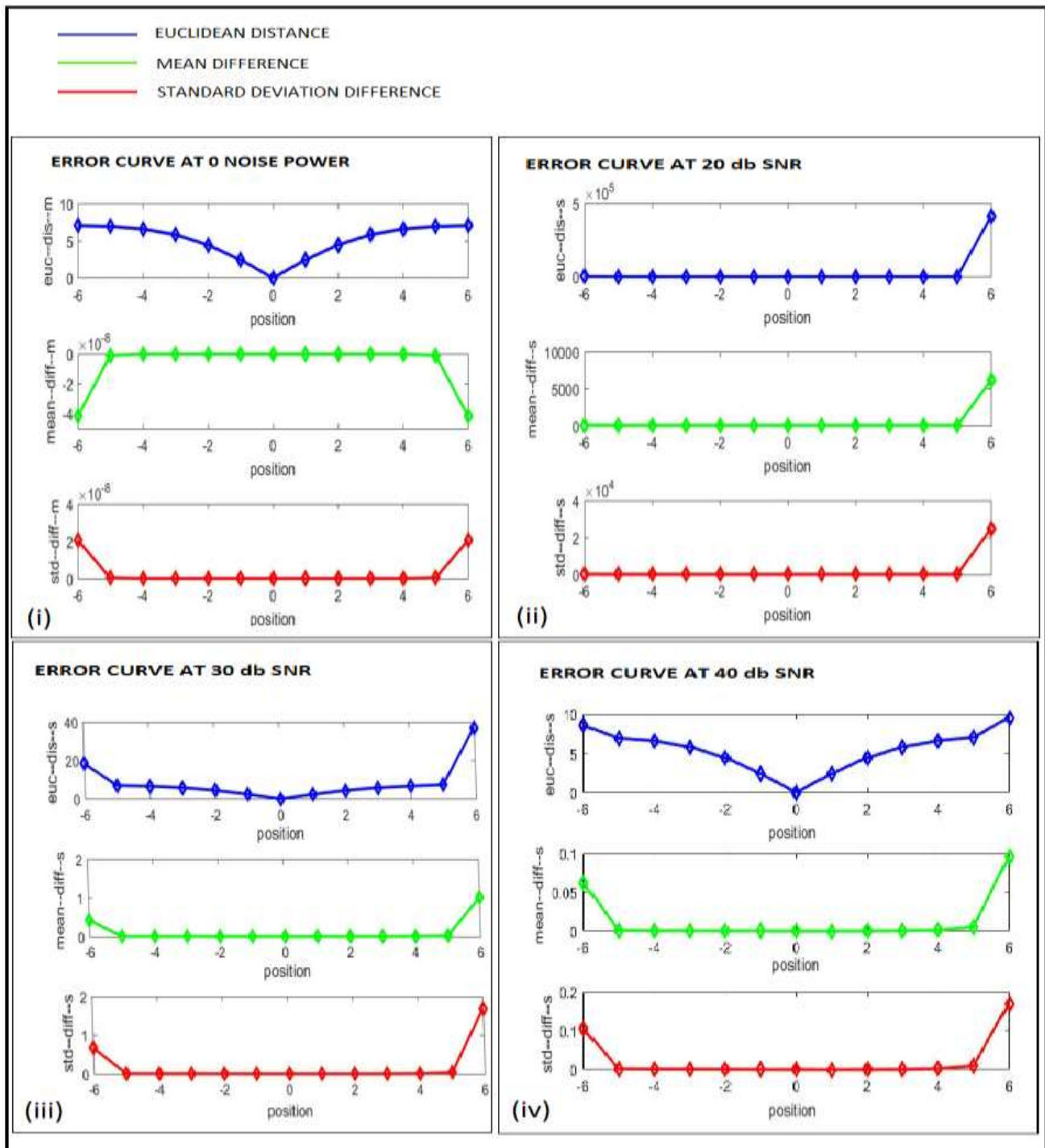

Fig - 8: Error analysis of Gaussian fitted curves at showing the performance of the system at different SNRs.

Figure 8 graphs the analysis of the errors in curve fitting and centroid detection capability of the system for different signal-to-noise ratios. The figure 8(i) shows the expected error in curve fitting when there is no noise in

system, and figure 8(ii), figure 8(iii) and figure 8(iv) shows the error curves at 20db, 30db, 40db of SNR respectively. The error curves are generated by determining the total Euclidian distance, mean difference and standard deviation of the curve plotted for beam with noise at different SNRs and comparing it to Euclidian distance, mean difference and standard deviation of the curve plotted for beam without noise.

The angle-of-arrival can be computed using formula

$$A - o - A = F\_o\_V_{MAX} \ \frac{L}{L_{MAX}} \qquad (10)$$

Where $A\text{-}o\text{-}A$ is the angle-of-arrival, $F\_o\_V_{MAX}$ is the maximum field of view, L is the distance of the centroid from the origin and L_{MAX} is the maximum possible distance of the centroid from the origin.

Further, it is found that the maximum entrance angle for the setup is ±27. 5° i.e. maximum angle-of-arrival possible is ±27. 5°. The maximum possible centroid distance from origin L_{MAX} increases when SNRs decreases for both +A-o-A and –A-o-A. The system takes samples at nine points using nine photo-detectors having 5.36 mm pitch between centers of active areas of two consecutive photo-detectors.

Table - 1: L_{MAX} at different SNRs for given $F\_o\_V_{MAX}$

$F\_o\_V_{MAX}$	SNR 20db	SNR 30db	SNR 40db
-27.5°	104.252mm	42.210mm	34.711mm
+27.5°	224.905mm	45.64576mm	35.531mm

Therefore, at an SNR of 40db angle-of-arrival is

$$A - o - A = 27.5° \ \frac{L}{35.531mm} \qquad (11)$$

The angular resolution of the system can be computed by using the formula

$$R_\theta = \frac{F\_o\_V_{MAX}}{S} \qquad (12)$$

Where R_θ denotes the angular resolution and S denotes the number sampling instants in the Gaussian fitted curve. The sampling rate used in the experiment is 261, and the angular resolution of the system is 0.105°.

5. CONCLUSION

With the fitting of Gaussian curves at different signal-to-noise ratio for the incident laser beam, it is found that the location of centroid shifts away from mean position as the SNR decreases. This results in the different position of the centroid for maximum and minimum A-o-A at different SNRs. It is found that the position of centroid of the Laser beam is different at different angle-of-arrival and moves correspondingly with the change in angle-of-arrival. The system works with the assumption that laser beam has a Gaussian intensity profile. Theoretically, the system is able to measure the angle-of-arrival with the very high angular resolution, depending upon the number of sampling instants in the Gaussian fitted curve. The resolution can be increased by increasing the sampling instants in the Gaussian fitted curve. With 261 sampling points the resolution of the system is 0.105°. Since minimum three detectors are to be illuminated for Gaussian curve fitting, total field of view achieved is 55°. This is less than other techniques mentioned in introduction section. The minimum SNR required for the operation of system is 30db as below this the errors in curve plotting increase drastically. This technique can be used as a simple approach for designing angle-of-arrival sensors for finding the direction of incoming laser radiation on combat vehicles.

REFERENCES

[1] David S.Dunavan & Richard babish (Feb. 21, 1989), "optical direction of arrival sensor with cylindrical lens". Us4806747

[2] David S.Dunavan, Rene Abren, Stephen Messelt & Edward Slebert (aug.15, 1989), "optical direction sensor having grey code mask spaced from a plurality of interdigitated detectors". Us4857721

[3] Jacques Dubois, André Cantin & Maurice gravel (Jun. 27, 1995), "digital high angular resolution laser irradiation detector (HARLID)". Us5428215

[4] Jacques Dubois, André Cantin & Maurice gravel (Feb. 18, 1997), "analog high-resolution laser irradiation detector (HARLID)". Us5604695

[5] Patrice Marquet, Jean-Mare Lemaire & Philippe Dunouvion (aug.7, 1990), "optoelectronic device for detecting and locating a radiant source". Us4946277

[6] Bandres Miguel a., Gutierrez-Vega & Julio c. (2004). "Ince Gaussian beams". Opt. Lett. Osa.   144–146. Bibcode:2004optl...29..144b. Doi:10.1364/ol.29.000144. Pmid 14743992.

[7] https://www.academia.edu/15180935/optical_specifications_gaussian_beam_propagation_material_properties_optical_coatings.

[8] Jacobs Donald h. Fundamentals of optical engineering. Mc Graw-hill book co., 1943

[9] R. Paschotta, article on 'cylindrical lenses' in the encyclopedia of laser physics and technology, 1. Edition October 2008, Wiley-vch, ISBN 978-3-527-40828-3

[10]   Pastuchová Elena & Zákopčan Michal. (2015), "Comparison of Algorithms for Fitting a Gaussian Function Used in Testing Smart Sensors". Journal of Electrical Engineering. 66. 178-181. 10.2478/jee-2015-0029.

BIOGRAPHIES

Anand Kumar Rai
M.Tech Electronics Design Technology (pursuing), NIELIT Aurangabad.
B.Tech Dr. A.P.J. Abdul Kalam Technical University, Lucknow.

Saurabh Bansod
Scientist 'B', NIELIT Aurangabad.
M.Tech in Electronics & Instrumentation, N.I.T. Rourkela.
B.E. in Electronics

Satya Prakash
Scientist 'F', LASTEC, DRDO.
B.tech in Electronics Communication Engineering, Delhi College of Engineering.

Ajay Rathee
Scientist 'E', LASTEC, DRDO.
M.Tech in Computer Technology, IIT Delhi.
B.Tech, M.D.U Rohtak, Haryana.