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Determination of Center of Mass and Radius of Gyration of Irregular Buildings and its Application in Torsional Analysis

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Abstract – *The geometrical properties plays an important role in the design and analysis of building structures and its components. It is a key element in determining the strength and stability of the structures. Center of Mass (CM) and radius of gyration are two of the important geometrical properties of the building which are considered as the basic determiner of the stability of the building under static, dynamic and torsional actions. Determination of these parameters based on manual calculation is easy for the regular building. For irregular buildings, the manual calculation becomes cumbersome and commercial software is preferred for determining these parameters. However, the use of computer software can be considerably resource intensive. In this paper, different existing methods of manual calculation of the position of center of mass and radius of gyration are introduced, their reliability is checked for real building cases, their application is reviewed and the most robust and accurate method applicable for all type of irregular buildings is recommended. Finally, the application of center of mass and radius of gyration in the torsional analysis is discussed.*

Key Words: **Center of Mass, Radius of Gyration, Irregular Buildings, Torsion, Torsional Stability**

1. INTRODUCTION

Irregular buildings (buildings with an asymmetrical plan) are becoming more popular day by day and are commonly used in the construction industries. Irregular and complex shapes are often preferred for aesthetics and optimising architectural functionality. It is also preferred sometimes by the clients to represent their distinctive identity, values and culture, and for achieving extraordinary futuristic impressions compared to typical regular buildings [1]. In urban areas, the irregular building is also considered to resolve the issues of the asymmetrical shape of the plots. Constructing a building that matches the shape of the plot can provide an optimal function to the building and economic values to the client. However, building with irregular shapes are not favoured by structural engineers because of the difficulty in structural analysis and design. There is also a lack of simple and proven manual calculation methods for analysing such complex structures. Hence, commercial software are mostly used for the design and analysis of these buildings. However, the use of commercial software can be resource-intensive. Similarly, the lack of proper knowledge of the software can lead to inaccurate analysis and design. This emphasizes the requirement and importance of the manual methods of structural design and analysis. Manual calculations are also valuable for quick design checks, during the conceptual design, and the verification of the results of the commercial software. In this regard, this paper provides insight into the manual calculation of two major building geometrical properties: Center of Mass (CM) and radius of gyration (r_g) and provides ways to use these parameters in determining torsional stability of the building.

1.1 Center of Mass

In the context of structural engineering, the Center of Mass (CM) is the point in the building where the entire mass of a building is assumed to be concentrated. It is also considered as the location of application of the gravitational and the inertial force in the building. Generally, the geometric center or the centroid of the building is considered as the CM of the building assuming the uniform mass density. For regular shapes without hollow or open building plan, the CM is located inside the perimeter of the building. However, for building with irregular plans such as L-shaped building plan and hollow round/circular buildings, it may be located outside of the building plan. The location of center of mass is considerably valuable for structural analysis and design of the building and its components. It helps in determining the location of the application of lateral loads, building eccentricities, and various phenomenon related to the stability of the building. Torsional action is one of the major phenomena in building with irregular floor plans. Torsional action in the building can induce significant stresses, especially in the case of lateral loads such as wind and earthquake. The response of such building subjected to lateral force tends to be significantly stronger due to twisting of the building. In building with regular floor plans, these effects are not dominant and the lateral load mainly cause the translational displacement of the building [2, 3].

1.2 Mass Radius of Gyration

Mass Radius of gyration or simply radius of gyration of the building is defined as the radial distance from the center of mass, about which the moment of inertia of the total concentrated mass of the building is equal to the moment of inertia of the building's actual distribution of mass. It is mathematically equal to the square root of the mass moment of inertia divided by the mass of the building. In structural engineering, the radius of gyration is used to describe the distribution of cross-sectional area in structural members, estimation of stiffness, and torsional rigidity. In this paper, only the importance of radius of gyration in torsional rigidity is considered. Similar to the center of mass, the two-dimensional radius of gyration is mostly considered during the structural analysis, assuming that the building has uniform mass density. Because of this assumption, the radius of gyration is determined based on the area instead of mass. In term of area, the radius of gyration is the result of the square root of the area moment of inertia divided by area of the building. For a vertical axis, the radius of gyration is the result of the square root of the polar moment of inertia divided by area of the building. The radius of gyration about the vertical axis of the building plays an important role in determining the torsional rigidity or stability in the building. When the torsional stiffness radius about the center of mass of the building is smaller than the radius of gyration of the building, the building is considered to be torsionally flexible or unstable. Similarly, when the torsional stiffness radius of the building about its center of mass is greater than the radius of gyration of the building, the building is considered as torsionally stable or torsionally stiff/restrained [4].

2. LITERATURE REVIEW

The torsional response of building with irregular shape under lateral force makes the design of building more complicated compared to the design of a more regular building. Design of irregular building for lateral loads such as wind and earthquake is still considered as a new area in research. Several codes have recommended the use of accidental and dynamic eccentricities for considering the effect of torsion during the equivalent static method of analysis. Recently, countries in the higher risk of wind and earthquake actions have introduced dynamic methods instead of an equivalent static method for the design of irregular buildings. Due to the complexity of the torsional behaviour of the building, the use of commercial software is mainly preferred over the manual calculation for the torsional analysis. However, several studies have shown that the determination of approximate torsional stability in the building is quite straight forward once the radius of gyration, translational stiffness and the rotational stiffness of the building is known [5, 6, and 7]. However, it is very important to understand that the determination of torsional stability in the multi-storey building is not straight forward because of the difference in the position of the center of mass and center of rigidity in each storey of the building. Another recent study has provided a more accurate generalised solution for the determination of the torsional stability in multi-storey building [8]. In all of the above methods, the radius of gyration of the building is the key parameter in determining the torsional stability of the building. The radius of gyration of the building must be calculated accurately to accurately determine the torsional behaviour in the building. To determine the radius of gyration, the position of the building's centre of mass, and it's polar moment of inertia need to be determined. The most common method for determining the location of the centre of mass of irregular building are the Plumb Line Method [9], Geometric Decomposition, and Coordinate Method [10]. Similarly, the most common method for determining the polar moment of inertia of the irregular buildings are Integral Method [11], and Coordinate Method [12]. These methods are discussed in Section 3 of this paper.

3. METHOD AND METHODOLOGY

The Center of Mass and radius of gyration of the two case study buildings are calculated using the Geometric Decomposition Method, Integral Method [11] and Coordinate Method [10]. The simplicity of the process is assessed and the results are compared with the results from commercial software SPACE GASS to find out the most reliable method.

3.1 Calculation of Center of Mass of the Irregular Building

The center of mass or centroid of the building can be determined mainly by three methods:

a. **Plumb Line Method**

In this method, a cardboard piece of the model of the building is required. The center of mass is then determined experimentally by using a pin and plumb line. The cardboard is held by the pin in such a way that it can rotate freely. Then a line is drawn along the plumb line and a similar procedure is repeated by placing the pin in another position of the cardboard to draw another line. The intersection of the two lines becomes the center of mass of the building plan. If a cardboard model is prepared during the conceptual design stage of the building, this method can be used with ease. As this method is not feasible and accurate without the preparation of an exact scaled model made of homogenous material and thickness, it is not used in this paper for the comparison.

b. **Geometric Decomposition Method**

In this method, the classical equation for the centroid is used by breaking the irregular shapes into a number of regular shapes. For example, for a non-uniform C-shaped building, the building is divided into blocks of web and flanges as shown in Fig- 1 and the position of the centre of mass (c_x, c_y) of the building is calculated using Equation 1 (a) and (b).

$$c_x = \int_{i=1}^{n} \frac{a_i \times x_i}{A} \qquad (1a)$$

$$c_y = \int_{i=1}^{n} \frac{a_i \times y_i}{A} \qquad (1b)$$

Where,

a_i is the area of block '*i*',

x_i and y_i are the X and Y-coordinates of the centroid of the block '*i*', and

A is the total area of the building.

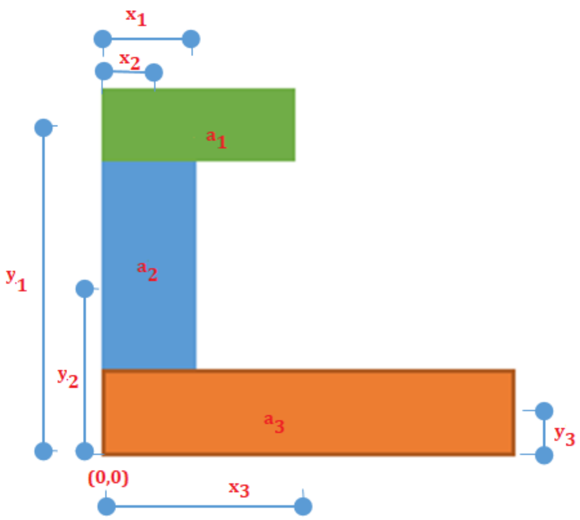

Fig -1: Determination of centroid by Geometric Decomposition Method

c. Coordinate Method

In this method, the centroid of the irregular building is determined by assigning the coordinates to each vertex of the building plan. Based on the coordinates, the position of the center of mass is determined either by Equation 2(a) and 2(b) or Equation 3 (a) and 3(b) given by [10]. For this equation to be used, the building plan needs to be a non-self-intersecting closed area.

$$c_x = \int_{i=0}^{n-1} \frac{x_i}{n-1} \qquad (2a)$$

$$c_y = \int_{i=0}^{n-1} \frac{y_i}{n-1} \qquad (2b)$$

$$c_x = \frac{1}{6A} \int_{i=0}^{n-1} (x_i + x_{i+1})(x_i\, y_{i+1} - x_{i+1}\, y_i) \quad (3a)$$

$$c_y = \frac{1}{6A} \int_{i=0}^{n-1} (y + y_{i+1})(x_i\, y_{i+1} - x_{i+1}\, y_i) \quad (3a)$$

Where,

x_i is the x-coordinate of vertex 'i' of the building,

y_i is the y-coordinate of vertex 'i' of the building,

A is the total area of the building calculated using the Equation 4.

$$A = \frac{1}{2} \int_{i=0}^{n-1} (x_i\, y_{i+1} - x_{i+1}\, y_i) \qquad (4)$$

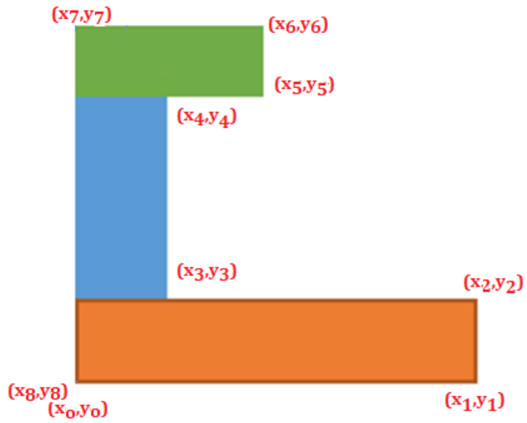

Fig -2: Determination of centroid by Coordinate Method

3.2 Calculation of Polar Moment of Inertia of the Irregular Building

To calculate the polar moment of inertia of the building, it's principal area moment of inertia are required. There are mainly two proven methods for calculating the area moment of inertia: Integral Method and Coordinate Method.

a. Integral Method

In this method, the area moment of inertia of the building (I_x, I_y) is determined by the product of the area of the building multiplied by the distance from the reference point as shown in Equation 5 (a) and (b).

$$I_x = \iint y^2 dx\, dy \qquad (5a)$$

$$I_y = \iint x^2 dx\, dy \qquad (5b)$$

$$I_z = I_x + I_y \qquad (6)$$

Where,

dxdy is the area of the building,

x and *y* are the X and Y-axis distance from the reference point, and

Ix, Iy and *Iz* are the area moment of inertia about X and Y-axis and polar moment of inertia about vertical Z-axis respectively.

For a rectangular building with length 'L' and width 'B' as shown in Fig-3, the above integral equation becomes:

$$I_x = \int_{-L/2}^{L/2}\int_{-B/2}^{B/2} y^2 dy\, dx = \int_{-L/2}^{L/2} \frac{b^3 dx}{4\times 3} = \frac{LB^3}{12}$$

$$I_y = \int_{-B/2}^{B/2}\int_{-L/2}^{L/2} x^2 dx\, dy = \int_{-B/2}^{B/2} \frac{l^3 dy}{4\times 3} = \frac{BL^3}{12}$$

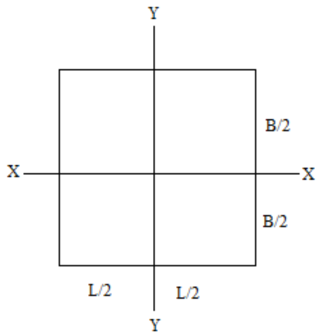

Fig -3: Building plan with length 'L' and width 'B'

b. Coordinate Method

In this method, the area moment of inertia of the irregular building from the reference point (0,0) is determined by assigning the coordinates to each vertex of the building plan. Based on the coordinates, the moment of inertia is determined using Equation 7(a) and 7(b) given by [12]. For this equation to be used, the building plan needs to be a non-self-intersecting closed area.

$$I_x = \frac{1}{12}\int_{i=0}^{n-1} ({x_i}^2 + x_i x_{i+1} + {x_{i+1}}^2)(x_i\, y_{i+1} - x_{i+1}\, y_i \qquad (7a)$$

$$I_x = \frac{1}{12}\int_{i=0}^{n-1} ({y_i}^2 + y_i y_{i+1} + {y_{i+1}}^2)(x_i\, y_{i+1} - x_{i+1}\, y_i) \qquad (7b)$$

The moment of inertia about the centroid is determined using the parallel axis theorem:

$$I_{xg} = I_x - A{c_x}^2 \qquad (8a)$$

$$I_{yg} = I_y - A{c_y}^2 \qquad (8a)$$

Where,

x_i is the x-coordinate of vertex 'i' of the building,

y_i is the y-coordinate of vertex 'i' of the building,

I_{xg} and I_{yg} are the area moment of inertia about the centroid of the building, and

A is the total area of the building determined from Equation 4.

3.3 Calculation of Radius of Gyration of the Irregular Building

The radius of gyration (r_g) is calculated using Equation 9, based on the polar moment of inertia determined from Equation 6 and the area of the building.

$$r_g = \sqrt{\frac{I_{zg}}{A}} \qquad (9)$$

3.3 Determination of Center of Mass and Radius of Gyration from SPACE GASS

SPACE GASS is a commercial structural engineering design and analysis software. It provides the feature of calculation of the centroid and the radius of gyration of any shape. The building shape needs to be defined in the shape builder window and then it automatically calculates the geometrical properties based on the provided building shape. The case studies building are modelled in SPACE GASS to determine the center of mass and radius of gyration and the results are used to determine the accuracy of the methods discussed above.

4. RESULTS AND DISCUSSION

Two real case study buildings are used for the assessment of the accuracy of the results of the manual calculation methods. The floor plan of these buildings are shown in Fig- 4 and Fig-5.

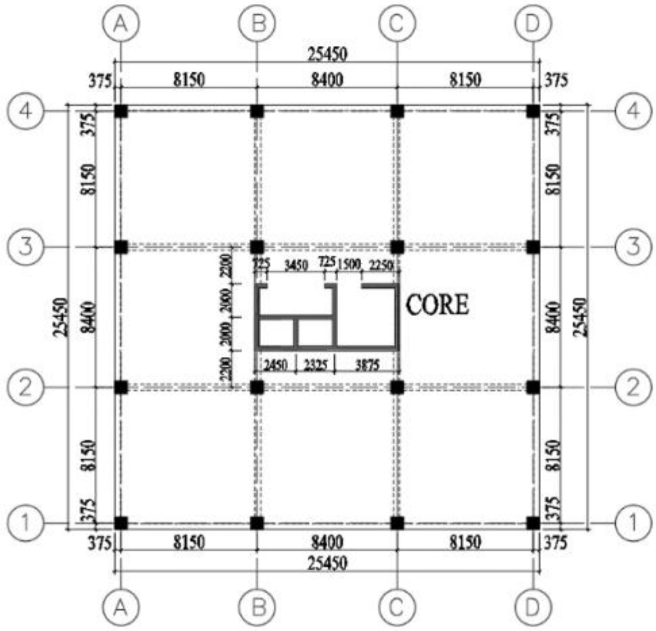

Fig -4: Floor plan of case study building 1

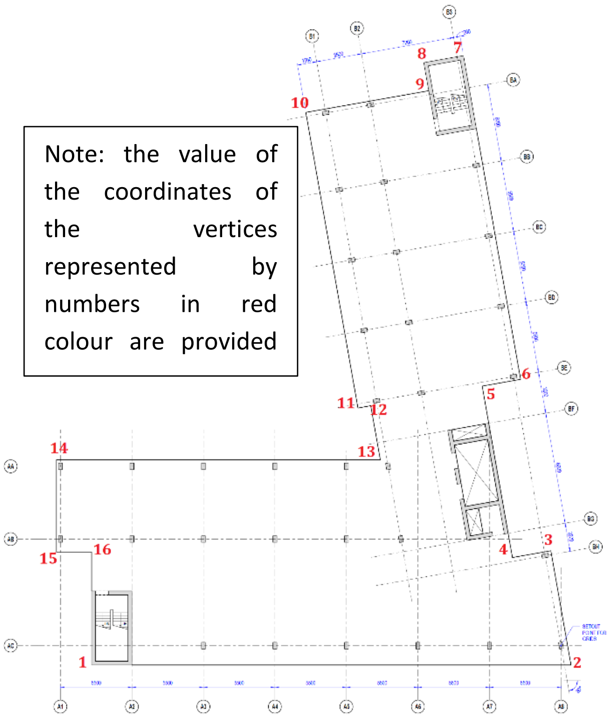

Fig -5: Floor plan of case study building 2 [13]

From the floor plan layout, the coordinates of the vertices of the building are determined, and the position of the center of mass is calculated using Geometric Decomposition Method (GDM), Coordinate Method (CoM) and from SPACE GASS as discussed in section 3. The results are shown in Table 1 and Table 3 for the case study building 1 and 2 respectively. Similarly, the moment of inertia of the buildings are calculated using Integral and Coordinate Methods and are summarized in Table 2 and Table 4. The calculation of the radius of gyration and its comparison are shown in Table 5 and 6. As Integral Method is cumbersome to use for irregular buildings, only Coordinate Method is used for the case study building 2.

Table-1: Result of the position of center of mass of case study building 1 from different methods

no	x_i (m)	y_i (m)	Location of center of mass (C_x, C_y) (reference point is A1)			
			GDM	CoM 1 (Eq 2)	CoM 2 (Eq 3)	SPACE GASS
1	0	0	C_x= (25.45 /2) =12.725 m =C_y	C_x= [(0+25.45+25.4+0)/4] = 12.725 m =C_y	C_x=[(25.45×25.45×50.9 + 25.45×25.45×25.45/(6×25.45×25.45)) =12.725 m = C_y	C_x = 12.725 m = C_y
2	25.45	0				
3	25.45	25.45				
4	0	25.45				
5	0	0				

Table-2: Result of the moment of inertia of case study building 1 from different methods

Moment of Inertia			
Integral Method		Coordinate Method (reference point is A1)	
I_x (m^4)	I_y (m^4)	I_x (m^4)	I_y (m^4)
= 25.45× 25.45³ /12 = 34959.9	=I_x =34959.9	[(25.45×25.45 ×(2×25.45²+ 25.45×25.45) + 25.45×25.45×(25.45²)]/12 = 139839.51	=I_x =139839.51

Table-3: Result of the position of center of mass of case study building 2 from different methods

no	x_i (m)	y_i (m)	Location of center of mass (C_x, C_y) (reference point is A1-AC)			
			GDM	CoM 1 (Eq 2)	CoM 2 (Eq 3)	SPACE GASS
1	2.4	-1.5	-	(22.55, 19.33)	(22.09, 15.25)	(22.09, 15.25)
2	39.3	-1.5				
3	37.7	7.6				
4	34.7	7.1				
5	32.4	20.0				
6	35.4	20.6				
7	31	45.6				
8	27.9	45.1				
9	28.3	43.0				
10	18.6	41.3				
11	22.6	18.3				
12	24.1	18.6				
13	24.8	14.6				
14	-0.4	14.6				
15	-0.4	7.9				
16	2.4	7.9				
1	2.4	-1.5				

Table-4: Result of the moment of inertia of case study building 2 from different methods

Moment of Inertia (reference point is A1-AC)			
Integral Method		Coordinate Method	
I_x (m^4)	I_y (m^4)	I_x (m^4)	I_y (m^4)
-	-	528597.3	363354.8

Table-5: Result of the radius of gyration (r_g) of case study building 1

Integral Method		Coordinate Method	
I_{zg} (m^4)	r_g (m)	I_{zg} (m^4)	r_g (m)
34959.9 ×2	$\sqrt{\frac{34959.9 \times 2}{25.45^2}}$ = 10.4	(139839.5 1- 25.45^2 × 12.725^2) × 2 =34959.9× 2	$\sqrt{\frac{34959.9 \times 2}{25.45^2}}$ = 10.4
The value obtained from SPACE GASS is 10.4 m.			

Table-6: Result of the radius of gyration (r_g) of case study building 2

Coordinate Method 1		Coordinate Method 2	
I_{zg} (m^4)	r_g (m)	I_{zg} (m^4)	r_g (m)
528597. 3+3633 54.8- (917.31 × 22.55^2 +917.31 × 19.33^2) =82745. 8	$\sqrt{\frac{82745.8}{917.31}}$ = 9.5	528597.3+ 363354.8- (917.31× 22.09^2 +917.31× 15.25^2) =231002.2	$\sqrt{\frac{231002.2}{917.31}}$ = 15.87
The value of r_g obtained from SPACE GASS is 15.87 m.			

From the comparison of the results of the location of the center of mass and radius of gyration, it is found that the Coordinate Method 2 using equations 3, 7, 8 and 9 give the accurate result as similar to the SPACE GASS. This method is simple to use for building with a lower number of vertices. For more irregular building with multiple vertices, an excel spreadsheet may be used. The Geometric Decomposition Method is only efficient for calculating the centroid of the regular building. Likewise, the Integral Method is best suited for square/rectangular or more regular building structures. Coordinate Method 1 which uses Equation 2 to determine the centroid of the building is only accurate for regular structures and for irregular structures, it's is least accurate. It gave 40% lesser values of radius of gyration compared to Coordinate Method 2 (using Equation 3 to determine centroid). For regular building structures, all the methods are found to be accurate.

The calculation of the center of mass and radius of gyration of the building are required to determine the torsional stability in the building. An example of determining torsional stability in the building based on [4] for case study building 2 is provided below.

Assuming a mono-symmetric single-storey system, for torsionally rigid/stable building the following condition should be satisfied.

$$\frac{k_{torsional}}{k_{translational}} + e^2 > r_g^{\,2} \qquad (10)$$

Where, $k_{torsional}$ is the torsional stiffness of the building, $k_{translational}$ is the lateral stiffness of the building, e is the distance between center of mass and Center of rigidity (which is considered 0 for this example), and r_g is the radius of gyration of the building about the vertical axis.

For case study building 2, the main lateral load-carrying elements are structural walls. The flexural lateral stiffness of these walls can be considered as equivalent to the moment of inertia of the walls because the other parameters cancel each other when calculating the ratio of torsional and translational stiffness. The torsional stiffness can be calculated using Equation 11.

$$k_{torsional} = \sum (k_{translational,y}\, x_i^{\,2} + k_{translational,x}\, y_i^{\,2}) \qquad (11)$$

Where x_i and y_i are the X-axis and Y-axis distance of walls from the center of mass of the building.

The building has three core walls with the moment of area and distance from CM as shown in Table 7.

Table-7: Moment of inertia and distance from CM of the core walls of the case study building 2

	I_x (m^4)	I_y (m^4)	x_i (m)	y_i (m)	$k_{torsional}$
Wall 1	18	7.5	-18	-14	18×14²+7.5×18² =5958
Wall 2	49	11	10	3	1541
Wall 3	18	7.5	8	27	13602
Sum	85	26	-	-	21101

Substituting the values of the sum of translational stiffness and torsional stiffness into Equation 10 gives:

About X − direction,

$$\frac{21101}{26} + 0 = 812 > r_g^{\,2}(= 252)$$

About Y − direction,

$$\frac{21101}{85} + 0 = 248 < r_g^{\,2}(= 252)$$

It signifies that the building is torsionally stable/stiff in X-direction and it is torsionally flexible in Y-direction. However, considering Eurocode [14], which state that if the result in the left-hand side in the above calculation is greater than $(0.8\, r_g)^2$ the building is torsionally stable. For such a scenario, the case study building 2 is torsionally stable in both directions.

5. CONCLUSIONS

The position of Center of Mass (CM) and the radius of gyration are important geometrical properties of the building. These properties determine the stability of the building against static, dynamic and torsional actions. As the irregular building is highly impacted by the torsional actions, the position of center of mass and radius of gyration of these buildings are more important than regular buildings. The position of center of mass can influence eccentricity in the building resulting in the amplification of design actions. Understanding of these geometric properties of the building is important in achieving design efficiency. In this paper, mainly three methods of manual calculation of the location of CM and the radius of gyration were discussed. These methods were used to determine the CM and radius of gyration for two case study buildings and the accuracy of these methods were checked with the results from SPACE GASS software. All the methods considered in this paper are found accurate for a regular building case. Whereas for irregular buildings only Coordinate Method 2 is found to be the robust and accurate method. Calculation of accurate position of center of mass and the value of the radius of gyration plays important role in determining the torsional stability in the building.

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BIOGRAPHY

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