COLOURING OF GRAPH AND ITS PROPERTIES

Dr. Pankaj Kumar Chaudhary

Assistant Professor, Department of Mathematics, Dr. APJ Abdul Kalam Women's Institute of Technology, L. N. Mithila University, Darbhanga, Bihar, India

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ABSTRACT:- Let G be a graph. The assignment of colours to the vertices of G, one colour to every vertex, in order that the adjacent vertices are assigned different colours is called vertex colouring or colouring of the graph G. A graph G is n-colourable if there exists a colouring of G which apply n colours. The minimum number of colours required to paint a graph G is called the chromatic number of G and is denoted by $\chi(G)$. It is concerned with the upper bound on the chromatic number for graphs of maximum vertex degree.

KEYWORDS: Chromatic number, Vertex degree, Spanning tree, Connected graph, Circuit

INTRODUCTION

It deals with a subdiscipline of graph theory known as graph colouring. While the word graph is common in mathematics courses as far back as introductory algebra, usually as a term for a plot of a function in graph theory the term takes on a different meaning. In the context of graph theory, a graph is a collection of vertices and edges, each edge connecting two vertices.

Theorem 1

If the maximum vertex degree of a graph G is Δ, then $\chi(G) \leq \Delta + 1$.

Proof. We elect any arbitrary vertex in G and colour it with one among of the $\Delta + 1$ available colour. We then pick any uncoloured vertex in G and colour it with a colour that has not been assigned to any of the vertices adjacent to it. We then repeat this last step until every vertex in G is coloured. Because any given vertex v is connected to at the most Δ vertices, there are often at the most Δ distinct colours already used on the vertices adjacent to v, so there will always be at least one colour available to colour v.

Theorem 2

For any graph, G, there is an order that can be assigned to the vertices of G for which the greedy colouring algorithm will use the graph's chromatic number of colours to properly colour G.

Proof. By the definition of the chromatic number, we know that at least one such colouring exists. Assign an arbitrary order to the colours used in the graph. Now, we consider the set of all graph colours that use the chromatic number of colours. Of these, we consider the subset of these colourings which use the first colour the maximum number of times. Of those colourings, we consider the subset of colourings which use the second colour the maximum number of times, and so on, until you are left with one colouring. In this colouring, we all know that any vertex coloured with the second colour must be adjacent to a minimum of one vertex coloured with the first colour. Similarly, any vertex coloured with the third colour will be adjacent to both a vertex of the first colour and a vertex of the second colour, and so on. In this graph, order the vertices as follows: place any vertex coloured with the first colour first in the order. Locate any unordered vertex coloured with the lowest-ordered colour obtainable next within the vertex order. Continue this process until all the vertices within the graph are ordered. Now, applying the greedy colouring algorithm to this ordered graph using the same colour order as used previously, we'll recreate the original graph colouring, which uses the graph's chromatic number of colours.

Theorem 3

Every connected graph has a minimum of one spanning tree.

Proof. If the graph contains no circuits, it is already a tree and is therefore its own spanning tree. If the graph contains at least one circuit, remove one edge from each circuit in the graph. The graph will still be connected, as a circuit implies that for any two vertices on the circuit, two distinct paths exist between those vertices. By removing one edge from the circuit, we remove one of these paths, which still leaves one path between those two vertices. Further, the removal of an edge from a circuit ensures that the remaining edges won't form a circuit. Now, because the new graph is connected and contains no circuits, it's a tree of the original graph. As we did not remove any of the original graph's vertices in this process, the new graph is a spanning tree of the original graph.

Theorem 4

For any connected graph, G, there's an order during which one can place the vertices of G such every vertex features a higher ordered neighbour, with the exception of the last vertex within the order.

Proof. Consider a spanning tree of G, which we'll call G_t. G_t contains all of the vertices of G, and any vertices that are adjacent in G_t are also adjacent in G. Now assume that all of the edges in G_t are given a weight, which we will call length. Furthermore, let each edge up G_t have a length of 1. Now choose any vertex v_0 in G_t. as long as vertex, we order the vertices of G_t, also because the corresponding vertices in G, as follows:

1. Find the vertex for which the path from that vertex to v_0 has the greatest length. If more than one such vertex exists, choose any of these vertices. Place this vertex first in the order.

2. Find the unordered vertex for which the path from it to v_0 has the greatest length. If more than one such vertex exists, choose any of these vertices. Place this vertex next in the order.

3. Repeat step 2 until all vertices in G_t except v_0 are ordered.

4. Place v_0 last in the order. It can easily be shown that there is only one distinct path between any two given vertices in a tree, as two paths between those vertices would imply that a circuit exists. Given this, by the above ordering algorithm, all the vertices in the ordered graph of G_t, and thus within the ordered graph of G, will have higher-ordered neighbours, except for v_0, which appears last in the order.

CONCLUSION

In this paper it is concerned with the upper bound on the chromatic number for graphs of maximum vertex degree. A graph is a collection of vertices and edges, each edge connecting two vertices. For any connected graph, G, there's an order during which one can place the vertices of G such every vertex features a higher ordered neighbour, with the exception of the last vertex within the order.

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