Weighted Fraction Method (WFM)

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Abstract - *In this paper, a new solution is proposed within the scope of the implicit methods used in the Multi- Attribute Decision Making (MADM). The proposed method is called Weighted Fraction Method (WFM).*

The proposed method aims to increase the capacity between the lowest and highest value in the results to make the ranking of the available options more clearly, thus reducing the convergence of results that appears in classic methods sometimes.

Two coefficients were proposed, coefficient of excellence (COE) and coefficient of anomalous (COA).

By using the proposed method, capacity in the analyzed examples was doubled up to 1.5 times by comparison with the three classical methods (WSM, WPM and TOPSIS), thus providing more space for the superlative between the options available to take the suitable decision

The proposed coefficients through results of example, proved their superiority over the other three methods, as well as proposed method show that its free from anomalies that found in other methods sometimes.

Key Words: MADM, TOPSIS, WSM, WPM, AHP

1. Research problem:

Multi Attribute Decision Making (MADM) is one of the methods adopted by decision makers in the world when there are multiple criteria that effect on decision making [1,2], but sometimes it is some convergence in the results [3].

List of Options	WSM
O8	0.05784
O4	0.05783
O3	0.04028
O7	0.04026
O13	0.04023

Figure-1: illustrates some convergence of results sometimes [3]

In the Technique for Order of Preference by Similarity to Ideal Solution method (TOPSIS), there is the problem of difficulty of consistency of judgment, as well as the Weighted Sum Method (WSM) does not always reflect the real situation [4].

In either of these two methods, there are anomalies in results, which make the decision to arrange them according to preference is unclear.

While some researchers point out that the weighted product method (WPM) is more effective than the other two methods for ease of application in cases with high subjective elements [5], so a new method was proposed in which the results are made more capacity than (WPM) and other two methods, making the appropriate decision clearer and easier.

The proposed method results in terms of rankings are identical to the (WPM) but with a wide range.

2. INTRODUCTION

The Multi Attribute Decision Making (MADM) method is proposed by Saaty Thomas. It is used when there are multiple parameters affecting on the end product with a limited number of predefined alternatives [6].

These methods require both comparisons outside and inside parameters to judge or make an unbiased decision suitably to solve the problem.

The MADM decision-making system is based on three main components:

1 - Alternatives (may be a set of experiments or goods of various types to be bought, car or mobile phone or others).

2 - Attributes (the resulting parameters or available specifications such as capacity, performance factor, the amount of production, price and storage capacity or others).

3. Weight (impact index of variation), or the strength of each resulting parameter (attribute) and measure the performance of each set of these attributes.

Sometimes the parameters of different weight, such as the results of experiments such as temperature, capacity and efficiency, each of its own importance as the need of the experimenter, and sometimes the parameters are equal weight, as a person wants to buy a mobile phone and each of these parameters such as brand, price, storage capacity and camera accuracy, all of equal importance to the user.

The common methods used under MADM [7] are:

2.1 Weighted sum method (WSM).

It is the simplest method in the group. In this method, the attributes and their weight are determined by the values given, if the large value is the most important, such as the storage capacity, for example, each value given within this parameter is divided by largest value to find the ratio of each criterion, according to the following formula:

$$X_{ij}^{*} = \frac{X_{ij}}{X_{max}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (1)$$

If the smallest value is the most important, this value is divided by the other values to find the ratio of each criterion according to the following formula:

$$X_{ij}^{*} = \frac{X_{min}}{X_{ij}} \dots\dots\dots\dots\dots..\dots\dots\dots\dots\dots\dots. (2)$$

The weight of each parameter (wj) is calculated as will be explained later.

The final values of the WSM method are calculated using the sum of the values (X*) after multiplying by the weight of each parameter as in the following equation:

$$C_i = \sum_{j=1}^{N} w_j (X_{ij}^{*}) norm \dots\dots\dots\dots.. (3)$$

The results are rearranged for (Ci) in descending order, the highest value represents the optimal choice, then the next, and so on.

2.2 Weighted Product Method (WPM).

This method is similar to the previous method (WSM) but differs only in the final equation.

In this method, the values of (X*) are multiplied after they rise to the power (wj), instead of adding it as in WSM.

$$C_i = \prod_{j=1}^{N} [(X_{ij}^{*}) norm]^{w_j} \dots\dots\dots\dots\dots. (4)$$

2.3. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS).

This method is more complex than the previous two methods, and summarizes the following steps:

i - The attributes and their significance are determined by the values given as previously, but in this way the root of the sum of its squares is calculated as follows:

$$X_{avg} = \left[\sum_{j=1}^{N} X_{ij}^{2}\right]^{0.5} \dots\dots\dots\dots\dots\dots.. (5)$$

ii - Each value given within this parameter is divided by (Xavg) to find the ratio of these values:

$$X_{ij}^{*} = \frac{X_{ij}}{X_{avg}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (6)$$

iii- After completing the calculation of the previous step, we get the weighting matrix from the following equation:

$$B_{ij} = w_j X_{ij}^{*} \dots\dots\dots\dots\dots\dots\dots\dots\dots.\dots.. (7)$$

iv- For each column in the matrix, which represents the status of the value of this parameter we extract the best case is ideal (B +) and the worst case (B-) and depending on their impact.

In the selection of the price or cost parameter will be the lowest value in the column for this parameter is the ideal value (B +) and the highest value in the column is the worst (B-), but if the parameter is for power or amplitude, the highest value in this column will be the ideal (B +) and the lowest value will represent the worst case (B-).

v- Calculate the square root of each parameter for the ideal and worst case of the row within the array as follows:

$$A^{+} = \left[\sum_{j=1}^{N} (B_{ij} - B^{+})^{2}\right]^{0.5} \dots\dots\dots (8)$$

$$A^{-} = \left[\sum_{j=1}^{N} (B_{ij} - B^{-})^{2}\right]^{0.5} \dots.\dots\dots (9)$$

vi- Find the value of (Ci) from the following equation, which represents the final values of the decision after sorting them in descending order as in the other ways.

$$C_i = \frac{A^{-}}{A^{-} + A^{+}} \dots\dots\dots\dots\dots\dots\dots. (10)$$

3. Weight of attributes:

As before, the weight of each parameter or attribute (wj) is necessary to complete solution by the previous three methods.

There are two types of weight calculation of parameters, as follows:

3.1. Similarly important parameters:

The person who determines the importance of the attributes is the beneficiary, It means that the experimenter, the party wishing to purchase or others, they determine the importance of each parameter depending on the nature of the issue.

To find the weight of each parameter in a set of equal importance, its weight will be the inverted of attribute number.

$$w_j = \frac{1}{N} \dots\dots\dots\dots\dots\dots\dots\dots\dots (11)$$

3.2. Different important parameters:

One of the most common methods to find the weight of different parameters is the Analytic Hierarchy Process (AHP); this method is summarized in the following steps:

i- Creating goals and evaluating the attributes, that which is done through the evaluation of the decision makers or the person wishing to buy.

ii- Finding the relative importance of the attributes, resulting in the A1 matrix as follows:

	b1	b2	b3
b1	1	b12	b13
b2	b21	1	b23
b3	b31	b32	1

Noting that b21 is the inverse of b12.

iii- Find the relative weight of each attribute through the following equation:

$$GM_i = \left[\prod_{j=1}^{N} b_{ij}\right]^{1/N} \dots\dots\dots\dots\dots\dots (12)$$

Where: i = 1 ⋯ N

The resultant is as the following matrix:

$$\begin{bmatrix} GM_1 \\ \vdots \\ GM_N \end{bmatrix}$$

iv- The weight of the parameters is found by dividing the relative weight of each attribute by the total relative weight of the parameters, as follows:

$$w_j = GM_j / \sum_{j=1}^{N} GM_j = \mathrm{A}_2 \dots\dots\dots (13)$$

Important Note: In both methods to calculate the weight of the parameters, the sum of weights of the parameters must be equal to one.

$$\sum_{j=1}^{N} w_j = 1$$

After finding the weight of the parameters in the second way, this resultant must be checked as followed by the proposed Satty method.

The matrices A3 and A4 are calculated as follows:

A3 = A1×A2

A4 = A3 / A2

The matrix A2 represents the weight of the parameters calculated from equation (13).

$$A2 = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

Find the maximum lambda value (λmax) of the A4 matrix and represent the average value of the elements of the A4 matrix.

Calculate the conformity index *CI* as follows:

$$CI = \frac{(\lambda_{max} - N)}{(N-1)} \dots\dots\dots\dots\dots\dots (14)$$

Where N represents the number of rows of the A4 matrix, and the smallest *CI* value represents the deviation.

Calculate the ratio of consistency *CR* as follows:

$$CR = {CI}/{RI} \dots\dots\dots\dots\dots\dots\dots\dots (15)$$

Where: *RI* is a random index that is extracted from the following table [8, 9]:

Table -1: Random index proposed by satty.

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

It should be noted that the value of *CR* must be less than (0.1)

When this condition is proved, the weight of the wj parameters calculated in equation 13 is acceptable for analysis according to the three methods explained above; otherwise the values of the attributes must be re-evaluated.

4. Proposed method:

The suggested method is named weight fraction method (WFM), and this method is based on a new technique that expands the differences between the results, to make the results clearer than the three methods (WSM, WPM and TOPSIS) described above.

The solution of this method is according to the following steps:

i- After the attributes and their weights are determined by the given values, and as previously explained in the (WSM), find the difference between the largest and the lowest value of each attribute given in the problem.

$$X_{dif} = X_{max} - X_{min} \quad \dots\dots\dots\dots\dots\dots (16)$$

ii- If the highest value for the attribute is the most important, each given value within this parameter is divided by X_{dif} to find X^* according to the following equation:

$$X^*_{ij} = \frac{X_{ij}}{X_{dif}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (17)$$

If the minimum value for the attribute is the important, the X_{dif} value is divided by each given value to find X^* according to the following formula:

$$X^*_{ij} = \frac{X_{dif}}{X_{ij}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (18)$$

iii- The final result of the proposed method is found as follows:

$$C_i = \prod_{j=1}^{N} \left[(X^*_{ij}) norm \right]^{(N * w_j)} \quad \dots. (19)$$

Where: N represents the number of available attributes.

The results are rearranged for (Ci) in descending order, the highest value represents the optimal choice, then the next, and so on, as in previous methods.

Example:

The following experiments were done to find the temperature difference ΔTc, coefficient of performance COP and cold air mass ratio μ of the Vortex Tube [7].

Table – 2: experimental results for ΔTc, COP, and μ

Experimental results			
Exp	ΔTc	cop	μ
N1	66.4	0.103644	0.923076923
N2	52.2	0.090148	0.896551724
N3	62.7	0.093995	0.846153846
N4	49.0	0.083057	1
N5	50.6	0.083068	0.941176471
N6	48.1	0.305182	1
N7	55.5	0.230179	1
N8	59.9	0.190488	1
N9	63.4	0.169185	1
N10	64.8	0.149352	1
N11	46.9	0.106654	0.947368421
N12	45.1	0.104375	0.8
N13	54.5	0.092640	0.925
N14	41.8	0.095466	0.928571429
N15	54.5	0.095466	0.857142857
N16	40.7	0.14284	0.756756757
N17	44.5	0.122115	0.904761905
N18	45.5	0.102501	0.979591837
N19	47.2	0.092424	0.98245614
N20	40.4	0.093052	0.967741935
N21	41.0	0.139263	0.5
N22	31.0	0.100402	0.916666667
N23	50.4	0.116153	0.380952381
N24	44.8	0.087671	0.806451613
N25	59.0	0.083825	0.580645161

After solving the example according to the four methods described, the results are as shown in the table (3) below:

Table -3: ranking of WSM, WPM, and TOPSIS compared with the proposed method.

Ranking	No.	WSM	No	WPM	No	TOPSIS	No	WFM
1	N 6	0.97114	N 6	0.96681	N 6	0.96628	N 6	2.9299
2	N 7	0.82626	N 7	0.82001	N 7	0.66847	N 7	1.78752
3	N 8	0.75035	N 8	0.7327	N 8	0.49803	N 8	1.27542
4	N 9	0.71141	N 9	0.68344	N 9	0.40954	N 9	1.03484
5	N 10	0.67222	N 10	0.6327	N 10	0.33055	N 10	0.82123
6	N 1	0.55946	N16	0.54507	N16	0.27957	N16	0.52493
7	N 17	0.55876	N17	0.52141	N21	0.24895	N17	0.45951
8	N 16	0.5578	N 1	0.49233	N 17	0.21469	N 1	0.38691

Ranking	No.	WSM	No	WPM	No	TOPSIS	No	WFM
9	N 11	0.54128	N 11	0.48672	N 11	0.17407	N 11	0.37377
10	N 18	0.53872	N 21	0.48226	N 1	0.17002	N 21	0.36359
11	N 19	0.52111	N 18	0.47717	N 18	0.16933	N 18	0.35216
12	N 13	0.51824	N 12	0.45768	N 19	0.15212	N 12	0.31084
13	N 3	0.51362	N 3	0.44963	N 22	0.15104	N 3	0.29478
14	N 4	0.50892	N 13	0.44922	N 20	0.14850	N 13	0.29396
15	N 20	0.50790	N 19	0.44878	N 4	0.14742	N 19	0.29306
16	N 14	0.50502	N 14	0.44580	N 23	0.14739	N 14	0.28727
17	N 2	0.50205	N 22	0.44468	N 14	0.14488	N 22	0.28509
18	N5	0.49629	N20	0.44172	N13	0.14364	N20	0.27940
19	N12	0.49561	N2	0.43597	N12	0.14170	N2	0.26861
20	N22	0.4952	N15	0.4291	N5	0.1362	N15	0.2561
21	N15	0.49295	N4	0.42283	N3	0.13495	N4	0.24507
22	N21	0.48448	N5	0.41769	N2	0.13381	N5	0.23631
23	N24	0.46194	N24	0.41012	N15	0.12545	N24	0.22366
24	N23	0.42031	N23	0.40923	N24	0.10954	N23	0.22218
25	N25	0.41798	N25	0.37685	N25	0.07106	N25	0.17355

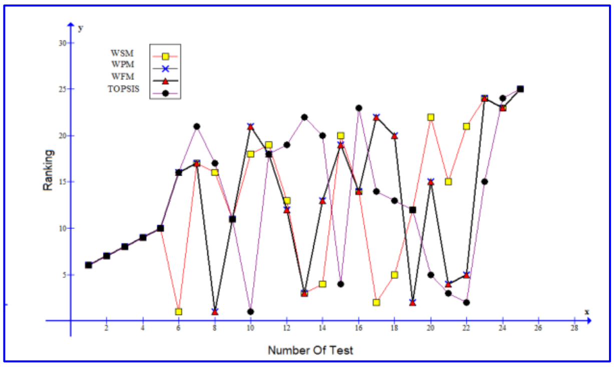

Figure- 2: the ranking of tests as optimize

5. Discussion of results:

In order to weigh the best method in choosing the optimal solutions, two proposed criteria were selected:

5.1. Coefficient of Anomalous:

It is the single results that appear in the comparison table between the four methods and there is no similarity in the same field of the results of the other methods, which is a negative status in the method.

The Anomalous Degree (AD) is calculated as follows:

i- 100 % is given for single anomalous result appears in front of three similar results.

ii- 50 % is given for each of dual anomalous results appears in front of two similar results.

iii- 0.0 % is given for no anomalous result appears.

$$COA = \sum_{j=1}^{N} \frac{AD}{N} \times 100\% \ldots\ldots\ldots\ldots\ldots (20)$$

Where: AD the anomalous degree

2- Coefficient of Excellence:

It is the ratio of difference between the highest and the lowest values of the results divided by the highest value in the results of that method, and the greater of this value means there is a large space to compare the results, which is a positive status in the method.

$$COE = \frac{(max.value - min.value)}{max.value} \times 100\% \ldots\ldots(21)$$

Where: COE = coefficient of (expand) excellence

By applying the two proposed criteria to evaluate the performance of the four methods in the previous example, as shown in the table (4):

Table- 4: Resultant of WSM, WPM, and TOPSIS compared with the proposed method.

method	WSM	WPM	TOPSIS	WFM
COE	57 %	61 %	92.6 %	94 %
COA	24 %	0.0 %	44 %	0.0 %

By application of the two proposed criteria, COE and COA to the results of the three methods, and compared with the proposed method in this research, it`s found that the COE in the three methods was 57 % at WSM, 61 % at WPM, and 92.6 % at TOPSIS while the COE in the proposed method was 94 %, and the COA in the three methods was 24 % at WSM, 0.0 % at WPM, and 44 % at TOPSIS while the COE in the proposed method was 0.0 %, as shown in table (4) , these results are shown that the proposed method and WPM were completely free of the anomalies, but the COE at proposed method has about 1.5 doubled in front of WPM.

6. CONCLUSIONS

The conclusions are summarized as follows:

1 - The proposed method gives more space for a preference between results than the other methods adopted in the multi-attribute decision-making method (MADM), which gives more clarity to make the appropriate decision.

2 - The results showed that the three methods in (MADM) have more a convergence result than the proposed method.

3. The increasing in the number of available options causes an increase in convergence results and the greater the need to apply the proposed method.

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