Total Chromatic Number of Middle and Total Graph of Net graph

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Abstract - *In this paper, we obtained the total coloring and the total chromatic number of net graph* N *, middle graph of net graph* $M(N)$ *, total graph of net graph* $T(N)$ *.*

Key Words: Middle graph, Total graph, Net graph, Total coloring, Total chromatic number.

1. INTRODUCTION

All graphs considered in this paper are nontrivial, simple and undirected. Let G be a graph with vertex set V and edge set E . The notation of total coloring was introduced by Behzad in 1965 and also he came out with new ideology that the total chromatic number of complete graph and complete bi-partite graph. A total coloring of a graph G, is a function $f:S \to C$, where $S \to V(G) \cup E(G)$ and set of colors C, satisfies the given conditions,

(i) No two adjacent vertices assigns the same colors,

(ii) No two adjacent edges assigns the same colors &

(iii) No edges and its end vertices assigns the same colors

The total chromatic number $\chi''(G)$ of G is the minimum cardinality k such that a graph G may have a total coloring by k colors. Behzad [1] conjectured that for every simple graph G has $\Delta(G)+1 \le \chi''(G) \le \Delta(G)+2$, where $\Delta(G)$ is the maximum degree of graph G, this is called the Total Coloring Conjecture (TCC). Rosenfeld [9] verified the total coloring conjecture, for any graph G with maximum degree ≤ 3. Total coloring have been extensively studied in different families of graphs. Muthuramakrishnanand et al [8] showed that Total Chromatic Number of Line, Middle and Total graph of star and square graph of Bistar graph. In this paper, we obtain the total chromatic number of $M(N)$ and $T(N)$.

2. PRELIMNARIES

Definition (1) The net graph or 3-sunlet graph is the graph on 6 vertices obtained by attaching 3 pendent edges to a cycle graph C_3 and is denoted by N .

Definition (2) Consider the vertex set $V(G)$ and the edge set $E(G)$ in a graph G . The middle graph of G , denoted by $M(G)$ is defined in the following way. $V(G) \cup E(G)$ be the vertex set of $M(G)$. consider x, y be two vertices of $M(G)$ are adjacent in $M(G)$ in case one of the following conditions holds:

(i) x, y are in $E(G)$ and x, y are adjacent in G .

(ii) x is in $V(G)$, y is in $E(G)$ and x, y are incident in G .

Definition (3) Consider the vertex set $V(G)$ and the edge set $E(G)$ in a graph G. The total graph of G, denoted by $T(G)$ is defined as follows. $V(G)\cup E(G)$ be the vertex set of $M(G)$. Consider x, y be two vertices of $M(G)$ are adjacent in $M(G)$ in case one of the following conditions holds:

(i) x, y are in $V(G)$ and x is adjacent to y in G.

(ii) x, y are in $E(G)$ and x, y are adjacent in G.

(iii) x is in $V(G)$, y is in $E(G)$, and x, y are incident in G.

3. TOTAL CHROMATIC NUMBER OF NET GRAPH

Theorem 1. Let $M(N)$ be the middle graph of net graph *N.* Then $\chi''(M(N))=7$

Proof:

Let the vertices of net graph be $\{u_1,u_2,u_3,\ldots,u_n\}$ and $\{v_1,v_2,v_3,\ldots,v_n\}$ and let the edges of the net graph be $\{e_1,e_2,e_3,\ldots,e_n\}$ and $\{e_1^{'},e_2^{'},e_3^{'},\ldots,e_n^{'}\}$, where $e_i=v_iv_{i+1}$ for $1\le i\le n-1$ be the edges, $e_n=v_nv_1$ and $e_i^{'}=v_iu_i$ for $1\le i\le n$ be the edges. By the middle graph definition, the vertices $\{v_i^{'}:1\le i\le n\}$ and $\{u_i^{'}:1\le i\le n\}$ subdivided each edges $\{e_i^{'}=v_iu_i:1\le i\le n\}$, $\{e_i=v_iv_{i+1}:1\le i\le n-1\}$ and $\{v_nv_1\}$ in $M(N)$. Hence, the vertex set and the edge set is given by $V(M(N))=\{v_i:1\le i\le n\}\cup\{u_i:1\le i\le n\}\cup\{v_i^{'}:1\le i\le n\}\cup\{u_i^{'}:1\le i\le n\}$, where $u_i^{'}=e_i^{'}(1\le i\le n)$ and $v_i^{'}=e_i(1\le i\le n-1)$,

$$E(M(N))=\begin{cases}(v_iv_i^{'}:1\le i\le n)\cup(u_i^{'}v_i^{'}:1\le i\le n)\cup(v_i^{'}u_{i+1}^{'}:1\le i\le n-1)\cup\\(v_i^{'}v_{i+1}:1\le i\le n-1)\cup(v_iu_i^{'}:1\le i\le n)\cup(v_n^{'}u_1^{'})\cup(v_n^{'}v_1)\cup\\(u_iu_i^{'}:1\le i\le n)\cup(v_i^{'}v_{i+1}^{'}:1\le i\le n-1)\cup(v_n^{'}v_1)\end{cases}$$

Let $f:S\to C$, where f is the total coloring, $S=V(M(N))\cup E(M(N))$ and $C=\{1,2,3,4,5,6,7\}$, while coloring the vertices and edges of a graph we obtain total coloring, ie)

$$f(v_i)=1;\ f(u_i^{'})=4;\ f(u_i)=6 \text{ for } 1\le i\le n$$

$$f(v_i^{'})=\begin{cases}7 \text{ if } i\equiv 1\,(\text{mod}2)\\6 \text{ if } i\equiv 0\,(\text{mod}2)\end{cases} \text{ for } 1\le i\le n-1$$

$$f(v_n^{'})=5$$

$$f(v_i^{'}v_{i+1}^{'})=\begin{cases}2 \text{ if } i\equiv 1\,(\text{mod}2)\\1 \text{ if } i\equiv 0\,(\text{mod}2)\end{cases} \text{ for } 1\le i\le n-1$$

$$f(v_n^{'}v_1^{'})=6;\ f(v_iv_i^{'})=4 \text{ for } 1\le i\le n$$

$$f(v_i^{'}v_{i+1}^{'})=3;\ f(v_n^{'}v_1)=3 \text{ for } 1\le i\le n-1$$

$$f(u_i^{'}v_i^{'})=\begin{cases}6, \text{ if } i\equiv 1\,(\text{mod}2)\\7, \text{ if } i\equiv 0\,(\text{mod}2)\end{cases} \text{ for } 1\le i\le n$$

$f(v_i^{'}u_{i+1}^{'}) = 5 \text{ for } 1 \le i \le n-1$

$f(u_i u_i^{'}) = 3;\ f(v_i u_i^{'}) = 6 \text{ for } 1 \le i \le n,\ f(v_n^{'}u_1^{'}) = 7;$

The above graph *M(N)* is properly colored with 7 colors by using the rule of total coloring.

Therefore the total chromatic number of the middle graph of net graph M(N) is 7,

ie) $\chi^{''}(M(N)) = 7$

Theorem 2. Let $T(N)$ be the middle graph of net graph N. Then $\chi''(T(N)) = 7$.

Proof:

Let the vertices of net graph be $\{u_1, u_2, u_3, \ldots, u_n\}$ and $\{v_1, v_2, v_3, \ldots, v_n\}$ and let the edges of the net graph be $\{e_1, e_2, e_3, \ldots, e_n\}$ and $\{e_1^{'}, e_2^{'}, e_3^{'}, \ldots, e_n^{'}\}$, where $e_i = v_i v_{i+1}$ for $1 \le i \le n-1$ be the edges, $e_n = v_n v_1$ and $e_i^{'} = v_i u_i$ for $1 \le i \le n$ be the edges. By the total graph definition, the vertices $\{v_i^{'} : 1 \le i \le n\}$ and $\{u_i^{'} : 1 \le i \le n\}$ subdivided each edges $\{e_i^{'} = v_i u_i : 1 \le i \le n\}$, $\{e_i = v_i v_{i+1} : 1 \le i \le n-1\}$ and $\{v_n v_1\}$ in $T(N)$. Hence, the vertex set and the edge set is given by $V(T(N)) = \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\} \cup \{v_i^{'} : 1 \le i \le n\} \cup \{u_i^{'} : 1 \le i \le n\}$, where $u_i^{'} = e_i^{'}(1 \le i \le n)$ and $v_i^{'} = e_i(1 \le i \le n-1)$,

$$E(T(N)) = \begin{cases} (v_i v_i^{'} : 1 \le i \le n) \cup (u_i^{'} v_i^{'} : 1 \le i \le n) \cup (v_i^{'} u_{i+1}^{'} : 1 \le i \le n-1) \cup \\ (v_i^{'} v_{i+1} : 1 \le i \le n-1) \cup (v_i u_i^{'} : 1 \le i \le n) \cup (v_n^{'} u_1^{'}) \cup (v_n^{'} v_1) \cup \\ (u_i u_i^{'} : 1 \le i \le n) \cup (v_i^{'} v_{i+1}^{'} : 1 \le i \le n-1) \cup (v_n^{'} v_1) \end{cases}$$

Let $f : S \to C$, where f is the total coloring, $S = V(T(N)) \cup E(T(N))$ and $C = \{1,2,3,4,5,6,7\}$, while coloring the vertices and edges of a graph we obtain total coloring, ie)

$f(v_i) = 1;\ f(u_i^{'}) = 4;\ f(u_i) = 6 \text{ for } 1 \le i \le n$

$$f(v_i^{'}) = \begin{cases} 7 \text{ if } i \equiv 1 \pmod 2 \\ 6 \text{ if } i \equiv 0 \pmod 2 \end{cases} \text{ for } 1 \le i \le n-1$$

$f(v_n^{'}) = 5;\ f(u_i v_i) = 2 \text{ for } 1 \le i \le n$

$$f(v_i^{'} v_{i+1}^{'}) = \begin{cases} 2 \text{ if } i \equiv 1 \pmod 2 \\ 1 \text{ if } i \equiv 0 \pmod 2 \end{cases} \text{ for } 1 \le i \le n-1$$

$f(v_n^{'} v_1^{'}) = 6;\ f(v_i v_i^{'}) = 4 \text{ for } 1 \le i \le n$

$f(v_i^{'} v_{i+1}) = 3;\ f(v_n^{'} v_1) = 3 \text{ for } 1 \le i \le n-1$

$$f(u_i^{'} v_i^{'}) = \begin{cases} 6, \text{ if } i \equiv 1 \pmod 2 \\ 7, \text{ if } i \equiv 0 \pmod 2 \end{cases} \text{ for } 1 \le i \le n$$

$f(v_i^{'} u_{i+1}^{'}) = 5 \text{ for } 1 \le i \le n-1$

$f(u_i u_i^{'}) = 3;\ f(v_i u_i^{'}) = 6 \text{ for } 1 \le i \le n,\ f(v_n^{'} u_1^{'}) = 7;$

The above graph *T(N)* is properly colored with 7 colors by using the rule of total coloring.
Therefore the total chromatic number of the middle graph of net graph *T(N)* is 7,

ie) $\chi^{''}(T(N)) = 7$

4. CONCLUSIONS

In this research paper, we find out the total chromatic number of middle graph and total graph of net graph.

1. $\chi^{''}(M(N)) = 7$
2. $\chi^{''}(T(N)) = 7$

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