Equivalence Relation on Fuzzy Meet Hyperlattice

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ABSTRACT---In this paper, we introduce the notion of fuzzy join hypercongruence and we derive the connection between a fuzzy join hypercongruence on a fuzzy meet hyperlattice and a join hypercongruence on the associated meet hyperlattice.

KEYWORDS--- Hyperlattice, fuzzy meet hyperlattice, fuzzy join hypercongruence.

INTRODUCTION:

Lattice theory [1] is the recent trend in mathematics. The fuzzy set has been combined with hyperlattice. And now we are going to introduce the equivalence relation on fuzzy meet hyperlattice. And also we introduce the notion of fuzzy join hypercongruence. Hence we derive the connections between join hypercongruence on a fuzzy meet hyperlattice.

I. PRELIMINARIES:

In this section we discuss about some of the basic definitions that we use throughout this paper.

Definition 1.1:

Let L be a non-empty set with two hyperoperations $\wedge$ and$\vee$. Then (L, $\vee$, $\wedge$) is called as hyperlattice [4], if the following identities holds for all a, b, c $\in$ L.

1) $a \in a \wedge a$ and $a \in a \vee a$
2) $a \wedge b = b \wedge a$ and $a \vee b = b \vee a$
3) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ and $(a \vee b) \vee c = a \vee (b \vee c)$
4) $a \in a \wedge (a \vee b)$ and $a \in a \vee (a \wedge b)$.

Definition 1.2:

Let L be a non-empty set with two hyperoperation $\vee$ and$\wedge$. Then (L, $\vee$, $\wedge$) is called a fuzzy hyperlattice [2], if the following identities holds for all a, b, c $\in$ L.

1) $(a \vee a)(a) > 0$ and $(a \wedge a)(a) > 0$
2) $a \vee b = b \vee a$ and $a \wedge b = b \wedge a$
3) $(a \vee b) \vee c = a \vee (b \vee c)$ and $(a \wedge b) \wedge c = a \wedge (b \wedge c)$
4) $(a \vee (a \wedge b))(a) > 0$ and $(a \wedge (a \vee b))(a) > 0$.

Definition 1.3:

Let L be a non-empty set, $\vee$: L ⊠L $\longrightarrow$ F*(L),be a fuzzy hyperoperation and $\wedge$: L ⊠L $\longrightarrow$ L be a operation. Then, (L, $\vee$, $\wedge$) is a fuzzy meet hyperlattice if for all x, y, z $\in$ L the following conditions holds:

1) $(x \vee x)(x) > 0$ and $(x \wedge x)(x) > o$
2) $x \vee y = y \vee x$ and $x \wedge y = y \wedge x$
3) $(x \vee y) \vee z = x \vee (y \vee z)$ and $(x \wedge y) \wedge z = x \wedge (y \wedge z)$
4) $(x \vee (x \wedge y))(x) > 0$ and $(x \wedge (x \vee y))(x) > 0$.

Definition 1.4:

Let (L, ∨, ∧) be a fuzzy meet hyperlattice and ρ be an equivalence relation on L. For any u, v ∈ F*(L), we say that u $\bar{\rho}$ v if the following condition holds:

(1) For all a ∈ L, if u(a) > 0, then there exists b ∈ L, such that v(b) > 0 and a ρ b;
(2) For all x ∈ L, if v(x) > 0, then there exists y ∈ L, such that u(y) > 0 and x ρ y.

II. EQUIVALENCE RELATION ON FUZZY MEET HYPERLATTICE

In this section we introduce the notion of fuzzy join hypercongruence [3], and we derive the connections between a fuzzy join hypercongruence on fuzzy meet hyperlattices and a join hypercongruence on the associated meet hyperlattice.

Definition 2.1:

An Equivalence relation ρ on a fuzzy meet hyperlattice (L, ∨, ∧) is said to be a fuzzy join hypercongruence on (L, ∨, ∧) if for all a, a', b, b' ∈ L, the following conditions holds:

1) a ρ a' implies that (a ∨ b) $\bar{\rho}$ (a' ∨ b') and
2) b ρ b' implies that (a ∧ b) $\bar{\rho}$ (a' ∧ b').

The following theorem depicts that a homomorphism of fuzzy meet hyperlattices can induce fuzzy join hypercongruence on a fuzzy meet hyperlattice.

Theorem 2.2:

Let (L_1,$\vee_1$, $\wedge_1$) and (L_2, $\vee_2$, $\wedge_2$) be two fuzzy meet hyperlattices. A map f : $L_1 \rightarrow L_2$ is a homomorphism of fuzzy meet hyperlattices, then ρ = ker f = {(a, b) ∈ L_1 ◻ L_1| f(a) = f(b)} is a fuzzy join hypercongruence on (L,$\vee_1$, $\wedge_1$).

Proof:

We know that, ρ = ker f is an equivalence relation on L_1.

For all a, a', b, b' ∈ L_1, let a ρ a' and b ρ b', then,

f(a) = f(a') and f(b) = f(b').

then we get that f(a $\vee_1$, b) = f(a) $\vee_2$ f(b) = f(a') $\vee_2$ f(b').

Hence for all x ∈ L_1, let (a $\vee_1$ b) (x) > 0, then

f (a $\vee_1$ b) f(x) = sup {(a' $\vee_1$ b') (x')| f(x') = f(x), x' ∈ L_1} ≥ (a $\vee_1$ b) (x) > 0.

Therefore we have f(a' $\vee_1$ b') f(x) > 0, which implies that sup{(a' $\vee_1$ b') (x')| f(x') = f(x), x' ∈ L_1} > 0.

The above inequality implies that there exists x'∈ L_1 such that,

(a' $\vee_1$ b') (x') > 0 and

f(x') = f(x), implies (a' $\vee_1$ b') (x') > 0 and x ρ x'.

Conversely,

For all s ∈ L_1, let (a' $\vee_1$ b') (s) > 0, then there also exist t ∈ L_1, such that

(a $\vee_1$ b) (t) > 0 and s ρ t.

Hence (a ∨ b) $\bar{\rho}$ (a' ∨ b').

Similarly, we can show that (a ∧ b) $\bar{\rho}$ (a' ∧ b').

Therefore, ρ = ker f is a fuzzy join hypercongruence on $(L, \vee_1, \wedge_1)$.

Now we introduce the fuzzy strong join hypercongruence on fuzzy meet hyperlattices.

Definition 2.3:

Let (L, ∨, ∧) be a fuzzy meet hyperlattice and ρ be an equivalence relation on L. For any u, v ∈ F*(L), we say that u $\bar{\bar{\rho}}$ v, if the following two identities hold:

(1) For all a ∈ L, if u(a) > 0, then there exists b ∈ L, such that v(b) > 0, u (a) = v (b) and a ρ b;
(2) For all x ∈ L, if v(x) > 0, then there exists y ∈ L, such that u(y) > 0, u (y) = v (x) and x ρ y.

Definition 2.4:

An equivalence relation ρ on a fuzzy meet hyperlattice (L, ∨, ∧) is said to be a fuzzy strong join hypercongruence on (L, ∨, ∧), if for all a, a', b, b' ∈ L, then the following implications are satisfied:

1) a ρ a' implies (a ∨ b) $\bar{\bar{\rho}}$ (a' ∨ b') and
2) b ρ b' implies (a ∧ b) $\bar{\bar{\rho}}$ (a' ∧ b')

Theorem 2.5:

Let ρ be a fuzzy strong join hypercongruence on a fuzzy meet hyperlattice (L, ∨, ∧). If we define the following fuzzy join hyperoperation on the quotient set L/ ρ : for all $\bar{x}$, $\bar{y}$, $\bar{z}$ ∈ L/ ρ,

$(\bar{x} \vee' \bar{y})(\bar{z})$ = sup {(x' ∨ y') (z') | x' ∈ $\bar{x}$, y'∈ $\bar{y}$, z' ∈ $\bar{z}$} and

$(\bar{x} \wedge' \bar{y})(\bar{z})$ = sup {(x' ∧ y') (z') | x' ∈ $\bar{x}$, y'∈ $\bar{y}$, z' ∈ $\bar{z}$}, then (L/ ρ, ∨ ′, ∧ ′) is a fuzzy meet hyperlattice.

Proof:

We shall show that ∨ ′and ∧ ′ are well-defined.

For any x, x', y, y' ∈ L, let $\bar{x} = \overline{x'}$ and $\bar{y} = \overline{y'}$ then x ρ x' and y ρ y'.

Since ρ is a fuzzy strong join hypercongruence on (L, ∨, ∧), it follows that

(x ∨ y) $\bar{\bar{\rho}}$ (x' ∨ y') and (x ∧ y) $\bar{\bar{\rho}}$ (x' ∧ y').

This shows that for all a ∈ L, if (x ∨ y) (a) > 0, then there exists b ∈ L, such that (x' ∨ y') (b) > 0,

(x ∨ y) (a) = (x' ∨ y') (b) and a ρ b.

Conversely, for all s ∈ L, if (x' ∨ y') (s) > 0, then there exists some t ∈ L, such that (x ∨ y) (t) > 0,

(x ∨ y) (t) = (x' ∨ y') (s) and s ρ t,

Hence we can get that for all $\bar{z}$ ∈ L/ ρ, then

$(\bar{x} \vee' \bar{y})(\bar{z})$ = sup {(x' ∨ y') (z') | x' ∈ $\bar{x}$, y'∈ $\bar{y}$, z' ∈ $\bar{z}$}

= sup {(x'' ∨ y'') (z'') | x'' ∈ $\bar{x}$, y''∈ $\bar{y}$, z'' ∈ $\bar{z}$}

= sup {(x'' ∨ y'') (z'') | x'' ∈ $\overline{x'}$, y''∈ $\overline{y'}$, z'' ∈ $\overline{z'}$}

= $(\overline{x'} \vee' \overline{y'})(\bar{z})$, whence $(\bar{x} \vee' \bar{y}) = (\overline{x'} \vee' \overline{y'})$.

Similarly, we show that ($\bar{x} \wedge' \bar{y}$) = ($\overline{x'} \wedge' \overline{y'}$).

Therefore, $\vee'$ and $\wedge'$ are well-defined.

Theorem 2.6:

An equivalence relation ρ is a fuzzy join hypercongruence on a fuzzy meet hyperlattice (L, ∨, ∧), if and only if ρ is a join hypercongruence on corresponding meet hyperlatice (L, ⊗, ⊕).

Proof:

Set a ρ a', b ρ b', where a, a', b, b' ∈ L.

We have, (a ∨ b) $\bar{\rho}$ (a' ∨ b') and

(a ∧ b) $\bar{\rho}$ (a' ∧ b') if and only if the following conditions are satisfied:

1) for all x ∈ L, if (a ∨ b) (x) > 0, then there exists y ∈ L, such that (a' ∨ b') (y) > 0 and x ρ y;
2) for all s ∈ L, if (a' ∨ b') (s) > 0, then there exists t ∈ L, such that (a ∨ b) (t) > 0 and s ρ t;
3) for all x ∈ L, if (a ∧ b) (x) > 0, then there exists y ∈ L, such that (a' ∧ b') (y) > 0 and x ρ y;
4) for all s ∈ L, if (a'∧ b') (s) > 0, then there exists t ∈ L, such that (a ∧ b) (t) > 0 and s ρ t.

these conditions are equivalent to the following ones,

1) for all x ∈ L, if x ∈ a ⊗ b, then there exists y ∈ a' ⊗ b', such that x ρ y;
2) for all x ∈ L, if s ∈ a' ⊗ b', then there exists t ∈ a ⊗ b, such that s ρ t;
3) for all x ∈ L, if x ∈ a ⊕ b, then there exists y ∈ a' ⊕ b', such that x ρ y;
4) for all x ∈ L, if s ∈ a' ⊕ b', then there exists t ∈ a ⊕ b, such that s ρ t.

which shows that (a ⊗ b) $\bar{\rho}$ (a' ⊗ b') and (a ⊕ b) $\bar{\rho}$ (a' ⊕ b').

Therefore, ρ is a fuzzy join hypercongruence on (L, ∨, ∧), if and only if ρ is a join hypercongruence on (L, ⊗, ⊕).

III. CONCLUSION

Hence, in this paper we introduced the notion of fuzzy join hypercongruence and we derive the equivalence relation on fuzzy meet hyperlattice. We also depicted the connection between a fuzzy join hypercongruence on a fuzzy meet hyperlattice and a join hypercongruence on the associated meet hyperlattice.

IV. REFERENCES

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