International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 07 Issue: 06 | June 2020 www.irjet.net p-ISSN: 2395-0072

Reliability Assessment of Reinforced Concrete Continuous Beam Designed using Eurocode 2

Nura Shehu Aliyu Yaro[1], Amana ocholi[2], Aminu Darda'u Rafindadi[3], Ahmad Abdulaziz[4]

[1,2and 4]*Lecturer Department of Civil Engineering, Ahmadu Bello University Zaria, Kaduna state.*
[3]*Lecturer Department of Civil Engineering, Bayero University Kano, Kano state.*

Abstract - *This study presents the structural reliability of a reinforced concrete continuous beam designed to Eurocode 2 specifications. First Order Reliability Method (FORM) was employed in the analysis. The limit state functions were defined and some variables (width, span, effective depth, load ratio, and characteristic strength) were varied to assess the reliability indices and hence their significance to ensure a safe design. The analysis indicates that for all load ratios the safety indices **(β)** are directly proportional to the effective depth in bending failure within effective depths of 500 mm to 700mm and also inversely proportional to the width of the exterior span width of 200mm to 400mm. It was also observed that the safety index **(β)** increases with characteristics strength within values of 20N/mm² to 40 N/mm². For shear bending it was observed that for both interior and exterior support the safety index is directly proportional to both effective depth and width of the beam while it is inversely proportional to the beam span. The safety indices **(β)** for width beam and span width are all with the Euro code safety indices **(β)**. In conclusion, the influence of variation of some of the basic variables on reliability levels shows that effective depth, length, characteristic strength, breadth and load ratios are the most significant variables of the design safety levels criteria for bending and shear under the ultimate limit state of a reinforced continuous concrete beam.*

Key Words: **Reliability, Continuous beam, Assessment, Reinforced, Safety index**.

1. INTRODUCTION

The purpose of every design is the achievement of an acceptable probability that a structure will not become unfit for its intended use-that is, it will not reach a limit state. A limit state is a state at which a structure may cease to be unfit for use. Every design aims are to avoid such conditions being reached during the expected life of the structure [1]. Therefore, structural reliability theory is concerned with the rational treatment of uncertainties associated with the design of structures and with assessing the safety and serviceability of these structures [2]. A continuous beam is a horizontal rectangular beam resting on more than two incompressible simple bearings.

The approach utilized in this research is FORM which is also called the second-order method [3]. Despite limited information, the probability distribution of the design variable was prescribed. Though it still essential to identify that irrespective of the individual variate distribution. It is the safety margin distribution (R-S) that is significant in calculating the probability of failure P_f [4]. The important parameters for calculating the safety indices for the second-moment reliability method are the means and variance (first and second moments) [5]. It was important to know that in most previous researches the Coefficient of Variation COV (ratio of the standard deviation to mean value) is generally published [6,7]. Table 1 below shows the relevant parameter values with their corresponding distribution for the basic variable used in the research. The reliability analysis of the beam was carried out by integrating all the variables using a computer program, FORM5 which is based on the First Order Reliability Method.

Table 1.0 Parameters of Stochastic Model on a Beam Due to Bending and shearing

S/N	Parameters	Distribution type	COV	$E(x)_i$	$S(x)_i$
1	Characteristic strength of concrete, f_{ck}	Log- normal 3	0.15	30 N/ mm	4.5N/mm
2	width of beam, b_w	Normal 2	0.05	300 mm	15mm
3	Span of beam, L	Normal 2	0.05	5000mm	250mm
4	Effective depth, d	Normal 2	0.05	600mm	30mm
5	Imposed load	Log- normal 3	0.3	5.0Kn/m	1.5

A study conducted indicated that the steel strength and concrete strength are log-normally distributed, whereas the geometric variables like the span, breadth, and width of the beam are normally distributed[8].it is important to note that for this research all the variable were normalized in the application of the FORM5. The basic use of the reliability method is to help determine the probability that the structure in question will attain any of the known limit states within a specified period. Hence it becomes very crucial to assess the resistance of the reinforced concrete continuous beam in a probabilistic environment since its component materials must be dependent on each other even though they are individually varied to attain a beam structure that will act as one unit in resisting loads acting on it. Hence there is a need for developing a model for performance function for the limit state of the beam need to be established. For the beam considered the failure modes considered are bending and shear. Thus, for each safety margin as obtained from equating the resistance of the beam as proposed by the codes [9]. The load effect was calculated by simple structural analysis, the basic variables are recognized while signifying their statistical behavior from literature

International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 07 Issue: 06 | June 2020 www.irjet.net p-ISSN: 2395-0072

2. Basic for Reliability Analysis

Reliability analysis was defined according to Melchers [7] as the systematic calculation and prediction of the probability of limit state violation. In reliability analysis, the action **S**, and the resistance **R** are considered as random variables, and the structure is considered to have failed when **R** is less than S at any point [7]. The limit state function **g(x) is** formulated utilizing a model based on a physical understanding of the empirical data. Due to idealization, inherent physical uncertainties, and inadequate or insufficient data, the models themselves are the parameters entering the models such as materials properties and load characteristics are uncertain. Consequently, uncertainties are grouped into [7]

- Model uncertainties
- Statistical uncertainties
- Inherent uncertainties

Model uncertainty is associates with the crudeness and incompleteness of mathematical models that describe a phenomenon. Statistical uncertainties are associates with the statistical evaluation of the test results or observation. They may result from limited numbers of test which cause uncertainty. Inherent uncertainties refer to the randomness of a phenomenon. The randomness is a result of combining uncontrollable fluctuation of many different factors such as wind and snow. This uncertainty is also referred to as physical uncertainty [7]. The FORM is an analytical estimation in which the reliability index is taken as the minimum distance from the origin to the limit state surface in standardized normal space (u-space) and the most possible design point (failure point) is investigated by utilizing mathematical programming methods [10]. Though since the performance function g(X) is estimated by a linear function in U- space at the design point, hence, problems may arise when the **g(X)** is strongly nonlinear [11]. FORM has been designed for the approximate computation of general integral over a given domain with locally smooth boundaries but especially for probability integral occurring in structural reliability. The present version of FORM, FORM 5, is an updated version and is written in FORTRAN. For FORM it is required that X is at least locally continuously differentiable, i.e. the probability densities exist. The random variable X_1 and X_2 are called basic variables. The locally sufficiently smooth (at least once differentiable) state function is denoted by G(x) and is defined such that

g(x) > 0 corresponds to a favourable state.

g(x) = 0 corresponds to limit state or failure boundary.

g(x) < 0 corresponds to failure domain

In the context of FORM, it is convenient but necessary only locally that G(x) is a monotonic function in each component of *x* [11]. Generally, the performance function **g(X)** is given as the difference between the resistance R(X) and the demand or solicitation on the system **S(X)** that is., **g(X) = R(X) – S(X).** Although, in reliability engineering analysis, **g(X)** is usually expressed in terms of displacement, strain, stress, etc. The performance functions can be related to the following structural conditions [12]:

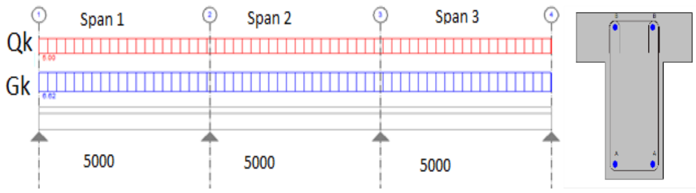
Ultimate limit state: under this limit state condition failure is highly related to the total collapse of the structure or part of it which implies that the structural safety ids highly affected. Serviceability limit state: In this limited state condition the failure events are associated with disruption of the normal use of the structure such as extreme vibration and deflection. It is convenient to describe failure events in terms of function relations, which if they are fulfilled, define that the failure event **F** occurs as in equation (1)

$$F = \{g(x) \leq 0\} \qquad [1]$$

Where g(x) is termed a limit state function. The component of the vector x is the realization of the so-called basic random variables X representing all relevant uncertainties influencing the problem at hand. The failure event F is defined as the set of realization of the limit state function g(x), which is zero or negative.

3. Design Information

A Three (3) span continuous beam shown in fig (3.1) transmitting a uniformly distributed load was designed using Euro code 2. The beam width and overall depths are 300mm and 600mm respectively. The beam forms part of beam arrangement in a structure, with spacing at 5m centers and transmitting loading from a 180mm thick slab. The beam carries characteristic dead and imposed loads of 6.62kN/m and 5.0 kN/m respectively



(a) (b)

Figure 1.1(a)and(b): Three-span Continuous Beam and section of beam.

The bending moment diagram (BMD) and Shear force (SFD) From Structural Analysis Using Prokon Analysis and design software are shown in Fig (3.2)

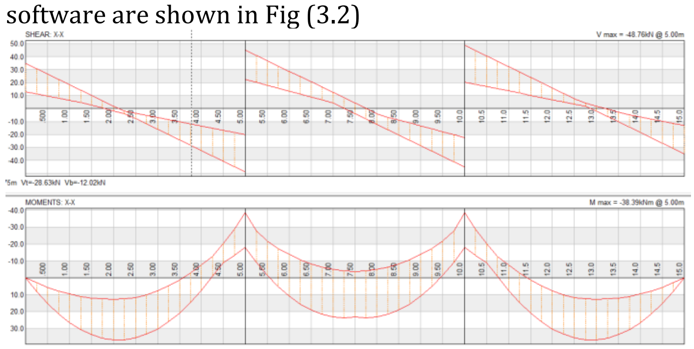


Fig 1.2: Continuous Beam BMD and SFD

3.1 PERFORMANCE FUNCTIONS FOR RELIABILITY ANALYSIS

Limit State Equations

The limit state equations for the continuous beam considering the failure mode in Eurocode2 are considered below:

(i)Bending Failure Mode

The condition of failure for the continuous beam is given as;

$M_R - M_A \leq 0$

Hence, the limit state function is given as

$G(x) = M_R - M_A \leq 0$ (3.35)

Where

Moment of resistance $(M_R) = KF_{CK} b_{eff} d^2$

Taken condition reinforcement required at compression zone,

$$k = k_{bal} = 0.167$$

To take care of all the uncertainty that may lead to failure after it has been designed as throughout the continuous beam

Implies $M_R = 0.167 f_{ck} b_{eff} d^2$ [2]

$b_{eff} = b_w + 2(0.2 \times 0.85l)$

$M_R = 0.167 f_{ck} (b_w + 2(0.2 \times 0.85l)] d^2 (3.37)$

Applied moment,

$$M_{AB} = 0.09FL$$

$F = (1.35G_K + 1.5Q_K)L$ (3.38)

$M_{AB} = 0.09 Q_K L^2 (1.35\alpha + 1.5)$ [3]

$$G_K / Q_K = \alpha = load\ ratio$$

At first and last span $M_{AB} = M_{CD}$

$G_1(X) = 0.167 f_{ck} [b_w + 2(0.2b' + 0.1 \times 0.85l] d^2 - [0.09 Q_k l^2 (1.35\alpha + 1.5)]$ [4]

At the second interior M_{BC}

Applied moment

$M_{BC} = 0.07FL$ (At middle of interior span)

$F = (1.35G_K + 1.5Q_K)L(3.41)$

$M_{BC} = 0.07 Q_K L^2 (1.35\alpha + 1.5)(3.42)$

$G_2(X) = 0.167 f_{ck} [b_w + 2(0.2b' + 0.1 \times 0.85l] d^2 - [0.07 Q_k l^2 (1.35\alpha + 1.5)]$ [5]

In equations [2] to [5], M_R and M_A are the moment of resistance and applied moment respectively; F_{CK} is the characteristic strength of concrete, b_{eff}, d, b_w, and l are the effective width, effective depth, the width of the beam section and span length respectively; are the characteristic dead and imposed loads respectively. In equation [4] and [5], dead to live load ratio may be expressed as (α) ALPHA.

(ii) Shear Failure Mode

The condition of shear failure for a continuous beam is given as;

The condition of shear failure for a continuous beam is given as;

$$V_{RD,max} \geq V_{EF}$$

$$V_{RD,max} - V_{EF} \geq 0$$

Hence, the limit state function is given by;

$G(x) = V_{RD,max} - V_{EF}$ (3.44)

$V_{RD,max} = 0.18 b_w d (1 - \frac{f_{ck}}{250}) f_{ck}$ [6]

Shear at face of support i.e. first and second interior support

$$V_{EF,A} = V_{EF,B} = 0.45F$$

$F = (1.35G_K + 1.5Q_K)L(3.46)$

$V_{EF,AB} = 0.45 Q_K L (1.35\alpha + 1.5)$ (3.47)

$G_3(x) = 0.18 b_w d (1 - \frac{f_{ck}}{250}) f_{ck} - \{0.45 Q_k L (1.35\alpha + 1.5)\}$ [7]

Shear at the interior and exterior support

$$V_{EF,C} = V_{,EFD} = 0.60F$$

$$F = (1.35G_K + 1.5Q_{KL})L$$

$V_{EF,C,D} = 0.60 Q_K L (1.35\alpha + 1.5)(3.49)$

$G_4(x) = 0.18 b_w d (1 - \frac{f_{ck}}{250}) f_{ck} - \{0.60 Q_K L (1.35\alpha + 1.5\})$ [8]

In equation [6] to [8] V_{RD} and V_{EF}, are characteristic values of shear strength and designed value of shear strength respectively. In equation [7] and [8], dead to live load ratio may be expressed as ALPHA (α)

4. RESULTS AND DISCUSSION

The results obtained and discussion arising are therefore are presented below in for practical applications, an algorithm for solving these equations has been developed and coded by researchers, the reliability of the beam is carried out using FORM with the parameter for stochastic model generated [13]. Conceptually, the so-called FORM5 is based on the work in previous scholars' studies [14, 15].

Table 4.1 Parameters of Stochastic Model on a Beam Due to Bending and shearing.

S/N	Parameters	Distribution type	COV	E(x)i	S(x)i
1	Characteristic strength of concrete, f_{ck}	Log- normal 3	0.15	30 N/ mm^2	4.5N/mm
	Characteristic strength of concrete, f_y	Log- normal 3	0.15	460N/mm^2	6.9N/mm
2	width of beam, b_w	Normal 2	0.05	300 mm	15mm
3	Span of beam, L	Normal 2	0.05	5000mm	250mm
4	Effective depth, d	Normal 2	0.05	600mm	30mm
5	Imposed load	Log- normal 3	0.3	5.0Kn/m	1.5

4.1 Bending Failure Mode:

The results of the reliability analysis considering bending failure at varying values of design variable in the stochastic model are shown in this section

Interior Span in Bending

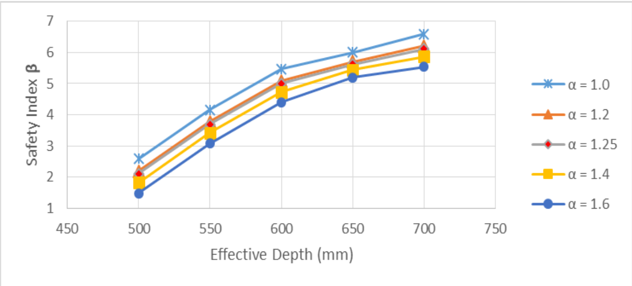


Fig 4.1: Safety Index for Various Effective Depth of Beam

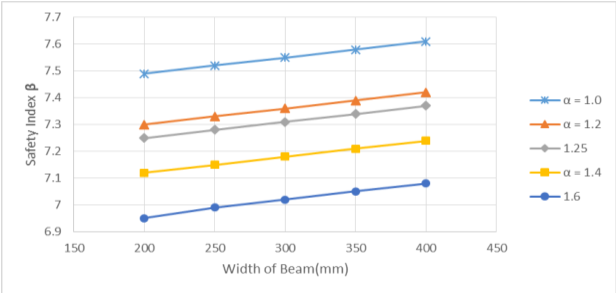


Fig 4.2: Safety Index for Various Width of Beam in Bending.

Exterior Span in Bending

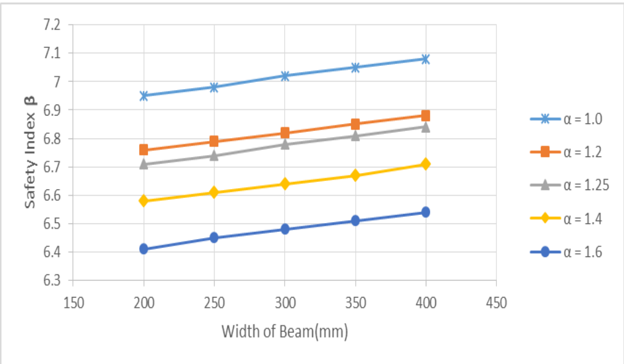


Fig 4.3: Safety Index for Various Width of Beam

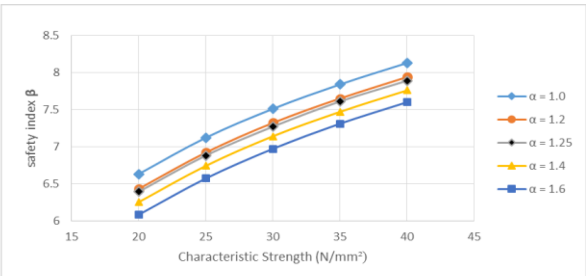


Fig 4.4: Safety Index versus Characteristic Strength 3.2

4.2 Shear Failure Mode

The results of the reliability analysis considering shear failure at varying values of design variables used in the stochastic model are shown in this section.

Interior Support in Shear

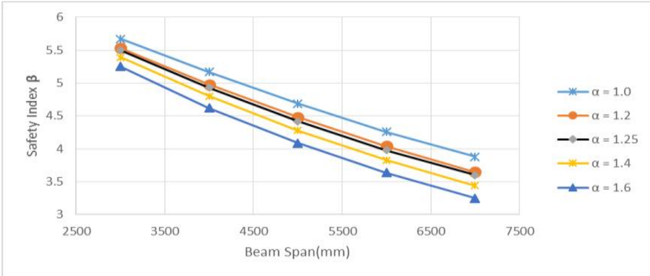


Fig 4.5: Safety Index for Various Span of Beam in Shear

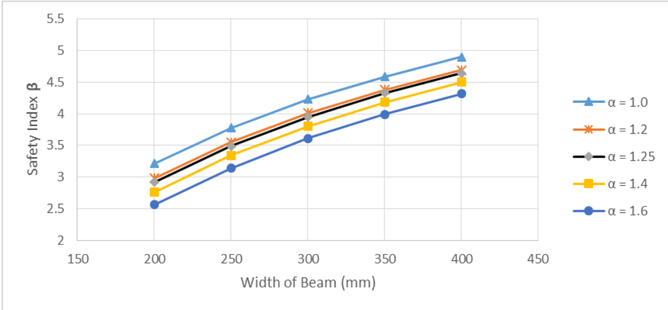


Fig 4.6: Safety Index for various width of the beam in Shear

Exterior Support in Shear

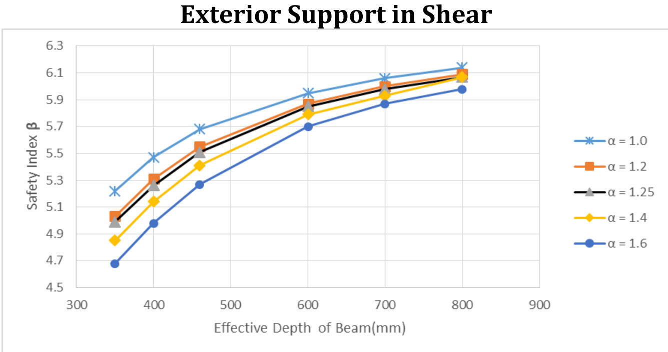


Fig 4.7: Safety Index versus Effective Depth in Shear

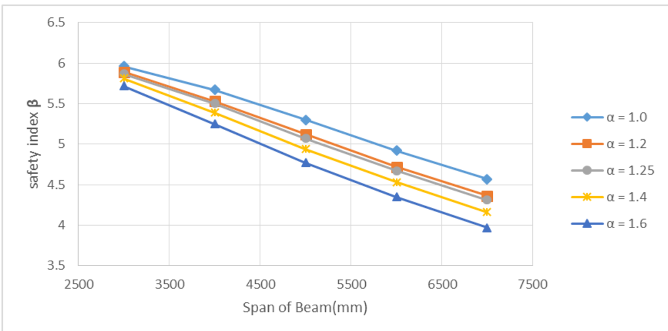


Fig 4.8: Safety Index for various Span of the beam in Shear

5. DISCUSSION OF RESULTS

5.1 Failure Due to Bending

Using reliability index of 3.8 as the target for safety index as stated in Eurocode(EN 1992,2005) for ordinary structures.it can be seen that for **α** =1.0, the effective depth for bending of interior span for the continuous beam is 520mm, for **α** =1.2 the minimum effective depth is 550mm, for **α** = 1.25 the minimum effective depth is 555mm for **α=** 1.4 the minimum effective depth is 565mm finally for **α** =1.6 the minimum effective depth is 575mm.

From Fig 4.2, 4.3, and 4.4 the safety index was observed to have increased with the increase in width of the beam at both interior and exterior and characteristic strength of concrete and decreases with an increase in load ratio. The acquired safety indices are all above the target reliability index of 3.8 as recommended by Eurocode (EN 1992, 2008) and that of the joint committee for structural safety target reliability of (JCSS, 2005). It can be seen that for all the load ratios and the characteristic strength of the concrete considered the safety index is above the standard safety index.

For the bending failure of the beam, the effective depth, width, characteristic strength of concrete, and span were considered as illustrated in Figs 4.1 to 4.4. When considering the effective depth, it is observed that the safety index increases with an increase in effective depth. This is because the effective depth is directly proportional to the resistance of the beam, which is in turn proportional to the safety index. Considering the width of the beam, it is observed that the safety factor **(β)** decreases as the width increases. Considering the characteristic strength, it is observed that the safety factor **(β)** increases as the strength increases. This is because the characteristic strength is directly proportional to the resistance of the beam, which is in turn proportional to the safety index. Considering the span of the beam, the safety index **(β)** observed to decreases with the increase of the beam span. This is because the span is proportional to the load of the beam, which is in turn inversely proportional to the safety index.

5.2Failure Due to Shear

From fig 4.5 Using reliability index of 3.8 as the target for safety index as stated in Eurocode (EN 1992, 2008) for ordinary structures.it can be seen that for **α** =1.0, the beam span in shear of interior support for the continuous beam is700mm, for **α** =1.2 the minimum beam span 6300mm, for **α** = 1.25 the minimum beam span is 6200mm for 1.4 the minimum beam span is 5750mm finally for **α** =1.6 the minimum beam span is 5400mm. From fig 4.6 as the width of the beam increases the safety index also increases. Also, from fig 4.7 the safety index was observed to have increased with the increase in effective depth of beam and decreases with an increase in load ratio. The acquired safety indices are all above the target reliability index. While From fig 4.7 the safety index was observed to have increased with the increase in effective depth of beam and decreases with an increase in load ratio. The acquired safety indices are all above the target reliability index of 3.8 as recommended by Eurocode (EN 1992, 2008) and that of (JCSS, 2005) standards.it can be seen that for all the load ratios and the beam depth considered, the safety index is above the threshold safety index.

For the shear failure of the beam, the effective depth, width, and span were considered as illustrated in figs 4.5 to 4.8. Considering the effective depth it was observed that the safety index increased as the effective depth increases. This is because the effective depth is directly proportional to the resistance of the beam, which is in turn proportional to the safety index. Considering the width of the beam, it is observed that the safety factor **(β)** increases as the width increases. This is because the width is directly proportional to the resistance of the beam, which is in turn proportional to the safety index. Considering the span of the beam, the safety index **(β)** observed to decreases with the increase of the span of the beam. This is because the span is proportional to the load of the beam, which is in turn inversely proportional to the safety index

5.3. CONCLUSIONS

From the results obtained, it is observed that an effective depth of 600 mm, a length of 5000 mm and a breadth of 300 mm are adequate for the proposed reinforced concrete continuous beam for a load ratio ranging from 1.0 to 1.25. Also, At higher load ratios considering the same parameters, the beam will experience failure due to the shearing of the interior support only while the rest of the parameters will be safe. Hence for load ratio less than or equal to 1.25, the beam is safe.

The uncertainties due to concrete strength and applied loads were accommodated, beam geometry was considered as deterministic; partial safety factor for both beam and load were also considered as deterministic. From the forgone variable it was observed that the ultimate failure mode of the beam yields lower reliability indices than the service mode hence the design is safe.

5.4 RECOMMENDATION

Based on the unpredictable and inconsistent levels of safety and unexpected failure of some structures due to many sources of uncertainties, reliability analysis using the FORM is mandatory in assessing the safety level of a structure. It is recommended that structural safety should always be assessed. Hence, an advance should be made toward producing simplified procedures and safety design functions for structures. However, due to structural inability to completely be free from the possibility of failure, design variables must, therefore, be in such a manner as to suitably fit the risk so that the goal of design, that is to design a safety margin so that the risk of failure is as small as possible is defected.

REFERENCES

[1] Bill Mosley, John Bungey and Ray H. (2007); Reinforced Concrete Design, Sixth Edition, Book Power with Palgrave Macmillan, New York.

[2] Mansour, A. E. (1989). An Introduction to Structural Reliability Theory. Mansour Engineering Inc Berkeley Ca.

[3] Ditlevsen, O and Madsen, H. O. (2005) Structural Reliability Methods. John Wiley and Sons Limited, England, 2005, pp 111 - 147.

[4] Ang, A. H. S., & Cornell, C. A. (1974). Reliability bases of structural safety and design. Journal of the Structural Division, 100(Proc. Paper 10777).

[5] Ayyub, B. M., & Haldar, A. (1984). Practical structural reliability techniques. Journal of Structural Engineering, 110(8), 1707-1724.

[6] Galambos, T. V., & Ravindra, M. K. (1978). Properties of Steel for Use in LRFD. Journal of the Structural Division, 104(9), 1459-1468.

[7] Melchers, R. E., & Beck, A. T. (2018). Structural reliability analysis and prediction. John Wiley & Sons.

[8] Fiorato, A. E. (1973). Geometric imperfections in concrete structures: a literature survey. National Swedish Institute for Building Research.

[9] EC2. Eurocode 2: Design of Concrete Structures. BS EN 1992-1- 2, European Committee for Standardization, CEN Brussels, 2008.

[10] Shinozuka, M. (1983). Basic analysis of structural safety. Journal of Structural Engineering, 109(3), 721-740.

[11] Tichý, M. (1994). First-order third-moment reliability method. Structural Safety, 16(3), 189-200.

[12] Ajimituhuo, J. L., Abejide, O. S., & Mangut, S. (2018). Reliability analysis of CFRP shear walls subject to blast loading. Nigerian Journal of Technology, 37(3), 626-632.

[13] Idris, A., & Ibrahim, A. (2017). Iso-Safety Design Charts for Singly Reinforced Concrete Sections to Eurocode 0 Recommended Target Safety Indices. Jordan Journal of Civil Engineering, 11(4).

[14] Rackwitz, R., & Flessler, B. (1978). Structural reliability under combined random load sequences. Computers & Structures, 9(5), 489-494.

[15] Hasofer, A. M., & Lind, N. C. (1974). Exact and invariant second-moment code format. Journal of the Engineering Mechanics Division, 100(1), 111-121.

[16] EC2. Eurocode 2: Design of Concrete Structures. BS EN 1992-1- 2, European Committee for Standardization, CEN Brussels, 2008.

[17] JCSS. Recommendations for Structural Safety. Joint Committee on Structural Safety, Luxemburg, 2005.