MOMENT CURVATURE AND SHEAR DISPLACEMENT HINGES FOR BEAMS

DonaGeegy[1], Dr. Mathews M Paul[2], Sinoy IJ[3]

[1]PG Student, Strucural Engineering, Dept. of Civil Engineering, Mar Athanasius College of Engineering, Kothamangalam, India.
[2]Professor, Dept. of Civil Engineering, Mar Athanasius College of Engineering,Kothamangalam.,India.
[3]Manager, Dept. of Structural Engineering, L&T Construction, Chennai, India.

---***---

Abstract - *Existing, reinforced concrete buildings should possess simple and well-established technique for satisfying seismic performance. They thus satisfy equilibrium criteria that should enable critical regions under the action of gravity loads and Earthquake induced forces to be readily identified. Once the elements of complete plastic mechanism are identified, plastic hinge locations in buildings are to be carefully detailed in a structure. The behaviour of member studies in element levels are done at different deformability levels. A control beam was kept for analysing member level flexure hinge. Hinge properties of beams and columns were identified for both flexure and shear. They are then characterised with the acceptance levels which include locating building at different limit states such as Immediate Occupancy (IO), Life Safety (LS), Collapse Prevention (CP). This gives inelastic ductile structural response of the structure. Performance Based Design (PBD) requires the maximum probable value of displacement induced forces to be estimated and compared with the demand curve to calculate the maximum probable drift taken by the structure*

Key Words: **Deformability Levels, Hinge Properties, Acceptance Levels, Performance-Based Design**

1. INTRODUCTION

Earthquake Resistant Design (ERD) Philosophy accepts damage in normal buildings during strong earthquake shaking, but targets to prevent collapse during severe level of shaking. Thus, quantifying the severity of an expected earthquake (seismic demand) is an important step in ERD. Further, the type of damage that is incurred depends on the type of structural system and design & detailing schemes adopted. For reinforced concrete (RC) moment frame buildings, structural damage, often leading to partial or even total collapse, is attributed to key deficiencies (in seismic capacity), like: (i) lack of regular structural grid, leading to asymmetry in mass and stiffness in plan, (ii) abrupt variations in sizes or discontinuance of vertical elements along height, causing discontinuity in load path, and stiffness & strength irregularity in elevation, and (iii) inadequate design and poor detailing of longitudinal & transverse steel reinforcement, leading to inadequate shear strength and curvature ductility of RC sections. In general, seismic safety of a RC building hinges critically on ensuring that it has adequate seismic capacity which surpasses the expected seismic demand, and thereby controlling the degree of damage incurred at the different levels of shaking.

Failure in building can be modelled through formation of hinges in beams and columns. A nonlinear analysis like this can predict the failure mode, maximum force and deformation capacity of the structure. But to do an accurate analysis nonlinear modelling of frame sections for flexure and shear is very important. Incremental lateral loads given to a structure linearly thus to derive the capacity plot of structure is important. This helps in study of failure criteria of each member. However, the nonlinear modelling of RC sections in shear and flexure for column and beam is not well understood. The current industry practice is to do nonlinear analysis for flexure only. The use of idealized moment-curvature curves and its properties, as proposed, offers a consistent and non-empirical approach compatible with actual section properties [8]. The highly popular model namely Mander's model is used for concrete stress strain relationship since it is simple and effective in considering the effects of confinement. The use of idealized moment-curvature curves [7]and its properties, as proposed, offers a consistent and non- empirical approach compatible with actual section properties.[4]Plastic hinge length and transverse reinforcement spacing found to have no influence on the base shear capacity but they have considerable effects on the displacement capacity of the frames.[6]

2. MOMENT CURVATURE CURVE FOR RC MEMBERS

Flexural behaviour of a reinforced concrete section can be studied with the knowledge of its moment-curvature relation. It is an important tool in generating moment field in a linear or nonlinear analysis as well as in predicting the complete nonlinear load-deflection behaviour of RC flexural members. This relation is non-linear mainly due to concrete cracking and steel yielding, which makes the analysis fairly complicated.

For this, accurate estimation is critical of initial flexural rigidity EI, bending moment capacity Mu, and curvature ductility $\mu\varphi$ of RC sections of these members, to obtain actual moment-curvature (M-φ) curves. In turn, the estimation of M-φ curves requires use of over-strength stress-strain (σ-ε) curves of concrete and reinforcing steel, along with geometric properties of RC sections, and not the design σ-ε curves given in codes.

2.1Moment– curvature for beam expirementally

A control beam was considered for deriving moment-curvature relationships of an RC beam. Experiments are conducted at L & T constructions, Research and Testing center on one simply supported RC rectangular beams subjected to symmetric two point loading simulating pure bending. Compressive values 39 Mpa and MOE values of specimen 32700 Mpa. Figure shows the photograph of the test specimens. The beam was tested under two point loads. Effective span (l) of the beam is 2000 mm. The load was applied at a distance of 300mm from both the ends, with the help of hydraulic jack. Thus providing a loading span of 700 mm. Deflections were recorded using LVDT (Linear variable displacement transducer) positioned at the midspan center, top and bottom. The geometrical and material properties of the test specimens are tabulated in Table 1.1 and Table 1.2. The yield strength of steel is 500N/mm2. The beams were casted using ready mixed concrete and cured for 28 days.

Table -1: Geometric Properties of Test Specimens

Breadth(B)	Depth (D)
200	300

Fig -1: Two point loading an beam specimen

The geometric details of beam is provided in Table 1.2, beam reinforced with 2- 12 mm diameter at the bottom and 3- 10 mm diameter bar as top reinforcement. Transverse reinforcements of 8 mm diameter at a spacing of 250 mm diameter was provided.

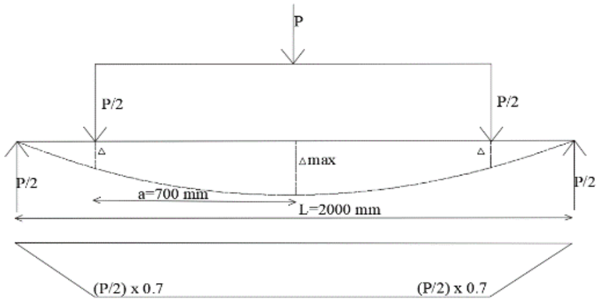

Fig -2: Loading Digram of beam[5]

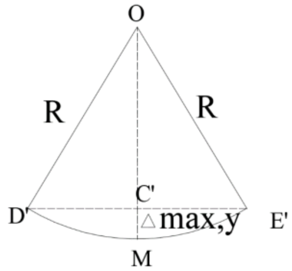

Fig -3: Central deflection [5]

Generation of moment-curvature relation from experimental load deflection data:

The deflection profile between the loading points takes shape of an arc of a circle with radius R and a constant drift from central line as shown in Fig. 7 in a pure bending region. Using theory of simple bending, the curvature can be calculated from the deflection data obtained from the experiments as explained below. Curvature of the beam section given by, $Ø = \frac{1}{R}$

Substituting in curvature for a given cross-section of a beam from the experimental load-deflection data can be calculated. Moment at mid-span of the simply supported beam from Fig 1.1 (determinate structure) can be calculated as,

$M = \frac{P \times 0.7}{2}$,Where, 700 mm was the loading span.

2.2 *Moment curvature relation by by mander stress- strain for confined concrete*

Confinement of concrete by transverse reinforcements in RC sections increases the limits of maximum compressive stress and compressive strain in concrete. Many models are available to quantify these effects and arrive at the σ-ε curve of confined concrete. Of the various models for estimating confinement of concrete, the Mander's Model offers a σ-ε curve of confined concrete , which is applicable to both circular and rectangular RC sections [Mander et al, 1988]. Strain levels in the extreme compression fiber of concrete and at center of layers of reinforcement steel bars are indicators employed to monitor change in behavior of RC sections (e.g., cracking of concrete in tension, yielding of longitudinal reinforcement bars in tension, spalling of cover concrete in compression, buckling of longitudinal reinforcement bars in compression, and crushing of concrete in compression, and fracturing of longitudinal reinforcement bars in tension.

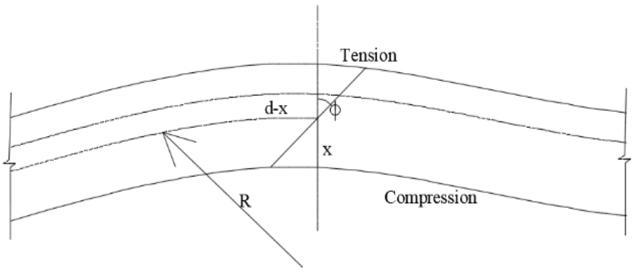

Fig -4: Pure bending curve[8]

Consider pure bending in beam specimen. Then,

$$\emptyset = \frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d-x} = \frac{\varepsilon_c+\varepsilon_s}{d} (1)$$

Where,

$$\emptyset = curvature$$

$$f_c = maximumstress \in compression \in concrete$$

$$\varepsilon_c = maximumstrain \in concrete$$

$$\varepsilon_s = maximumstrain \in steel$$

x=depth of neutral axis

d= effective depth of section

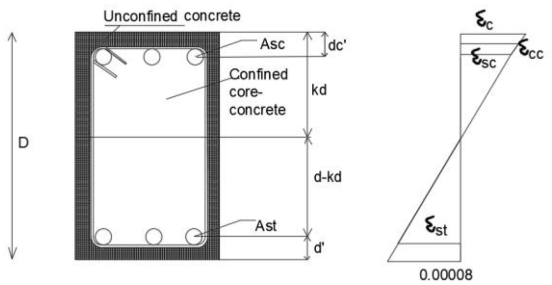

Fig -5: Cracking of concrete

Curvature after cracking,

Effective moment of inertia,

$$\emptyset at M_{cr} = \frac{M_{cr}}{EI_{eff}} \qquad (3)$$

Taking moment about compressive force due to concrete, yeild moment given by,

$$M_y = A_s \times f_y \times \left(d - \frac{kd}{3}\right) + A'_s \times f'_y \times \left(\frac{kd}{3} - d'_c\right)$$
(4)

Strain of $(0.002+\frac{E_s}{2\times10^5})$, $\emptyset at M_{yeilding} = \frac{0.002+\frac{E_s}{2\times10^5}}{d-kd}$
(5)

To accommodate non- linearity of stress- strain at higher limits, Whitney-block approximate the parabolic stress-distribution in concrete to equivalent rectangular stress- block representation. Equilibrium of tension and compressive force at higher limits,

Ultimate moment by taking about tension steel,

$$M_{u,spalling} = 0.36f'_c Cb\left(d - \frac{c}{2}\right) + A'_s f'_y (d - d')$$

$$\emptyset_u = \emptyset_{spalling} = \frac{0.0035}{c}$$

$$M_{u,end} = 0.36f'_c Cb\left(d - \frac{c-d'_c}{2}\right) + A'_s f'_y (d)$$
(7)

$$\emptyset_{ultimate} = \frac{0.01}{c-d'_c}$$

The sequence of reaching the limit states in both unconfined and confined under-reinforced sections are: (a) cracking of concrete in tension, (b) yielding of extreme layer of longitudinal reinforcement bars in tension, (c) spalling of cover concrete in compression, and (d) crushing of core concrete in compression, [7].

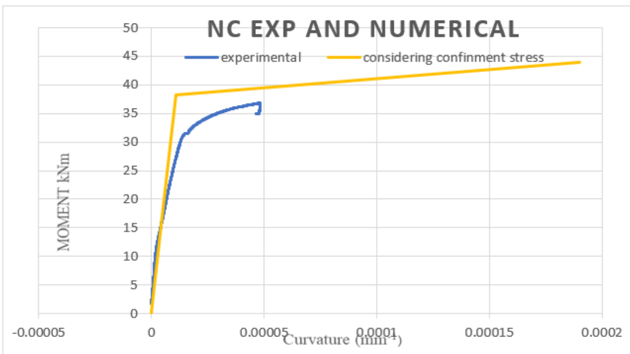

Chart -1: Comparison of Moment Curvature curves

3. SHEAR DISPLACEMENT MODEL

Major international design codes reviews with regard to the shear provision in RC section is discussed. This includes Indian Standard IS 456: 2000, British standard BS 8110: 1997 (Part 1), American Standard ACI 318: 2008 and FEMA 356: 2000. The shear capacity of a section is the maximum amount of shear the beam can withstand before failure.

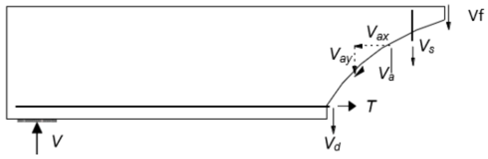

Fig -6: Shear on a section

Indian Standard (IS 456) and ACI standard 318(2008) [1]

As per IS 456:2000 total shear V_u resisted by beam is carried by two parts

Shear resisted by concrete V_c

Shear resisted by steel V_s

$$V_u = V_c + V_s \quad (8)$$

$$V_C = \delta \tau_c bd \left(where \delta = 1 + \frac{3P_U}{A_g f_{ck}} \leq 1.5 \right) (9)$$

$$V_s = 0.87 \times f_y A_{sv} \frac{d}{s_v} \text{ for vertical stirrups} \quad (10)$$

Where f_y = yield stress of transverse reinforcement

A_{sv}=Total cross-sectional area of one stirrup considering all legs

d= effective depth

S_v= spacing between two stirrups

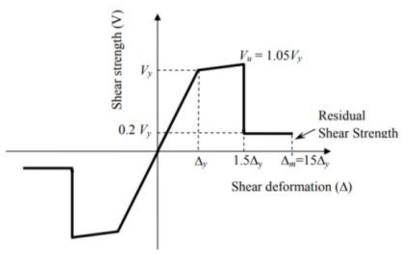

Fig 7 Shear strength vs deformation curve from ACI 318[1]

Shear displacement,Δ_{shear}

It is the shear displacement before and at the cracking point. This point is corresponding to the flexural cracking. Uncracked shear stiffness $K_{shear}$is defined as slope of the shear force versus shear displacement relation. $\frac{V}{\Delta_{shear}} = \frac{GA}{L}$ (11)

Where V = shear force, Δ_{shear} = shear displacement before cracking.

By ACI 318 (2008) V_U or the ultimate shear is assumed to be 5% higher than that of the yield shear V_y (Eqn 11)and the corresponding displacement to be 1.5 times Δ_{shear} as shown in fig3.2 .Thus, for a beam member V_U is given as per equation but neglecting the contribution of shear provided by the cracked concrete section, we neglect shear produced by concrete and will only consider V_s value. Therefore, beams section, $V_U = V_s$

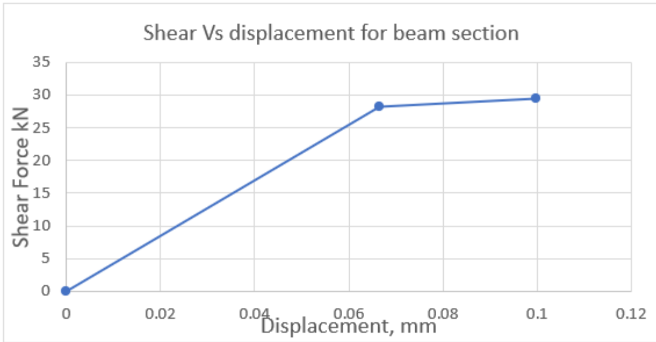

Chart -2: Shear force vs displacement for beam

4. ACCEPTANCE LEVELS OF PERFORMANCE

The idealised Moment curvature curves are converted into moment – rotation curves.These can be obtained with the help of plastic hinge length given by, l_p .In practice, an effective length lp is considered over which a given plastic curvature is assumed to be constant, instead of the physical length lpc [Park and Paulay, 1975]. This help assume a uniform curvature (throughout lp), to estimate the total rotation .l_p value is assumed to be half the depth of the beam or column section considered.[ATC 40]

$$l_p = \frac{D}{2} \quad (12)$$

$$\theta_P = (\emptyset | Pl_p)/3 \quad (13)$$

4.1 For flexure in member

From the acceptance chart given in FEMA 356 for a moment- frame building the levels deformation levels are obtained which is used for comparing the idealised curve obtained from the confined moment-rotation values. The initial value of cracking rotation depends only on (1/3) of the plastic hinge length. When the yield point stats forming the rotation, value proceed to depend on the whole plastic length which finally matches the total length of the element.

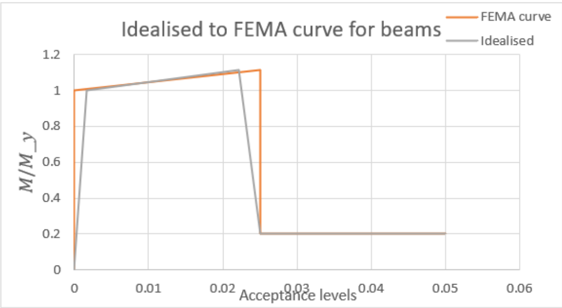

Chart -3: Flexure in beam member

4.2 For shear in beam member

The shear failure is a brittle failure. Therefore, it doesn't accommodate much displacement levels.

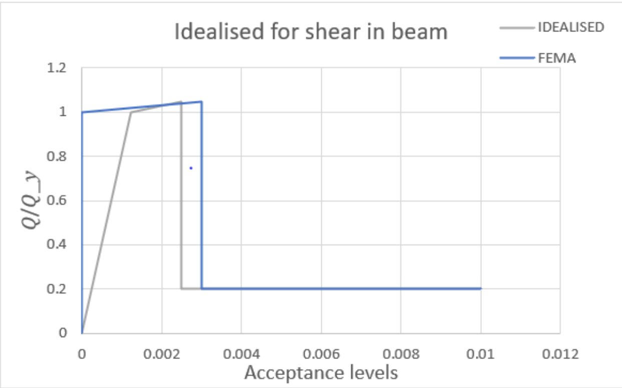

Chart -4: Shear in beam member

The moment curvature derived hinges gave near compactibility with the FEMA curve hinges. Thus, a tool for has been developed to define hinge property according to the section property, area of steel and transverse reinforcement.The excel tool allowes the maximum shear capacity and moment values of the section, thus analysis the failure of member sections in a building can be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

5. CONCLUSIONS

Typical over-strength σ-ε curves of concrete and reinforcement steel used in the study help capture M-φ, Q-y characteristics of RC sections reasonably well; Idealised bilinear M-φ curves for flexure developed using limit states, namely cracking of concrete in tension, yielding of extreme layer of longitudinal reinforcement bars in tension, spalling of cover concrete in compression and crushing of core concretein compression, closely represent flexural rigidity, flexural strength and curvature ductility capacity of RC sections. The Q-y curves represent the maximum shear capacity supported by the section for a compression controlled column and beam member. These curves are used to monitor damage states of RC buildings under earthquake shaking due to flexure and shear.

The acceptance curve provides a wide picture on user defined hinge model for each member in the building, which gives more approved results than a auto hinge model. The performance point of building can be thus made compatible with American code procedures identified in ATC-40.

REFERENCES

[1] "Seismic Evaluation and retrofit of concrete buildings" Applied Technology council,ATC-40,Report No. SSC 96-01,Volumes I and II, Seismic safety commission ,Redwood city,CA 1996.

[2] IS 1893 (Part I):2016,Indian Standard Code," Criteria for Earthquake Resistant Design of Structures-Part I",General Provision of buildings, Bureau of Indian Standards New Delhi.

[3] Akanshu Sharma, G.R. Reddy, K.K. Vazea, R. Eligehausenb, (2013) Pushover experiment and analysis of a full scale non-seismically detailed RC structure, Engineering Structures 46, pp. 218–233.

[4] Chopra & A K Goel ," A modal pushover analysis procedure for estimating seismic demands of buildings", Earthquake Engineering and Structural Dynamics,31,561-582,2002.

[5] RN Ingar, RK Chadha,K Balaji Rao ,"Development of probabilistic seismic hazard map of India", Report on National Disaster Management Authority,2010.

[6] N Lakshmanan, K Muthumani, GV Rama Rao, "Verification of Pushover analysis Method with static loading test", Earthquake hazard and mitigation,125,2008.

[7] P. Sunitha, Rupen Goswami and C.V. R. Murty,(2016), Idealised bilinear moment-curvature curves of RC sections for pushover analysis of RC frame buildings.,The Indian Concrete Journal, Vol. 90, Issue 4, pp. 43-54Federal Emergency Management Agency FEMA 273 (1997), Prestandard and Commentary for earthquake rehabilitation of building, USA.

[8] J. B. Mander, M. J. N. Priestley, and R. Park , *Theoretical Stress-Strain Model for Confined Concrete* , ASCE, J. Struct. Eng., 1988, 114(8): 1804-1826

BIOGRAPHIES

Dona Geegy
PG Student
Mtech Structral Engineering
MA College of Engineering

Dr. Mathws M Paul,
Proffessor
Mtech Structral Engineering
MA College of Engineering

Sinoy IJ,
Manager, R&D
Mtech Structural Engineering,
Larsen & Turbo Construction