INVESTIGATION ON RESPONSES OF LINEAR QUADRATIC REGULATOR CONTROLLED ADAPTIVE AIR SUSPENSION DURING BRAKING

D. Sundara Pandiyan[1], Dr.P.K. Palani[2]

[1] PG Scholar, Mechanical Engineering, Government College of Technology, Coimbatore, Tamilnadu, India.

[2] Professor, Mechanical Engineering, Government College of Technology, Coimbatore, Tamilnadu, India

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Abstract: *The main functions of the vehicle suspension system are to provide vehicle support, stability and directional control during handling manoeuvres and to provide effective isolation from road disturbances. A passive suspension does not provide a good ride comfort to the passenger. An active suspension continuously monitors the displacement of the car chassis and provides the required effort to control the vibrations of the vehicle, consequently improving the ride comfort. The mathematical model is developed with an air spring component as the primary restoring force providing element. The ride comfort is further affected during braking, as the weight transfer and pitching motion cause discomfort to the passengers. The effect of braking on the road excitations and passenger ride comfort is studied and a suitable control system using the Linear Quadratic Regulator (LQR) is designed to control the vibrations experienced by the passengers. Then the performance of the LQR controlled active air suspension is compared with the passive suspension by using MATLAB Simulation*.

Key Words: **Ride comfort, Mathematical model, Active suspension, Passive suspension, MATLAB, LQR**

1. INTRODUCTION:

Vehicle dynamics is the study of how the vehicle will react to driver inputs on a given road. The subject of "vehicle dynamics" is concerned with the movements of vehicles – automobiles, trucks, buses and special purpose vehicles on a road surface. The movements of interest are acceleration, braking ride and turning. A motor vehicle is made up of many components within its exterior envelope. Yet, for many of the more elementary analyses applied to it all components move together. That is, for acceleration, braking and most turning analyses, one mass is sufficient. For ride analysis, it is often necessary to treat the wheels as separate lumped masses. In that case the lumped mass representing the body is the "sprung mass", and the wheels are denoted as "unsprung mass".

1.1 Ride Model

Automobiles travel at high speed and as a consequence experience a broad spectrum of vibrations. These are transmitted to the passengers either by tactile, visual or aural paths. The term "ride" is commonly used in reference to tactile and visual vibrations, while the aural vibrations are categorized as "noise". Alternatively, the spectrum of vibrations may be divided up according to frequency and classified as ride (0-25 Hz) and noise (25-20000 Hz). The 25 Hz boundary point is approximately the lower frequency threshold of hearing, as well as the upper frequency limit of the simpler vibrations common to all motor vehicles. Mostly all vibrations are simultaneously present i.e., noise is usually present when lower-frequency vibrations are excited. The lower-frequency ride vibrations are manifestations of dynamic behaviour common to all rubber-tired motor vehicles. The predominant excitation sources are the road roughness, tire/wheel and driveline imbalances. Other than these sources, variation in the speed of the vehicle (acceleration/braking) also cause significant disturbances to the passengers. This is studied and evaluated using vertical vehicle ride models using lumped mass spring-mass-damper system models such as quarter car, half car and full car.

The half car model consists of representation of two wheels in the longitudinal direction i.e., front and rear wheels, as a lumped mass model. It is predominantly developed to study the response of the pitching motion of the vehicle in addition to the bounce of the car model.

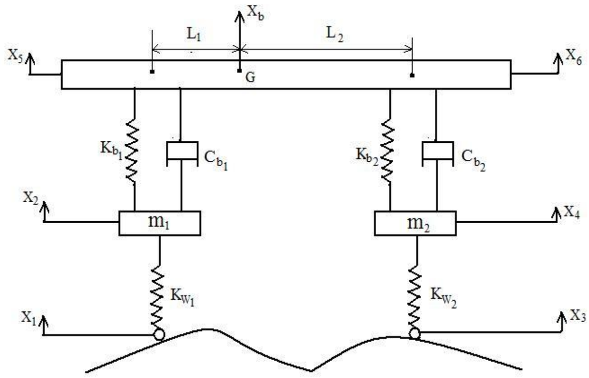

Fig -1: Half Car Model

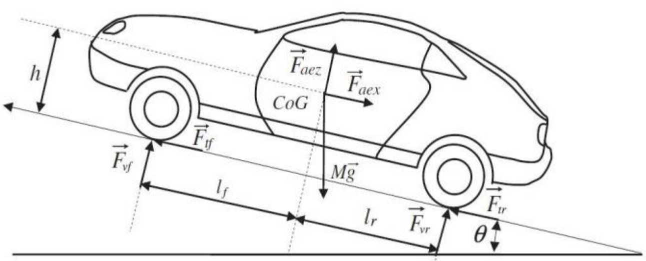

Fig -2: Longitudinal Dynamics Model

1.2 State Space Representations

In control engineering, a state-space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations or difference equations.

State variables are variables whose values evolve through time in a way that depends on the values they have at any given time and also depends on the externally imposed values of input variables. Output variables' values depend on the values of the state variables.

To abstract from the number of inputs, outputs and states, these variables are expressed as vectors. Additionally, if the dynamical system is linear, time-invariant, and finite-dimensional, then the differential and algebraic equations can be written in matrix form. The state-space method is known by significant algebraizing of general system theory. The state space representation is given by

$$X=AX+BU$$
$$Y=CX+DU$$

1.3 Linear Quadratic Regulator

The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic function is called the LQ problem. Like the LQR problem itself, the LQG problem is one of the most fundamental problems in control theory. One of the main results in the theory is that the solution is provided by the linear–quadratic regulator (LQR), a feedback controller whose equations are given below.

$$u=-KX$$

K is selected for minimizing the performance index given by,

$$J=\int_0^\infty (X^T QX+u^T Ru)\mathrm{dt}$$

The Q and R are weighting matrices which are to chosen to optimize the system. Elaborate optimization techniques are ideally followed to set the final Q and R values. The solution is given by a special equation known as the Algebraic Riccati equation (ARE), which is given by:

$$A'P+PA+Q-(PB+N)R^{-1}(B'P+N')=0$$

From the above Riccati equation, the matrix P is found by solving the equation. Then the solution is given by,

$$K=R^{-1}B^TP$$

1.4 Nonlinear Systems and Gain Scheduling

State feedback control systems can be only implemented in linear systems. Nonlinear systems need to be linearized about an equilibrium operating point. The control system also tries to maintain the system in the same equilibrium point. The control system is valid only in that respective localization. As the operation of the system moves away from the operating point the accuracy of the system falls rapidly. To overcome this effect two or more operating points are chosen and the corresponding linearized systems are developed. The control is scheduled between these various control points based on any of the states or the input. This process of creating a family of linearized system to replace a nonlinear system for linear control implementation is known as gain scheduling.

Gain scheduling is a common technique for controlling nonlinear systems with dynamics changing from one operating condition to another. Gain scheduling is used when a single set of controller gains does not provide desired performance and stability throughout the entire range of operating conditions for the plant.

1. Linearize nonlinear plant model at different operating conditions to obtain linear models that describe plant behaviour in the vicinity of the operating point that a linear model corresponds to.
2. Tune controller gains for all the linear plant models.
3. Implement a gain-scheduled controller architecture, where controller gains are "scheduled" with a scheduling variable, such as a measured output or a system state.

2. MATHEMATICAL MODELLING:

Mathematical modelling forms the first step in any study of dynamical systems. The model can be used to model in any of the programming tools for computational purposes. Mathematical models are developed based on simplified systems from the original complex systems. Then these simple physical models are used to develop the mathematical models based on the appropriate physical laws.

The ride comfort of a vehicle is analysed by using mathematical models which simplify the car system for feasible computational purpose. The model which falls into our domain of attention needs to capture the effect of pitching while braking. So a half car model is chosen. Four degree of freedom half car model is a comprehensive model which comprises the wheel and tire assembly. The wheel is termed as the unspring mass and is connected to the ground through a tire spring with relatively higher stiffness as compared to the suspension spring. Model is modelled with an active suspension configuration, where the removal of the actuator force will give the passive suspension model of the system. The graphical representation of the model is given below:

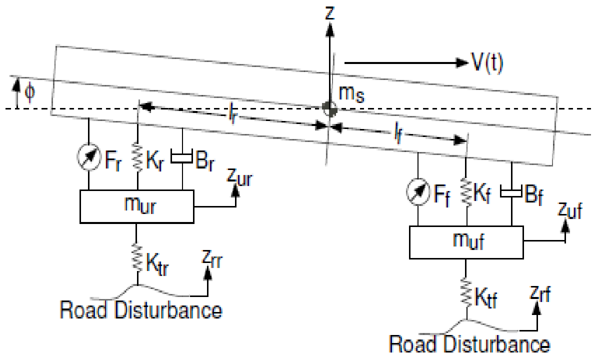

Fig -3: Half Car Model with Active Suspension

2.1 Linear Half Car Model

The mathematical model is derived from Newton's equation of motion. The notations are mentioned in the list of tables in the manuscript. The equations of motion that are derived are:

$$\ddot{z} = \frac{1}{m}\left[-k_f(z - z_{uf} - l_f\emptyset) - c_f(\dot{z} - \dot{z_{uf}} - \dot{l_f}\emptyset) - k_r(z + l_r\emptyset - z_{ur}) - c_r(\dot{z} + \dot{l_r}\emptyset - \dot{z_{ur}}) + F_r + F_f\right]$$

$$\ddot{\emptyset} = \frac{1}{I}\left[-k_f(z - z_{uf} - l_f\emptyset) - c_f l_f(\dot{z} - \dot{z_{uf}} - \dot{l_f}\emptyset) - k_r l_r(z + l_r\emptyset - z_{ur}) - c_r l_r(\dot{z} + \dot{l_r}\emptyset - \dot{z_{ur}}) + F_f l_f - F_r l_r + M_b\right]$$

$$\ddot{z_{uf}} = \frac{1}{m_{uf}}\left[k_f(z - z_{uf} - l_f\emptyset) + c_f(\dot{z} - \dot{z_{uf}} - \dot{l_f}\emptyset) - k_{tr}(z_{ur} - z_{rr}) + F_f\right.$$

$$\ddot{z_{ur}} = \frac{1}{m_{ur}}\left[k_r(z - z_{ur} - l_r\emptyset) + c_r(\dot{z} - \dot{z_{ur}} - \dot{l_r}\emptyset) - k_{tr}(z_{ur} - z_{rr}) + F_r\right.$$

2.2 Air Spring and Nonlinear Half Car Model

Similarly a mathematical model is developed for a ride model with nonlinear air spring stiffness. At first a stiffness equation for the air spring is developed. The relation for air spring stiffness was derived from adiabatic process equation from thermodynamics. This equation gives the relation between the pressure, nominal height, volume of air spring and its stiffness. From this stiffness, the magnitude of restoring force in both tension and compression can be evaluated.

The pneumatic spring is basically a column of confined gas in a container designed to utilize the compressibility of the gas to provide a restoring force to a suspended mass. Its ability is dependent on the effective area of the spring and the pressure inside of the spring. They provide an adjustable spring rate which helps in better controlling of the mass acceleration. They are mostly used in heavy SUVs and trucks in conjunction with leaf springs.

The common advantages of the air springs are:

1. Controllable spring rate
2. Adjustable load capacity
3. Simplicity of height control
4. Near constant frequency for various loads
5. Small amount of damping is present

Fig -4: Air Spring

Stiffness against the height of the air spring is represented below:

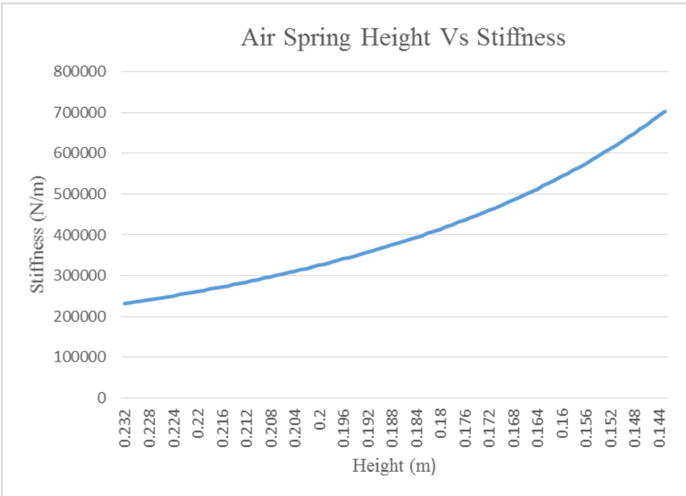

Chart -1: Air Spring Stiffness

Restoring force in both tension and compression against the nominal height is shown in the following figure:

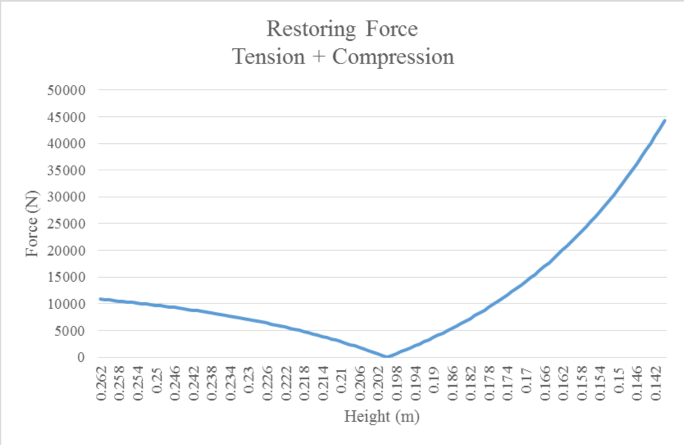

Chart -2: Air Spring Restoring Force

It can be seen that the air spring has higher stiffness in compression and lower stiffness in tension. This can have interesting effects in pitch motion of the vehicle. As the vehicle is relatively more compliant in tension and stiffer in compression, the diving part of the vehicle i.e., the front of the vehicle during nose dive in braking and rear of the vehicle during squat in acceleration, will require less control effort while the other part might require more control effort.
Thus the system of equations are given as

$$\ddot{z} = \frac{1}{m}\left[-k_{as}\left(z - z_{uf} - l_f\emptyset\right) - c_f\left(\dot{z} - \dot{z_{uf}} - \dot{l_f}\emptyset\right) - k_{as}\left(z + l_r\emptyset - z_{ur}\right) - c_r\left(\dot{z} + \dot{l_r}\emptyset - \dot{z_{ur}}\right) + F_r + F_f\right]$$

$$\ddot{\emptyset} = \frac{1}{I}\left[-k_{as}\left(z - z_{uf} - l_f\emptyset\right) - c_f l_f\left(\dot{z} - \dot{z_{uf}} - \dot{l_f}\emptyset\right) - k_{as}l_r\left(z + l_r\emptyset - z_{ur}\right) - c_r l_r\left(\dot{z} + \dot{l_r}\emptyset - \dot{z_{ur}}\right) + F_f l_f - F_r l_r + M_b\right]$$

$$\ddot{z_{uf}} = \frac{1}{m_{uf}}\left[k_{as}\left(z - z_{uf} - l_f\emptyset\right) + c_f\left(\dot{z} - \dot{z_{uf}} - \dot{l_f}\emptyset\right) - k_{tf}\left(z_{uf} - z_{rf}\right) + F_f\right.$$

$$\ddot{z_{ur}} = \frac{1}{m_{ur}}\left[k_{as}\left(z - z_{ur} - l_r\emptyset\right) + c_r\left(\dot{z} - \dot{z_{ur}} - \dot{l_r}\emptyset\right) - k_{tr}\left(z_{ur} - z_{rr}\right) + F_r\right.$$

3. STATE SPACE MODELLING

For design and modelling of control system, state space model is the most preferred way. The general state space model is poor in emulating the responses of the vehicle as it doesn't incorporate the excitations in the system. Therefore a better state space model is developed to better simulate the dynamic system of the vehicle. The disturbances which is the input to the system is added in the new model with a separate term, thus giving an accurate description of the system. For every plant\system to implement a control system, the plant needs to be a linear system. Because nonlinear control is very complex and often very expensive to implement. So a suitable linearization of the nonlinear equations are made through Jacobian linearization.

State Space models for both the systems are developed, and as mentioned, a modified state space form is developed to incorporate the external variables influencing the state of the system like road disturbances and pitching moments.

3.1 Linear Ride Model

The state space models for the active suspension is given below:

$$\dot{X} = AX + BU + GW$$
$$Y = CX$$

Where

A – State matrix; X – State vector; B – Control matrix;

U – Control force; G – Disturbance relation matrix

W – Disturbance vector; Y – Output vector;

C – Identity matrix

$$\begin{Bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\\ \dot{x_5}\\ \dot{x_6}\\ \dot{x_7}\\ \dot{x_8}\end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ \frac{-k_f - k_r}{m} & \frac{l_f k_f - l_r k_r}{m} & \frac{k_f}{m} & \frac{k_r}{m} & \frac{-c_f - c_r}{m} & \frac{l_f c_f - l_r c_r}{m} & \frac{c_f}{m} & \frac{c_r}{m}\\ \frac{k_f l_f - k_r l_r}{I} & \frac{-k_f l_f^2 - k_r l_r^2}{I} & \frac{-k_f l_f}{I} & \frac{k_r l_r}{I} & \frac{l_f c_f - l_r c_r}{I} & \frac{-c_f l_f^2 - c_r l_r^2}{I} & \frac{-c_f l_f}{I} & \frac{c_r l_r}{I}\\ \frac{k_f}{m_{uf}} & \frac{-k_f l_f}{m_{uf}} & \frac{-k_f - k_{tf}}{m_{uf}} & 0 & \frac{c_f}{m_{uf}} & \frac{-c_f l_f}{m_{uf}} & \frac{-c_f}{m_{uf}} & 0\\ \frac{k_r}{m_{ur}} & \frac{k_r l_r}{m_{ur}} & 0 & \frac{-k_r - k_{tr}}{m_{ur}} & \frac{c_r}{m_{ur}} & \frac{l_r c_r}{m_{ur}} & 0 & \frac{-c_r}{m_{ur}} \end{bmatrix} \begin{Bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\\ x_7\\ x_8\end{Bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ \frac{1}{m} & \frac{1}{m}\\ \frac{-l_f}{I} & \frac{l_r}{I}\\ \frac{-1}{m_{uf}} & 0\\ 0 & \frac{-1}{m_{ur}} \end{bmatrix} \begin{Bmatrix}u_1\\ u_2\end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ \frac{1}{I} & 0 & 0\\ 0 & \frac{k_{tf}}{m_{uf}} & 0\\ 0 & 0 & \frac{k_{tr}}{m_{ur}} \end{bmatrix} \begin{Bmatrix}w_1\\ w_2\\ w_3\end{Bmatrix}$$

4. SIMULINK MODELLING

The dynamic system is modelled using the Simulink environment of the MATLAB numerical computing software. The Simulink model provides versatility in recording and presenting the responses of the model for a wide variety of excitations. Simulink models are modelled based on the mathematical models. Both state space approach and raw equations of motion are used to develop separate models. The state space model is used for the simulation and study, because the control design is based on the state space approach.

4.1 Ride Model

The first model is based on the equations of motion. It is as follows:

International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 07 Issue: 06 | June 2020 www.irjet.net p-ISSN: 2395-0072

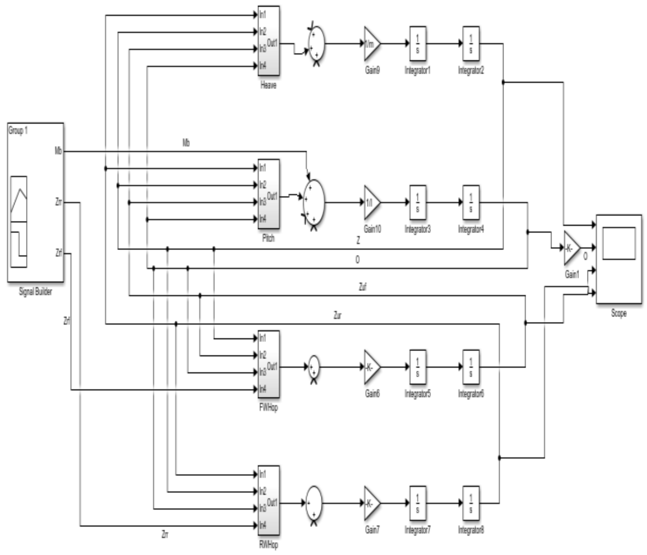

Fig -5: SIMULINK Model of 4 DOF Half Car Model

Each subsystem in the Simulink model is modelled after an equation of motion in the mathematical model. The four equation results in a total of four state variables namely heave or bounce (Z), pitch (Φ), front wheel hop (□□□), rear wheel hop (□□□)

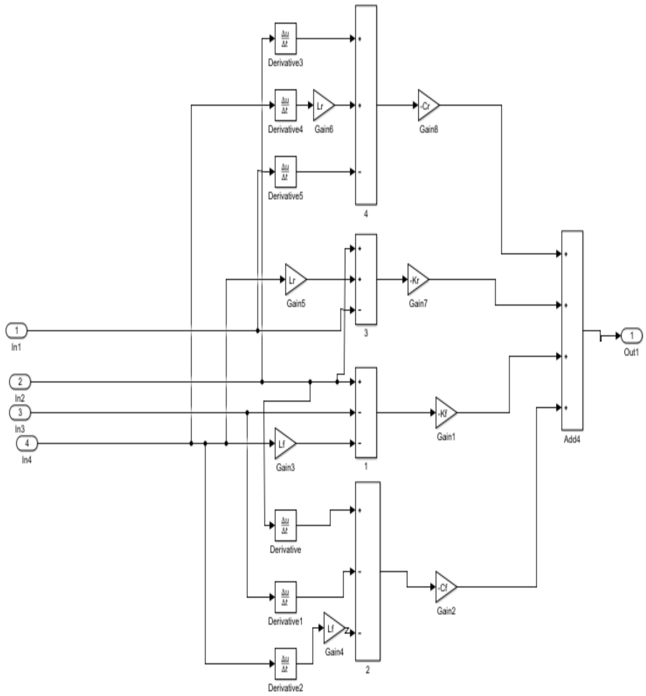

Fig -6: Spring Damper of 4 DOF Half Car SIMULINK Model

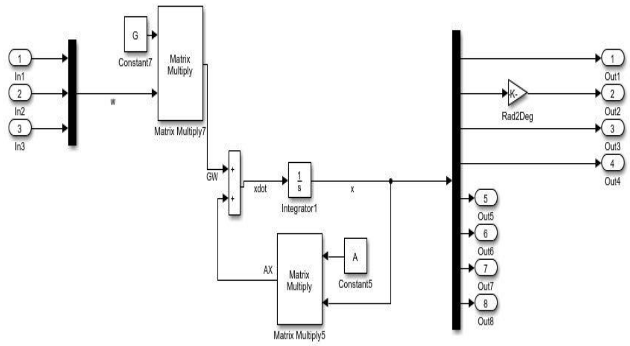

Fig -7: SIMULINK Model of State Space of Passive Suspension

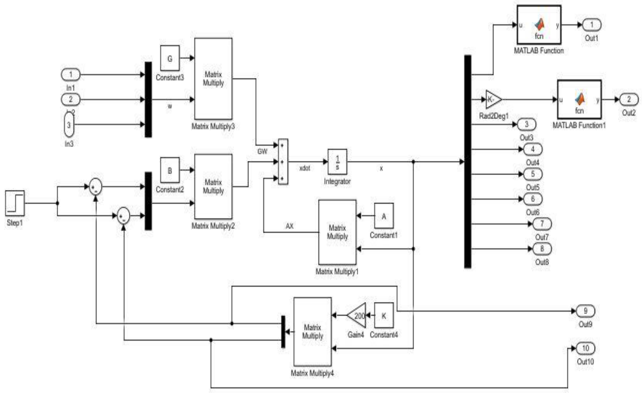

Fig -8: SIMULINK Model of State Space of Active Suspension

4.2 Full Vehicle Model

The longitudinal dynamics and the tire model are integrated with the ride model.

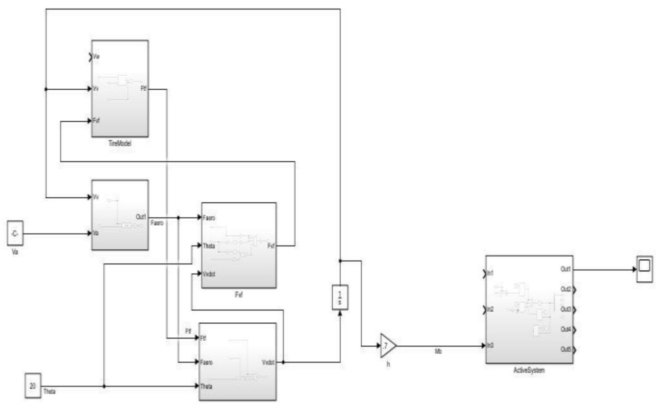

Fig -9: SIMULINK Model of Full Vehicle model

5. CONTROL SYSTEM

The control system uses a set of two sensors one on a wheel and other on the chassis on the wheel side. The relative displacement of the two-sensor position is used to estimate the road conditions and adjust the stiffness of the air spring to reduce the discomfort experienced by the passengers. This balance between softer spring required for smooth ride and harder spring required for better handling is effectively done by the controller aboard the vehicle.

To improve the experience of the system, two important things are carried out. To reduce the amount of energy consumed for the active control, an LQR controller is used to balance the trade-off between the ride comfort and the engine power used to run the compressor. In aiding and improving the effectiveness of the setup, the weighting matrices are optimized based on the Particle Swarm Optimization technique. Also, to facilitate the more accurate execution of the control of the nonlinear system, more than one operating points are taken. Each operating point have individual gain which are switched based on the state of the system or other signals based on the need. This is known as Gain Scheduling.

5.1 Gain Scheduling

The concept behind the proposed system is described below:

A linearized model is developed about multiple equilibrium points. For each developed model, a control system closed loop is made. The system is operated with the control system based on the closest operating point. Once the field variable changes within the proximity of another operating point, it changes to the corresponding closed loop system. Thus the entire operating range is covered with required number of control loops.

The following table gives the actual stiffness of the spring versus the linearized model for the operating point def = 0 m.

Table -1 Linearized Stiffness of Air Spring

Def(m)	K (Actual)	K (Linearized)	Deviation percentage
0.008	26726.42	29250	-9.4402
0.007	27024.53	29250	-8.2338
0.006	27327.36	29250	-7.0350
0.005	27635.02	29250	-5.8437
0.004	27947.63	29250	-4.6599
0.003	28265.31	29250	-3.4837
0.002	28588.19	29250	-2.3150
0.001	28916.38	29250	-1.1537
0	29250	29250	0.0000
-0.001	29589.17	29250	1.1463
-0.002	29934.01	29250	2.2851
-0.003	30284.64	29250	3.4164
-0.004	30641.17	29250	4.5403
-0.005	31003.74	29250	5.6568
-0.006	31372.45	29250	6.7659
-0.007	31747.43	29250	7.8675
-0.008	32128.79	29250	8.9618

It is advised that a single control system to be used within 10% range of the operating point and based on the convergence rate of the solution.

So as mentioned before a suitable second operating point of 0.01m is taken. This strategy of developing multiple linear models to define a nonlinear system and to schedule the respective gains for the state feedback system is known as Gain Scheduling. The two operating points are taken as 0.0 m and 0.1 m

The two different gains that are derived for the two operating points are used in a weighted transition method to allow for seam less transfer of control, otherwise it will result in a jump of control criteria once the system reaches the second operating point.
So gain is defined by:

$$K = \left(\frac{Z}{P_{eq}}\right)K_2 + (1 - Z/P_{eq})K_1$$

5.2 Optimization of LQR Tuning

The particle swarm optimization is a highly efficient method if compare with another. PSO mechanism started with population initialization to do random search and tracking. PSO consist of each particle that searching final solution on the determining range. Each particle will move consistent on their track on the surface of solution area. The best finding solution that encountered called global best, while the best position that have visited by the best particle is called local best.

The population will doing random searching over the solution surface in 30 iteration. The solution value range is 1 - 1000. The range value for Q and R are the same. Q must be semi definite matrix (□ ≥ 0) and (R > 0) positive definite. The PSO algorithm in this study is working independently. No feedback from the plant connected and affected the PSO tuning system.

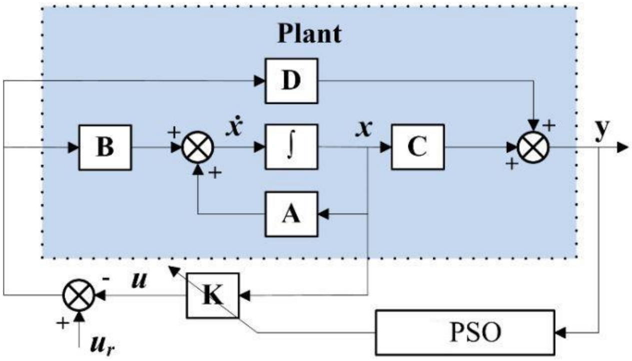

Fig -10: Schematic for Strategy for optimization

6. SIMULATION RESULTS AND DISCUSSION

The Simulation is carried out for the various models. The results of the state space model are alone studied in greater detail because only in state space model the active suspension can be implemented relatively with more ease. Many different models are studied for the project. First the half car model is studied with both passive and active suspension. Both the results are plotted in the same graph and evaluated using ISO standards of vibration evaluation. Similarly, half car model is modelled with both linear and nonlinear spring, the difference between the linear and nonlinear spring are well observed.

6.1 System Excitations

For any system, its characteristics are best studied by observing its response to various external, internal stimuli or excitations. Two excitations to the system are the road disturbances and the pitching moment due to braking. Each of these excitations are experienced by the system simultaneously.

6.1.1 Road Disturbances

Excitations to the system are the disturbances experienced by the vehicle during motion. When it is travelling at a constant velocity, the road profile is perceived by the vehicle in the same form with the frequency shift respective to the vehicle speed. But when the vehicle is accelerating, the road profile perceived by the vehicle is altered significantly from its actual form. A simple sine wave road profile is considered for the investigation of effect of varying velocity of the vehicle. The road profile of a 1.08Hz, 0.1 m amplitude road at both front and linear tire contact is:

□=□□□(2□□□□)

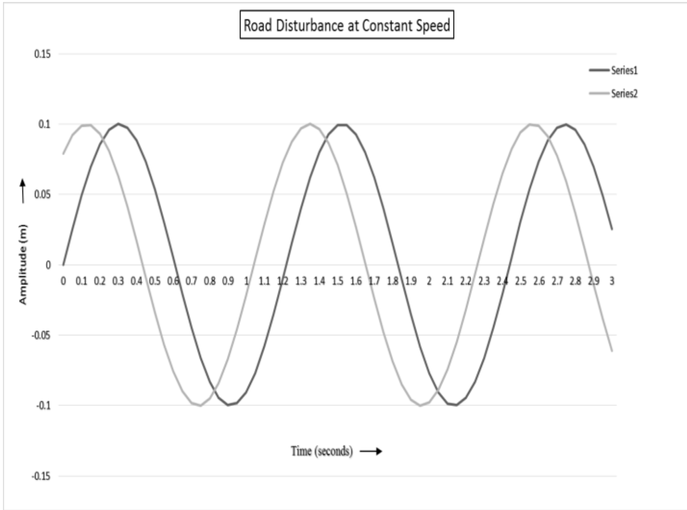

Chart -3: Road Disturbance at Constant Speed

The road excitation compared with constant velocity and two different cases of braking is compared to investigate the effect of braking. The two cases are:

1. Vehicle comes to full stop from travelling at 80 kmph in 5 s
2. Vehicle slows down from 60 kmph to 20 kmph in 3.5 s

The displacement at the tires for the same road profile but at braking condition of the case 1 (80 – 0 kmph in 5 s) is:

$$y = Asi(2\pi\gamma(u+at)t)$$

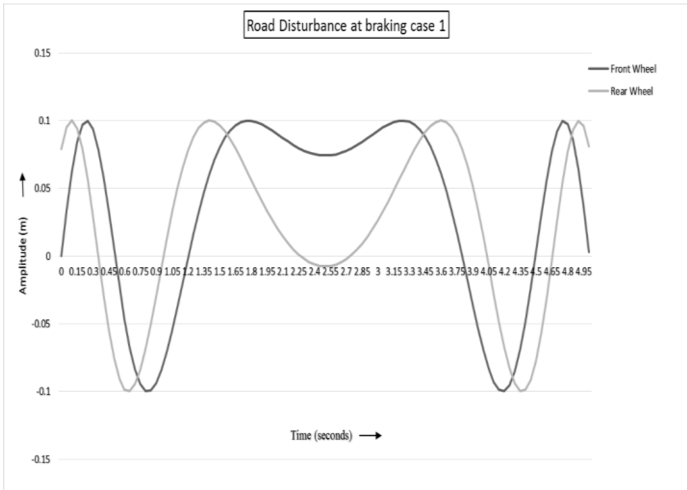

Chart -4: Road Disturbance at Case 1 Braking

The displacement at the tires for the same road profile but at braking condition of the case 2 (60 – 20 kmph in 3.5 s) is:

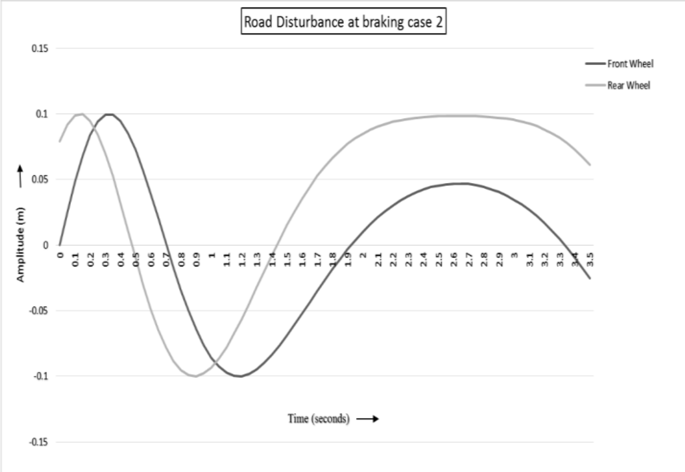

Chart -5: Road Disturbance at Case 1 Braking

Apart from the road disturbances, the vehicle also experience a pitching motion due to the braking. Braking causes the vehicle to experience a nose dive in and cause a load transfer to the front axle. The pitching moment is calculated by applying Newton's second law of motion. Considering two cases:

1. Vehicle comes to full stop from travelling at 80 kmph in 5 s
2. Vehicle slows down from 60 kmph to 20 kmph in 3.5 s

The equations are

$$F = m a$$

$$a = \frac{v-u}{t}$$

$$M_p = m \times a \times h$$

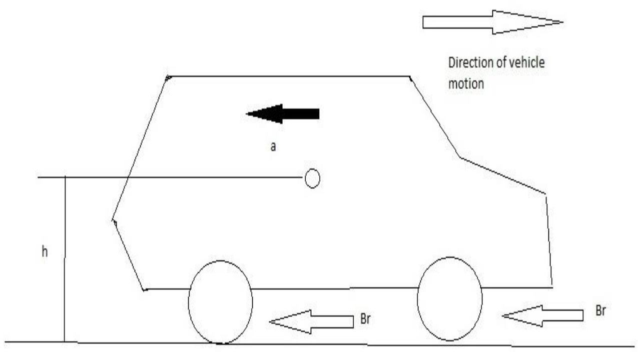

Fig -11: Vehicle Braking

Table -2 Braking Cases

Case Number	Deceleration	Pitch Moment	Time
1	4.44 m/s^2	8016.86 Nm	5 s
2	3.17 m/s^2	5723.75 Nm	3.5 s

6.1.2 Braking Effects

The effects due to braking are translated to the system by the driver's input. The driver either pushes on the accelerator pedal or the brake pedal. This consequently produces either a driving torque or a braking torque on the wheel causing it to accelerate or decelerate based on the longitudinal slip. When we press on the pedal, this causes the brake calliper to lock on the brake disc in the wheel hub assembly, this is again based on the type of the road and the tire conditions. Thus, the total inputs to the system are the road profile and the wheel torque input.

6.2 Active Suspension

The actuators provide smoothness and taken out the sharp edges and reduces the overshooting and improves the settling time. It has little effect on the rise time. The results are taken form a consolidated model. The gain matrix is updated with the optimized LQR controller.

6.2.1 Linear Model

The linear model are simulated by consolidated model of both active and passive suspension.

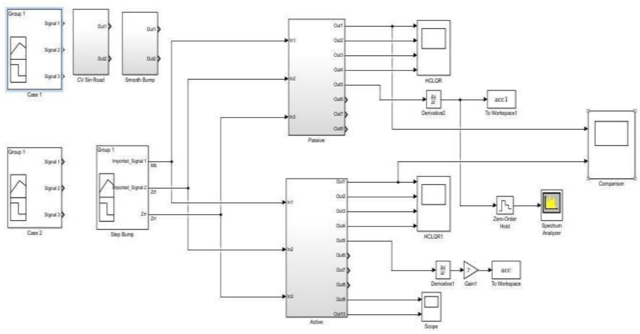

Fig -12: Consolidated SIMULINK Model of Passive and Active Suspension

The simulation shows the response of the vehicle for various excitations to the system. Both the passive and active suspension responses are recorded and super imposed to view the differences and control achieved by the controller. The simulation is first done for a bump input of 0.1m height to the vehicle wheels. The responses of the vehicle are as follows:

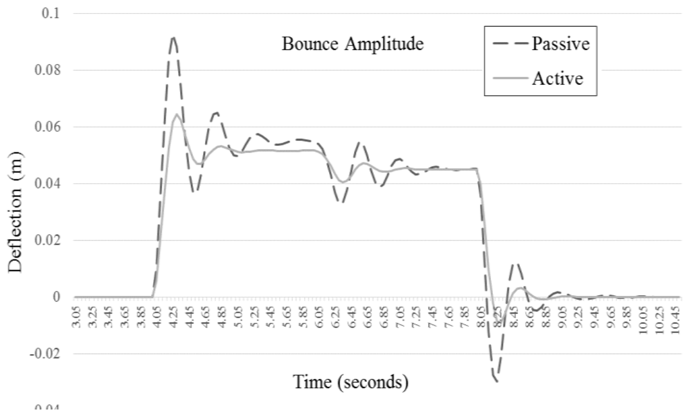

Chart -6: Vertical Deflection – Step Bump

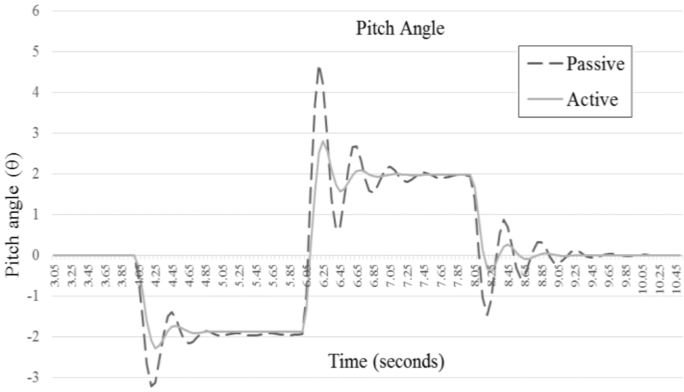

Chart -7: Pitch Angle – Step Bump

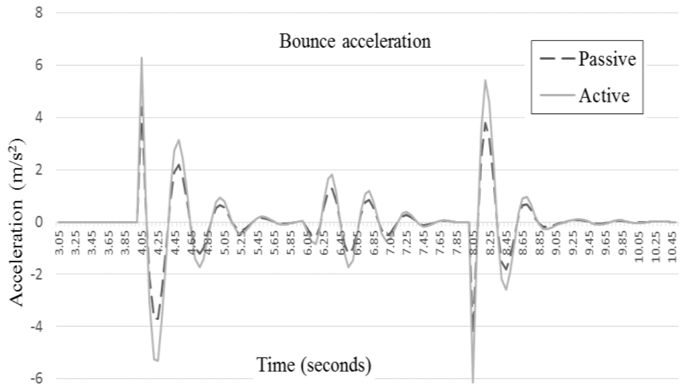

Chart -8: Vertical Acceleration – Step Bump

Now the excitations for the corresponding braking scenarios and their responses are as follows:

Case 1:

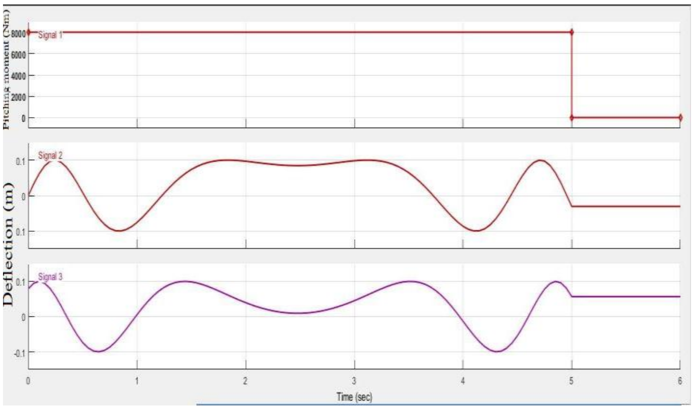

Fig -13: Braking Case 1: Disturbance Input

The responses are:

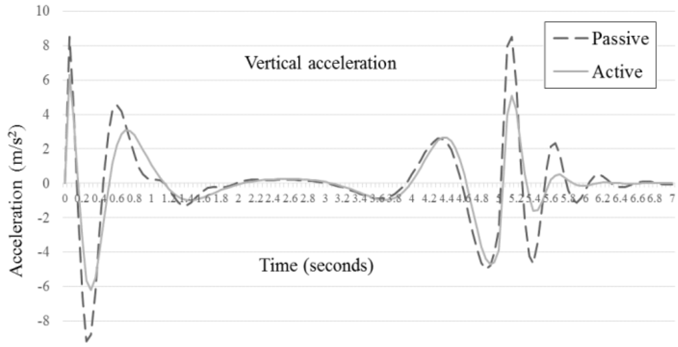

Chart -9: Vertical Acceleration – Braking Case 1

Case 2:

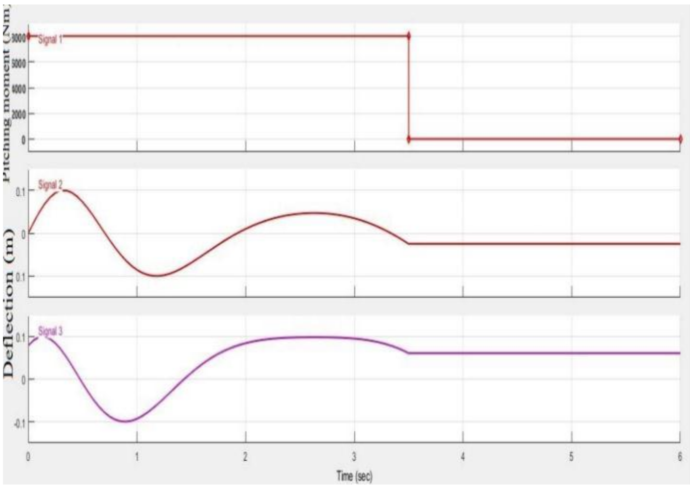

Fig -14: Braking Case 2: Disturbance Input

The corresponding responses are:

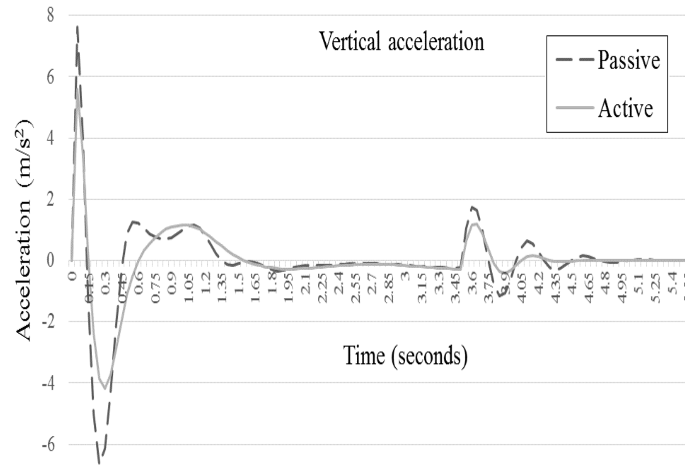

Chart -10: Vertical Acceleration – Braking Case 2

6.2.2 Nonlinear Model

Similarly, the same model is executed with the nonlinear air spring model in place of the linear spring. The difference between the linear and nonlinear effect is demonstrated with a single degree of freedom of system.

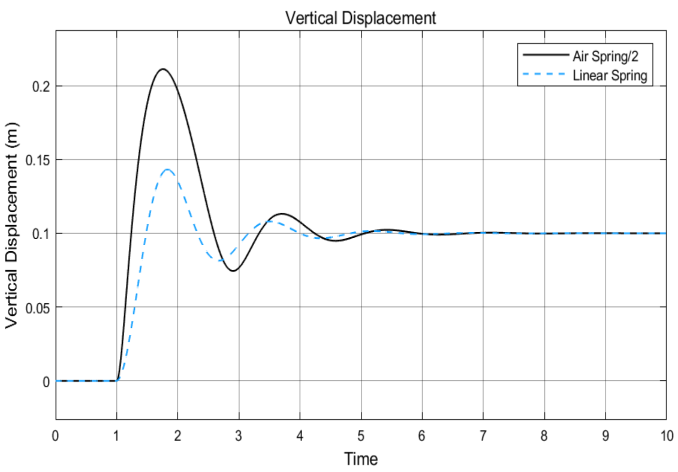

Chart -11: Linear Model vs. Nonlinear Model

From the above responses, the following observations are made.

1. With minimum amplitude of deflection, the sprung mass deflections are very similar.

2. With sudden deflection (like step bump), the air spring acts more rigid and the effect is more pronounced for larger amplitudes.

3. Even for larger amplitudes, for slow and gradual deflections, the air spring tracks a closer path to the linear spring response.

The bounce acceleration result for the nonlinear model is give below:

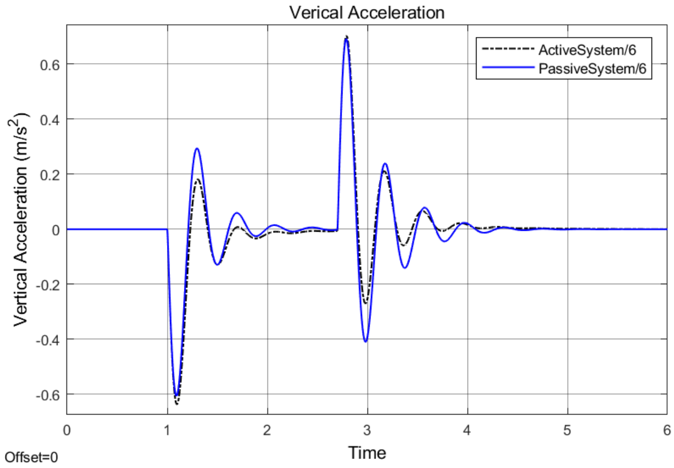

Chart -12: Vertical Acceleration – Nonlinear Model

6.3 Vibration Evaluation

The comfort of passenger during ride is a highly subjective evaluation parameter. It can be influenced by light level within the vehicle, humidity, driving environment, scenery among many other factors. The comfort level perception also differs from person to person. So a well-defined evaluation standard that is as objective as possible is required. There are many standards that have been proposed for vibration evaluation. Most of them propose a root mean square (RMS) values of the acceleration to define the comfortless. But as acceleration in different directions cause different levels of discomfort, a better evaluation standard was required. The frequency of the particular vibration also play an important level.

The most used and recommended evaluation standards are of the International Organization of Standardization (ISO). Based on 'ISO 2631-1: Mechanical vibration and shock – Evaluation of human body exposure to whole body vibrations. The standard has different weighting factors for vibrations of different frequencies and this weighted root mean square acceleration gives a single value for evaluation which is very convenient. There are also different sets of weighting factors for vibrations in different directions. The vibration for ride comfort perception during ride in a vehicle is determined by the weighted root mean square vertical acceleration given by the formula:

$$\square.\square.\square.\square\ \ddot{z} = \left[\frac{1}{\tau}\int_0^\tau a^2{}_w(t)dt\right]^{1/2}$$

The respective values are
<0.315 m/s2 - not uncomfortable
0.315 – 0.63 m/s2 - a little uncomfortable
0.500 – 1.00 m/s2 - fairly uncomfortable
0.800 – 1.6 m/s2 - uncomfortable
1.250 – 2.5 m/s2 - very uncomfortable
> 2 m/s2 - extremely uncomfortable

For vertical acceleration the Wd factor is taken for calculation of the weighted root mean square acceleration (w.r.m.s) value. A MATLAB code is developed to find out the respective w.r.m.s value of each case of simulation for both passive and active cases. The results are compared and the improvement of the active suspension is established. The MATLAB code for the calculation of w.r.m.s value is given in the appendix of this report.
The nonlinear model is evaluated using the subroutine. Though the simulation was conducted for a total of three scenarios including, a step bump and two scenarios of braking, only the step bump and case 1 of braking had significant level of acceleration to cause discomfort. The results of the two scenarios are then compared with their respective active suspension values. The results are summarized in the following table:

Table – 3: Vibration Evaluation Standards Results

INPUT	PASSIVE	ACTIVE
Step Bump	1.115 m/s^2	0.668 m/s^2
	uncomfortable	Fairly uncomfortable
Case – 1 Braking	0.9647 m/s^2	0.672 m/s^2
	Uncomfortable	Fairly uncomfortable

So in both scenarios, an improvement of one level in comfort level is registered showing the improved performance of the active suspension.

7. CONCLUSION

An active suspension for a pneumatic suspension was modelled and simulated. Linear Quadratic Regulator control strategy was used for the control system of the active system. The vehicle dynamic system was modelled in SIMULINK computing software. The model was verified by comparing results from literature Ahmed Esmael Mohan (2018). The improved performance of the active suspension compared to the passive suspension was established by comparing the results of three different scenarios in vehicle ride. Road excitation profile for braking conditions and the pitching moment induced on the sprung mass of the vehicle during the braking was also evaluated. The objective ride comfort level was also calculated by following the ISO standards for evaluation of effect of mechanical vibrations on the human body. For the arbitrarily chosen weighting matrices for the LQR system, an improvement of one level of comfort level was observed for the two scenarios.

Air spring model and the longitudinal model was integrated with the ride model to get an accurate picture of the braking. By evaluating the standards with the ISO vibration evaluation standards, it was proved that the ride comfort was improved using the LQR controller in the pneumatic suspension system.

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