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Extended State Observer based Double Hyperbolic Reaching Law for DC Motor

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Abstract - *This paper proposes an extended state observer (ESO) based double hyperbolic reaching law for position control of direct current (DC) motor. The double hyperbolic reaching law is made implementable when the complete state vector is not measurable. The closed loop stability is derived. The effectiveness of the proposed scheme is proved by considering different types of uncertainties and disturbances with various types of reference trajectories.*

Key Words: **Extended state observer, DC motor control, Reaching law, Double hyperbolic reaching law.**

1. INTRODUCTION

Sliding mode control has received attention from many researchers and industries due to its robust performance against external disturbances and parametric uncertainties [22]. Sliding mode control (SMC) has been proven to be an effective technique to control such systems and make their performance insensitive to parametric variations and external disturbances, when the system is in sliding mode [22]. It has found a variety of applications in the fields of robotics [28], aerospace vehicles [10], electrical drives [21] and automotive industry [12] to name just a few. The traditional sliding mode control has some problems or requirements:

- discontinuous control causes chattering problem
- the bound of uncertainties and disturbances should be known
- all states should be available for control law implementation

The problem of chattering is seriously limits the application of sliding mode control [1]. Many research has been carried out to alleviate chattering phenomenon in the literature, such as smoothing of discontinuous control [3], disturbance estimation based control [26], super twisting control [2], higher order sliding mode control [6], fuzzy sliding mode control [23], adaptive sliding mode control [24], reaching law based sliding mode control [8], etc.

The reaching law based sliding mode control has been investigate widely in the literature due to its ability alleviate chattering problem [8]. In the literature, many researchers have proposed reaching laws:

- Constant velocity reaching law [25],
- Exponential reaching law [27],
- Power reaching law [27],
- Single hyperbolic reaching law [14],
- Double hyperbolic reaching law [20].

In [7], an exponential reaching law is proposed to alleviate chattering and assure high tracking performance at the same time by changing switching gain magnitude. In [13], saturated non-switching term is introduced in adaptive reaching law is proposed for the electromagnetic formation flight. An exponential reaching law [17] was derived for improvement of robustness in the presence of external disturbances.

The double hyperbolic reaching law [20] has two hyperbolic functions having similar changing rate and opposite amplitude characteristics. One function has inverse hyperbolic sine characteristics which ensure fast convergence of sliding variable far away from sliding surface and other function has hyperbolic tangent characteristics which brings the sliding variable very closed to zero. Combination of both functions ensures fast convergence and chattering free motion.

The double hyperbolic reaching law for DC motor can be implemented, if the bound of uncertainties and external disturbances is known [20]. However, the control law also needs information of all states. In this case, it is necessary to obtain the uncertainty estimation as well as all states information. Several techniques were reported in the literature for state and/ or disturbance estimation. The disturbance observer [18], [16] estimates disturbance using binomial Q-filters and can be tuned by a single bandwidth parameter. The unknown input observer estimates the states and disturbance simultaneously using linear design model with linear disturbance model [9], [4]. The disturbance observer and unknown input observer still requires mathematical model. An extended state observer proposed in [11], [15] can estimate the state and disturbances simultaneously when the mathematical model of plant is not known.

In this paper, the double hyperbolic reaching law proposed in [20] made implementable when the complete state vector is not measurable. Additionally, the proposed observer-controller does not require the knowledge of bound of uncertainties and disturbances. The requirement of estimating uncertainty as well as all states in an unified manner, can be achieved by the extended state observer. The simulation has been carried out by considering various types of uncertainties and reference trajectories.

The rest of the paper is organized as follows. Section 2 briefs the modeling of DC motor. The extended state observer for present problem is derived in Section 3. The ESO based double hyperbolic reaching law is derived in Section 4. The closed loop stability is analyzed in Section 5. The simulation results of proposed observer-controller scheme is given in 6 and finally Section 7 concludes the paper.

2. MODELING OF DC MOTOR

In this paper, the problem of position control of a DC motor is considered.

2.1 A DC motor dynamics

The dynamics of a DC motor is given by [19],

$$\dot{\theta} = \omega \tag{1}$$

$$J_m \ddot{\theta} = K_t \psi - B_m \dot{\theta} - T_l \tag{2}$$

$$L_m \dot{\psi} = -K_b \dot{\theta} - R_m \psi + V_m \tag{3}$$

where θ is a motor shaft position, ω is the motor angular velocity, ψ is the armature current, V_m is the voltage across armature, T_l is a load torque applied mechanically on the motor shaft, J_m is the equivalent moment of inertia, B_m is the equivalent viscous friction, R_m is the equivalent armature resistance, L_m is the equivalent armature inductance, K_b is the back emf constant and K_t is the torque constant.

2.2 State space representation

In this paper, it is assumed that the motor shaft position $(y = \theta)$ is the only measurable signal of the system and considered as an output of the system. Selecting the motor shaft position $(x_1 = \theta)$, the shaft velocity $(x_2 = \omega)$ and the shaft acceleration $(x_3 = \dot{\omega})$ as state variables and the armature voltage $(u = V_m)$ as a control input of the system. The system $(1) - (3)$ can be written into state space form as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -\frac{K_t K_b}{J_m L_m} x_2 - \frac{B_m}{J_m} x_3 + \frac{K_t}{J_m L_m}(u + d) \end{aligned} \tag{4}$$

where, $d = -\frac{J_m L_m}{K_t}\left(\frac{K_t R_m}{J_m L_m}\psi + \frac{\dot{T}_l}{J_m} + \xi(x,t)\right)$ is the disturbance acting on the system (4), which includes parametric uncertainties and external disturbances acting on the system.

3. EXTENDED STATE OBSERVER

Consider an n –th order, single input-single output nonlinear dynamical system described by [11],

$$x^n = a(x, \dot{x}, \ldots, x^{n-1}, w) + bu \tag{5}$$

where $a(.)$ represents the dynamics of the plant and the disturbance, $w(t)$ is an unknown disturbance, u is the control signal, and x is the measured output. Let $a(.) = a_o(.) + \triangle a$ and $b = b_o + \triangle b$ where $a_o(.)$ and b_o are the best available estimates of a and b respectively and $\triangle a$ and $\triangle b$ are their associated uncertainties. Defining the uncertainty to be determined as $d = \triangle a + \triangle bu$ and designating it as an extended state, x^{n+1}, the dynamics (5) can be re-written in a state-space form as,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= x_{n+1} + a_o + b_o u \\ \dot{x}_{n+1} &= h \\ y &= x_1 \end{aligned} \tag{6}$$

where h is the rate of change of the uncertainty, i.e., $h = \dot{d}$ and is assumed to be an unknown but bounded function. By making d a state, however, it is now possible to estimate it by using a state estimator.

Consider a linear extended state observer of the form,

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + L_1(y - \hat{y}) \\ \dot{\hat{x}}_2 &= \hat{x}_3 + L_2(y - \hat{y}) \\ &\vdots \\ \dot{\hat{x}}_n &= \hat{x}_{n+1} + L_n(y - \hat{y}) + a_o + b_o u \\ \dot{\hat{x}}_{n+1} &= L_{n+1}(y - \hat{y}) \end{aligned} \tag{7}$$

where $e = y - \hat{x}_1 = x_1 - \hat{x}_1$ and $\hat{x}_{n+1}$ is an estimate of the uncertainty. Writing the observer dynamics (7) into compact form as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L(y - \hat{y}) \\ \hat{y} &= C\hat{x} \end{aligned} \tag{8}$$

where, $\hat{x} = [x_1 \quad x_2 \quad \cdots \quad x_n \quad x_{n+1}]^T$ is the state and disturbance estimation vector, $L = [L_1 \quad L_2 \quad L_3 \quad \cdots \quad L_n \quad L_{n+1}]^T$ is the observer gain matrix and $e = y - \hat{y}$ is the output estimation error of the ESO.

4. CONTROL LAW

In this section, first the double hyperbolic reaching law is described when the complete state vector is available. The control law is made implementable when the complete state vector is not available using the extended state observer.

4.1 Double hyperbolic reaching law

The sliding variable is defined as

$$s = Cx \tag{9}$$

where, $C = [c_1 \quad c_2 \quad c_3]$ are the user selected constant coefficients. The double hyperbolic reaching law [20] is given by

$$\dot{s} = -k_1 \tanh(as) - k_2 |s| \operatorname{asinh}(bs^q) \tag{10}$$

where, k_1, k_2, a, b are tuning parameters, q is a positive odd number, $\tanh(as)$ is the hyperbolic function and $\operatorname{asinh}(bs^q)$ is the inverse hyperbolic sine function.

The hyperbolic tangent function and its derivative are in the range of $[-1,1]$. It has almost linear property near zero. The inverse hyperbolic sine function has similar property but as it goes away from zero, the value is increasing monotonically.

When the sliding variable is away from the zero, the inverse hyperbolic sine function brings it near zero. When the sliding variable is near zero, the hyperbolic tangent function brings it very close to zero and thus eliminates the chattering problem.

4.2 ESO based double hyperbolic reaching law

The sliding surface is now defined using estimated states as

$$\hat{s} = C\hat{x} \tag{11}$$

The control law now is updated as

$$u = -\frac{1}{CB}\left(\hat{s}A\hat{x} + \hat{s}B\hat{d} + k_1 \tanh(a\hat{s}) + k_2 |\hat{s}| \operatorname{asinh}(b\hat{s}^q)\right) \tag{12}$$

which leads the sliding variable to

$$\dot{\hat{s}} = -k_1 \tanh(a\hat{s}) - k_2 |\hat{s}| \operatorname{asinh}(b\hat{s}^q) \tag{13}$$

In the next section, the closed loop stability is derived.

4. STABILITY ANALYSIS

In this section, the closed loop stability of the system (4) with the ESO based controller derived in (12) and (13). The ultimate boundedness of the closed-loop system is proved [5].

5.1 Observer error dynamics

Defining the state observer error dynamics as,

$$\tilde{x} = x - \hat{x}$$

$$\tilde{y} = C\tilde{x} \tag{14}$$

The observer error dynamics is obtained by subtracting (8) from (4) as,

$$\tilde{x} = (A - LC)\tilde{x} + Eh \tag{15}$$

Assuming that the pair (A, C) is observable and the rate of change of disturbance h is bounded. By selecting the eigenvalue of vector $(A - LC)$ in the left hand plane at appropriate location, the observer error dynamics can be stabilized.

On the similar lines for nonlinear observer dynamics can be obtained as

$$\tilde{x} = A\tilde{x} - LC\tilde{x} + Eh \tag{16}$$

Next the closed-loop stability of the system is derived.

5.2 Closed-loop stability

By assuming the pair (A, C) is observable, the error dynamics of ESO can be assured by placing the eigenvalue of $(A - LC)$ in the left half plane at appropriate location. By placing the observer and controller gains appropriately in the left half plane. Thus, the closed-loop stability of the system can be assured.

6. SIMULATION RESULTS

In this section, the effectiveness of proposed observer-controller scheme is evaluated by considering various scenarios i.e.

- Constant reference tracking with nominal parameters

- Sinusoidal reference tracking in the presence of constant load disturbance

- Constant reference tracking tracking in the presence of sinusoidal load disturbance

The nominal parameters of DC motor considered for all cases are shown in Table 1. For all scenarios, the controller and observer parameters are kept constant as shown in Table 2.

Table -1: Nominal parameters of DC motor

Parameters	Nominal value	Unit
R_m	6.898	Ω
L_m	27	mH
J_m	0.032	$\mathrm{kg/m^2}$
B_m	0.0022	Nm/rad

K_b	1.073	
K_t	1.073	

Table -2: Simulation parameters of observer-controller for all cases

Parameter	Value	Parameter	Value
a	5	k_1	100
b	10	k_2	20
C	[1 0 0]	q	3
L	[220 27690 -467650 166970]		

6.1 Case I. Constant reference tracking with nominal parameters:

In this case, the proposed scheme is tested with nominal parameters of DC motor. The initial states of the DC motor and the observer are set as $x_0 = \hat{x}_0 = [0\text{ rad} \quad 0\text{ rad/s} \quad 0\text{ rad/s}^2]^T$, and the reference position, velocity and acceleration are set to $[1\text{ rad} \quad 0\text{ rad/s} \quad 0\text{ rad/s}^2]$. The plots of plant states and disturbance and the observer states are shown in Fig.1. The observer estimates the plant states and disturbances very closely from the beginning of time as there is no mismatch between plant states and observer states.

6.2 Case II. Sinusoidal reference tracking in the presence of constant load disturbance:

In this case, the sinusoidal reference tracking of DC motor position $\sin(2t)$ in the presence of constant load disturbance of magnitude $7\ N$ were carried. The plot of plant states, load disturbance acting on the motor and the estimates of plant states and load disturbance are shown in Fig.3. The plant output and observer output is following the reference trajectory after 0.2 sec. from initial time as shown in Fig.2. The rest of the states and load disturbance are estimated accurately as shown in Fig.2-Fig.2. The plot of control input is shown in Fig.3. The plot of sliding variable is shown in Fig.3. The proposed scheme is working satisfactorily with constant disturbance and sinusoidal reference trajectory tracking.

6.3 Case III. Constant reference tracking in the presence of sinusoidal load disturbance:

In this case, the performance is evaluated in the presence of sinusoidal load disturbance of $6\sin(2t)\ N$ and constant reference tracking trajectory of magnitude 1 rad. The plots of plant states and observer states as shown in Fig.5. The output of plant and observer oscillate around the constant reference position with magnitude ± 0.07 rad. The reason for inexact tracking of reference trajectory is inaccurate estimates of plant states and sinusoidal load disturbance. As the load disturbance sinusoidal and the rate of change of load disturbance is non-zero, which leads non-zero states and disturbance estimation error as per (16). The plots of control input and sliding variable are shown in Fig.6. The performance of the proposed scheme is up-to the mark even in the presence of sinusoidal load disturbance. Similar results have been observed with parametric uncertainties and various load disturbances.

7. CONCLUSION

In this paper, the extended state observer based double hyperbolic reaching law has been implemented for DC motor. The proposed observer-controller scheme does not requires the complete state vector information for implementation, does not require the upper bound of disturbances. The proposed scheme is validated with various types of disturbances and different tracking trajectories.

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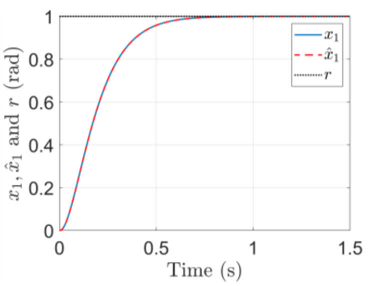

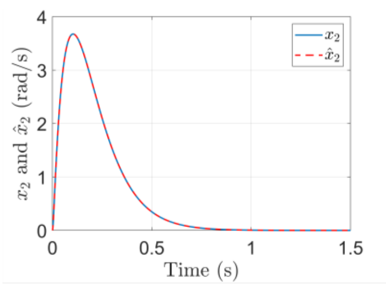

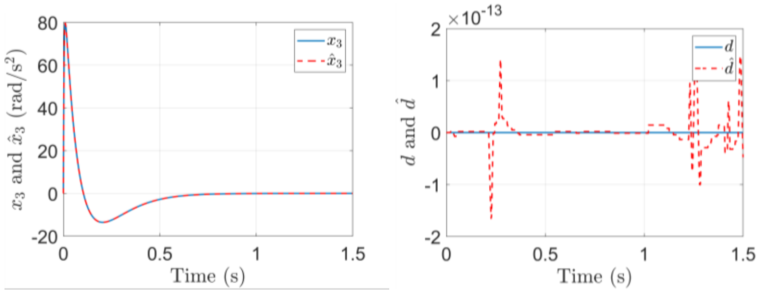

Fig -1: Case-I: Plant states and their estimation: (a) $x_1, \hat{x}_1$ and r, (b) x_2, and $\hat{x}_2$, (c) x_3 and $\hat{x}_3$, (d) d and $\hat{d}$.

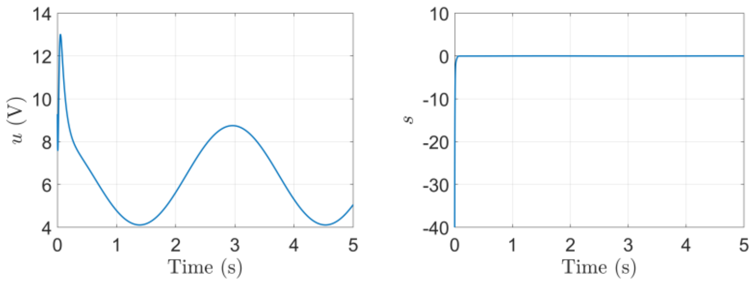

Fig -2: Case-I: Controller performance: (a) u, (b) s.

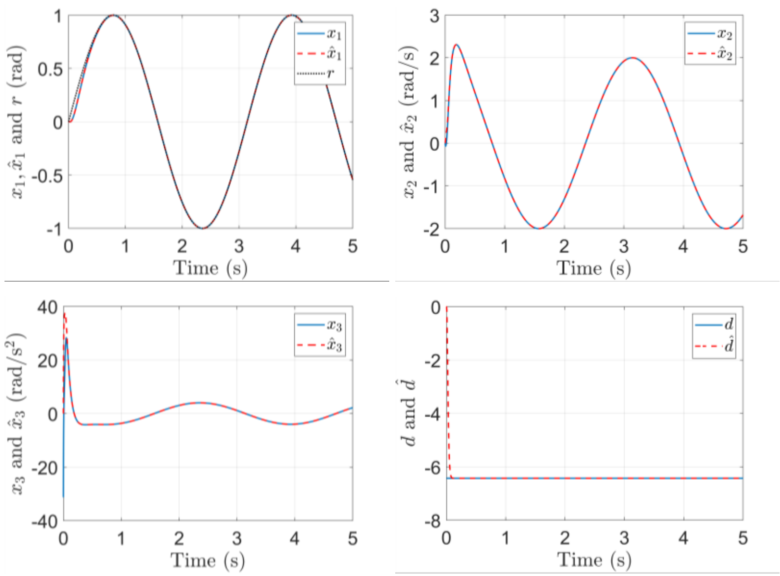

Fig -3: Case-II: Plant states and their estimation: (a) $x_1, \hat{x}_1$ and r, (b) x_2, and $\hat{x}_2$, (c) x_3 and $\hat{x}_3$, (d) d and $\hat{d}$.

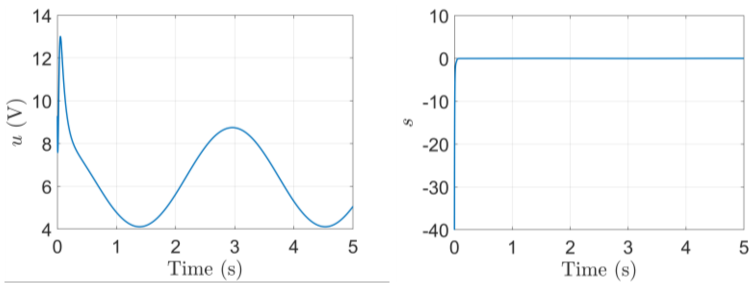

Fig -4: Case-II: Controller performance: (a) u, (b) s.

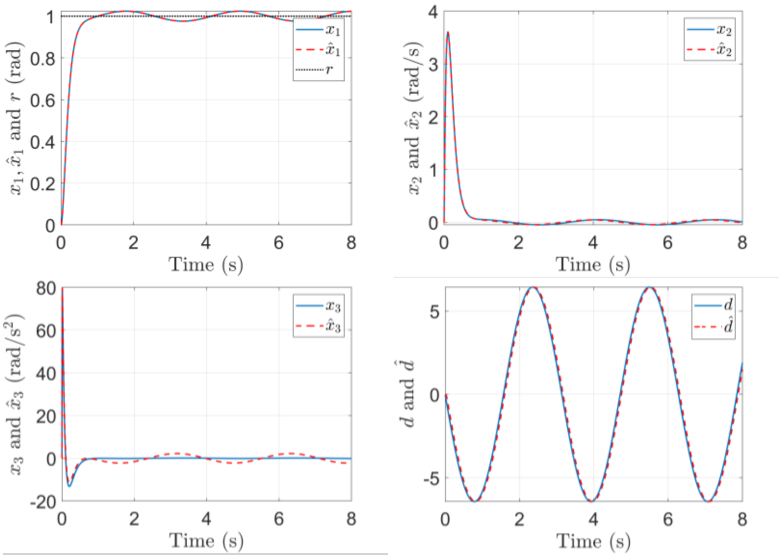

Fig -5: Case-III: Plant states and their estimation: (a) $x_1, \hat{x}_1$ and r, (b) x_2, and $\hat{x}_2$, (c) x_3 and $\hat{x}_3$, (d) d and $\hat{d}$.

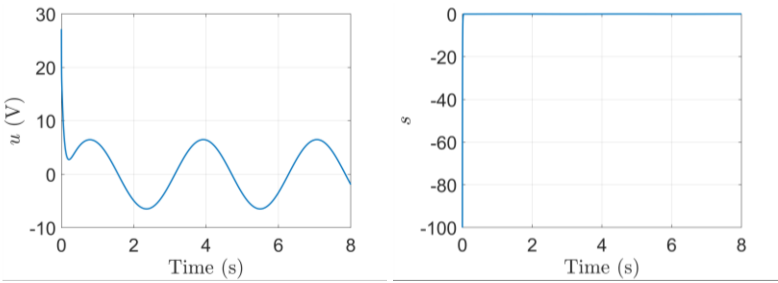

Fig -6: Case-III: Controller performance: (a) u, (b) s.