

Optimization of Selective Maintenance Priority Model for a Series-Parallel System using Enumerative Technique

Priteesh Mohan Garg and Alok Singh

¹Priteesh Mohan Garg M.Tech Scholar at Maulana Azad National Institute of Technology and Science, Bhopal, India

²Alok Singh Assistant Professor at Maulana Azad National Institute of Technology, Bhopal, India

Abstract – In industrial settings, defense services equipment and many other fields, the system or machinery utilized is expected to run many consecutive missions one after another with very short gap between them. In such cases the scope of a complete maintenance operation required to bring the system to its maximum possible working efficiency and reliability is impossible. In such situations a technique known as selective maintenance is used to only repair a selective number of components in the limited time available before the next mission. To facilitate this a novel approach is suggested in this paper. The problem is to find the best choice of components, maintenance actions and order of execution of those actions on their respective components for a series-parallel multi component system. As the number of components increase it becomes more difficult and time taking to sort them by human efficiency, that is where this technique comes in use. Finally, numerical example is taken to illustrate the solution technique proposed.

Key Words: Minimum threshold replacement reliability, maintenance priority, maintenance time available, mission duration, series-parallel system, maintenance decision, priority level.

1. INTRODUCTION

In many fast moving industrial, commercial, transport and defence systems, the systems run on missions which are consecutive in nature and a proper maintenance schedule is only possible after the systems have run a k number of missions. In such situation the only way to do maintenance is to apply selective maintenance approach. In this approach as defined by rice et. al.[1], the selective maintenance is an approach to only perform maintenance actions on a select few components while neglecting other components altogether. He used the approach of run to failure and then selective maintenance rather than preventive selective maintenance which was introduced by Cassady et. al.[2], also Chatelet et. al.[3] in his work used a model to optimize maintenance costs and a defined sequence that was obtained by a special ratio base on time dependent criteria. A scheduling model was developed for preventive maintenance by Pandey et. al.[4] and this work also defines the optimal number if periodic maintenance breaks for a defined finite maintenance break. Khatab et. al.[5] in his work defined the list of maintenance measures of different

levels for a system that performs multiple missions. This work considered those durations to be stochastic rather than fixed. Lust et. al.[6] used tabu search based programming to generate an order for maintenance actions. Similarly Liu et. al.[7] solved the problem of a resource constrained maintenance problem such as SPM by using ant colony optimization algorithm. But all these algorithms are either evolutionary algorithm, use fuzzy logic or are very complex for simpler computational devices.

Thus, in this work a model is proposed which is a simple decision based two-part solution model that uses enumerative technique to solve the problem of selective maintenance and is aimed to maximize individual component reliability.

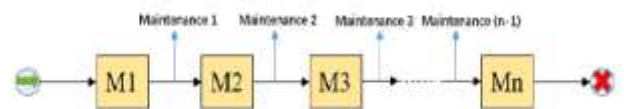


Fig-1. Schematic Diagram of consecutive mission with breaks

A system is used in section 2 to understand the proposed solution method and to describe the various parameters considered to be governing the system reliability and system state after each mission completion.

2. System Specifications

In today's environment most systems used are in a series-parallel conjecture to improve the chances of the entire system not failing. In such systems each component is attached to other components of the same subsystem by a parallel connection so in case of failure of a component the entire subsystem doesn't shut down and all subsystems are connected to each other in series connection and thus completing the system. The purpose of such a system to provide the system with redundancy so that the entire system doesn't fail at once. Hence for this work we consider a similar system and the aim of this work is to provide decision makers with a selective, accurately performing maintenance solution approach to produce an optimal maintenance set of components and subsystems which adhere to the given time limit of maintenance break between two consecutive missions.

2.1. Assumptions considered in the paper:

Fundamental assumptions taken in this paper to simplify the problem are:

1. The components in the system are assumed to have a binary nature state i.e. they are either failed or working.
2. Only three possible maintenance actions are considered, namely, failed replacement (FR), preventive replacement (PR) and minimal repair (MR).
3. The amount of resources is set before the calculations and are constant in nature. Also each components resource requirement is known for every possible action.

2.2. Reliability Calculations for Component, Subsystem and System

A series-parallel system is considered where 'i' denotes m number of independent subsystems are connected in series, and each subsystem j has n number of components connected in parallel.

The length of next mission is assumed as L and the system has returned after successfully completing k missions. Thus all components show some age based on their previous usage and the amount of maintenance they received in the previous break.

Let $X_{ij,k}$ and $Y_{ij,k}$ denote the component's state before the start of mission k and at the end of mission k respectively. The state of components can be given as:

$$X_{ij,k} = \begin{cases} 1, & \text{if } C_{ij} \text{ is in a working condition} \\ & \text{at the beginning of mission k} \\ 0, & \text{Otherwise} \end{cases} \quad 1$$

Similarly, at the beginning of a mission k, the state of the subsystem and the whole system is also indicated by {0,1}, where '0' denotes the failed state and '1' denotes the subsystem is in working state. Since each subsystem consists of n parallel components its state can be determined at the beginning of a mission k as:

$$X_{i,k} = 1 - \prod_{j=1}^{n_i} (1 - X_{ij,k}) \quad 2$$

At the start of a mission k the state of the entire system is defined as :

$$X_k = \prod_{i=1}^m \left(1 - \prod_{j=1}^{n_i} (1 - X_{ij,k}) \right)$$

Also, the condition of a variable can be written at the end of a mission k as:

$$Y_{ij,k} = \begin{cases} 1, & \text{if } C_{ij} \text{ is in a working state} \\ & \text{at the end of mission k} \\ 0, & \text{Otherwise} \end{cases} \quad 4$$

Similarly the state of subsystem and the entire system is given in equation 5 and equation 6 respectively:

$$Y_{i,k} = 1 - \prod_{j=1}^{n_i} (1 - Y_{ij,k}) \quad 5$$

$$Y_k = \prod_{i=1}^m \left(1 - \prod_{j=1}^{n_i} (1 - Y_{ij,k}) \right) \quad 6$$

Reliability calculation of individual components follow a Weibull distribution of lifespan thus reliability is given by equation 7:

$$R_{ij,k} = \exp\left(-\frac{\text{Time}^\beta}{\text{Scale}}\right) \cdot X_{ij,k} \quad 7$$

The reliability of each subsystem with n components in parallel is given by:

$$R_{i,k} = 1 - \prod_{j=1}^{n_i} (1 - R_{ij,k}) \quad 8$$

And for the entire system during a mission k reliability is given as:

$$R_k = \prod_{i=1}^m R_{i,k} = \prod_{i=1}^m \left(1 - \prod_{j=1}^{n_i} (1 - R_{ij,k}) \right) \quad 9$$

At the start of the next mission, the probability for the next mission can be remedied based on age, initial state and duration for each component.

Now for maintenance time calculation, Depending on the system reliability requirement, a component may or may not be selected for maintenance. But if a component is selected for maintenance it will consume some amount of time. The expression for time to perform maintenance can be estimated as follows:

$$T_{ij}(H_{ij,k}) = \begin{cases} 0, & H_{ij,k} = 0 \\ t_{ij,mr}, & H_{ij,k} = 1 \\ t_{ij,fr}, & H_{ij,k} = 2 \\ t_{ij,pr}, & H_{ij,k} = 3 \end{cases} \quad 10$$

From the equation above (10) it is clear that for variable time associated we can essentially segregate the time $T_{ij,h}$. Here $H_{ij,k}=0$ denotes that no time is consumed and thus total time is 0 as the component is not picked for maintenance.

- 3 Next $H_{ij,k}=1$ denotes minimum repair operation, where $t_{ij,mr}$ is the time to execute minimum repair; $H_{ij,k}=2$ denotes replacement of missed part judgment, where $t_{ij,fr}$ is the time to conduct corrective replacement operation; $H_{ij,k}=3$ denotes proactive replacement action to be taken, where $t_{ij,pr}$ is the time to perform proactive replacement action.

Table-1: Value assignment and effect

Type of maintenance policy	$V_{ij,k}$	$X_{ij,k+1}$	$B_{ij,k+1}$	$H_{ij,k}$
Do nothing	1	1	$\delta_{ij} + L_i$	0
Minimally repair failed component	0	1	$\delta_{ij} + L_i$	1
Replace failed component	0	1	0	2
Replace functioning component	1	1	0	3

It is therefore possible to approximate the relevant maintenance time of a C_{ij} portion for decision variable $H_{ij,k}$, and to calculate the overall maintenance period for the entire system as:

$$T_m = \sum_{i=1}^m \sum_{j=1}^{n_i} T_{ij}(H_{ij,k}) \tag{11}$$

From eq. 11 it can be inferred that for a particular decision variable of maintenance level $H_{ij,k}$, the respective time involved for system maintenance can be determined.

3. Decision Model for Maintenance

3.1. Objective Function

For complex systems in many industrial environments, how to perform selective maintenance as much as possible within a limited maintenance interval to improve system reliability over the mission period is a major problem. Many past researches on the topic have utilised used various approaches to solve the selective maintenance issue. One such new approach used in this thesis is to use 2 part solution technique to achieve at a feasible solution of a maintenance set which abides to the chosen constraints, all the constraints can be modified as per users requirement and industrial limitations and application needs.

Selective maintenance decision-making is done by maximizing task reliability under the restriction of the limited mission interval. Includes the necessary maintenance components and their respective maintenance actions. Moreover, the system component maintenance measures must be a feasible solution that is compatible with the real-time status. The objective decision function and related constraints of the analysis can be defined as follows:

$$\max(R_{k+1}) = \max \prod_{i=1}^m \left(1 - \prod_{j=1}^{n_i} (1 - R_{ij,k+1}) \right) \tag{11}$$

To achieve the above objective function we need to start at the base level that is we need to start by ensuring the highest reliability possible while abiding to constraints for individual components.

Thus inadvertently we have to focus on maximising equation 11, for that we will use the concept of limiting the lower limit

of reliability, that is we will set a limit of required reliability (R_{req}) which will help us in decision making and subsequently will keep the overall reliability levels high and upto standard of requirement. Also acting as one of the constraints to be followed.

For different models multiple constraints such as cost constraint, time constraint, manpower constraint, spare availability constraint, power availability constraint etc. all exist but For our work we are limiting to only Time available for maintenance during the scheduled break between missions as our only resource constraint, however multiple constraints can be applied on the base objective function to improve results and bring them closer to real scenario values. For now we will just consider time T_{max} as the sole resource constraint given as:

$$\sum_{i=1}^m \sum_{j=1}^{n_i} T_{ij}(H_{ij,k}) \leq T_{max} \tag{12}$$

Other Constraints applied are given as :

$$V_{ij,k} = \begin{cases} 1, & \text{if } H_{ij,k} > 0 \\ 0, & \text{Otherwise} \end{cases} \tag{13}$$

$$X_{ij,k+1} = \begin{cases} Y_{ij,k} + V_{ij,k}, & \text{if } Y_{ij,k} = 0 \\ Y_{ij,k}, & \text{otherwise} \end{cases} \tag{14}$$

Where $V_{ij,k}$ is the variable associated with the system state after its been through maintenance cycle, if a component has received maintenance (value of $H_{ij,k}$ is greater than 0) then it is expected to be working and hence assigned '1'; similarly if the component does not receive maintenance (value of $H_{ij,k} = 0$) it could either be working or failed at the end of the maintenance cycle/ break hence assigned a value of '0'.

Now the final state of the component before the next mission is denoted by $X_{ij,k+1}$ and it is obtained by the relation shown in equation 14.

3.2. Solution Methodology

As mentioned before the approach used is a 2 part solution technique which can be easily illustrated using the decision flow diagrams given ahead in figure 2 and figure 3.

Both the decision flow diagrams together are used to finally produce a suitable maintenance set of components with appropriate actions to be taken on them. As it is clear that the proposed selective maintenance model is a complex, non-linear and discrete problem. However due to the easiness of the model in use and adaptability to the problem this model is well suited to accommodate different constraint parameters.

The entire decision flow system is built and executed on a python program which takes numerical values and ".csv" file input to generate the desired output in ".csv" format only. The numerical input is basically the user based data and constraints that the user sets according to the mission's needs. Numerical input used in the current work are 1.

Minimum threshold of Replacement Reliability and 2. Time available of Maintenance (T_{max}).

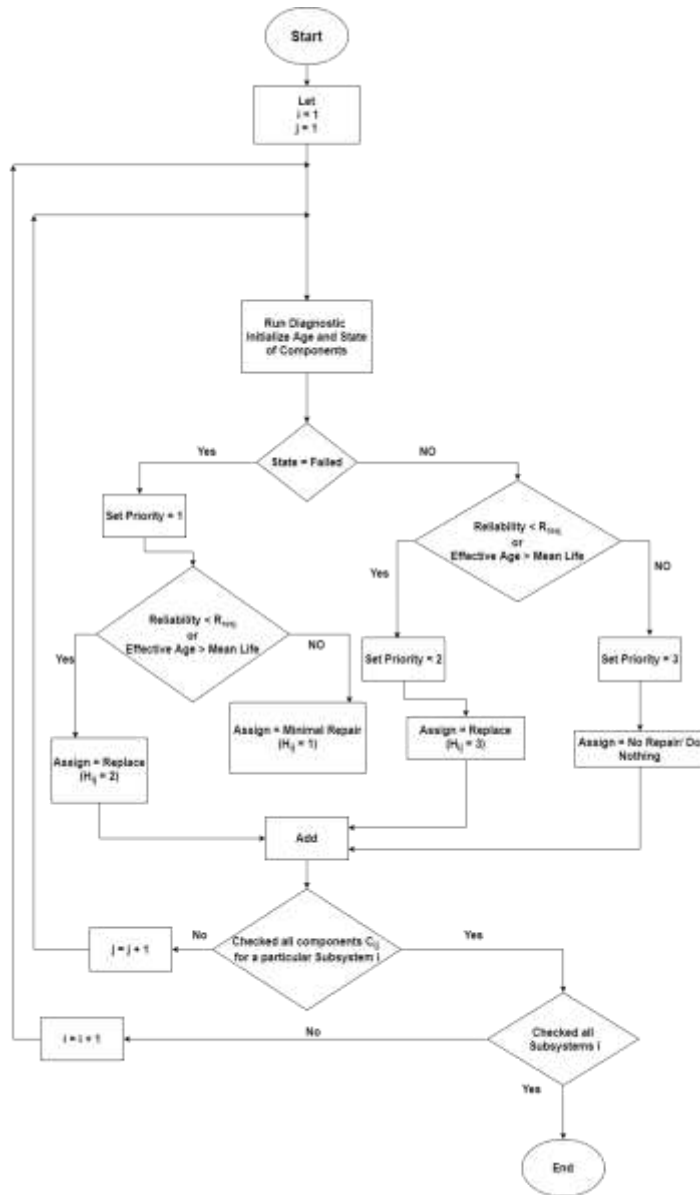


Fig-2: Part 1 of Decision Flow Model Diagram

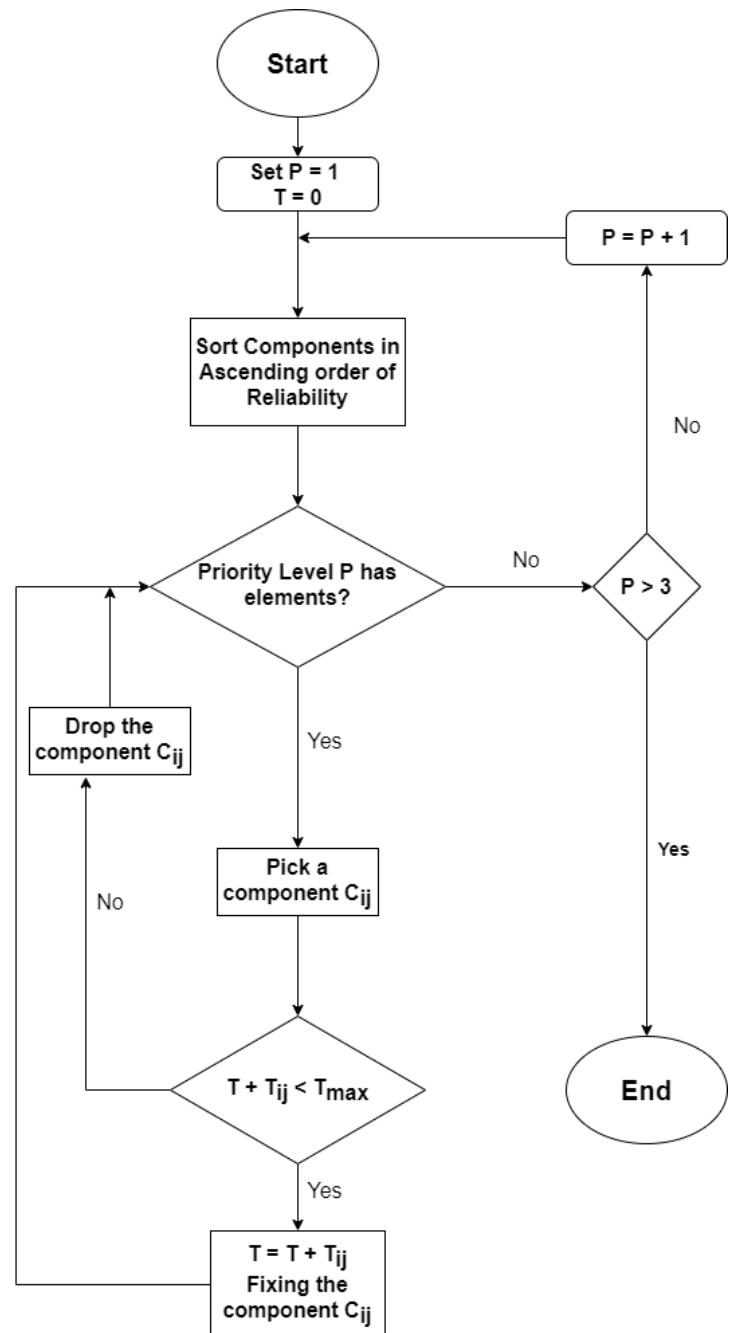


Fig-3: Part 2 of Decision Flow Model Diagram

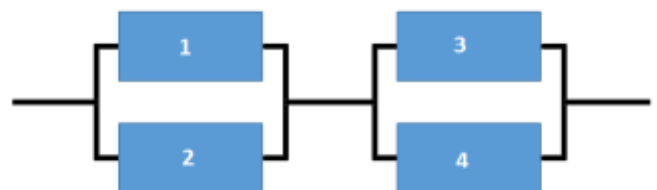


Fig-4: A schematic diagram of a series-parallel system

In the simple series-parallel system shown in fig. 4 each component is named as (C_{ij}) 1,2,3 and 4 where component 1 and 2 belong to subsystem $i=1$ and component 3 and 4 belong to subsystem $i=2$. The decision flow diagram 1 shown in fig. 2, initiates all values of i and j as 1, thus it picks up 1st component of 1st subsystem for running diagnostic operation at the start and then move forward in that particular order only i.e. the next component to go into diagnostic check-up will be 2nd component of 1st subsystem and so forth until all elements of that subsystem are checked after that it moves onto next subsystem and follows the same a pattern.

After running diagnostic, the components are scrutinised on the basis of their state that is whether they are failed or not. If a component is found to be failed it is immediately assigned priority level 1. Otherwise it goes into another decision maker which will be discussed ahead. Once a component is assigned priority level 1 it then is checked based on its condition i.e. age factor and its reliability to complete the next mission successfully. If either the reliability to complete next mission successfully or the age factor condition are not met successfully the component is assigned action of "Failed Replacement" and is replaced. If the above mentioned conditions are met, then the failed component is simply assigned "Minimal Repair" maintenance action and the component is just minimally repaired to bring it back to working state without improving its age and reliability rating.

For the working components the decision flow is shown on the right side of the diagram. Same as the failed components, the working components are also scrutinised and segregated based on the condition of reliability of completing next mission successfully and effective age factor. If the conditions shown are met, then Priority level is set as Priority Level 2 and the maintenance action along with it is also assigned as "Preventive Replacement". Otherwise the priority level is set as Priority Level 3 and those components are assigned no maintenance action or "Do Nothing/ No Repair".

Finally, once all the allocations of priority levels are done and each component is assigned a priority level then the decision flow chart ends.

As for decision flow diagram 2 shown in fig. 3 The sorting is done within a priority level and is a simple ascending order sort which puts the component with lowest reliability to complete the next mission $(R_{ij,k+1})$ at the highest execution rank and thus lower ranks are assigned with increasing order of reliability to complete next mission $(R_{ij,k+1})$. Once the sorting is complete for a priority level the decision flow moves into the execution loop, where for a priority level the components are picked one by one according to ranks assigned to them and then the maintenance action assigned to them and the resource value (in this case maintenance time) consumed by that action is compared to the maximum

resource value available for the current maintenance cycle (in this case maximum time available for maintenance or break duration). If the resource consumed is less than equal to total resource value, the maintenance action is executed and the resource value consumed is updated (T). Along with the resource value the component is also updated and the next component in the raking is then moved towards execution but this time the resource value consumed by its assigned maintenance action is added to the previously consumed value of resource and then compared with the total resource value, if it still satisfies the condition shown in the figure then the action is carried out and so on.

4. CASE STUDY AND NUMERICAL ANALYSIS

For the demonstration of the proposed method we will observe three different cases with three different system sets and conditions which will illustrate a variety of situations and will help bring out the efficacy related aspects of the proposed method and also show its drawbacks to be verified and rectified in future works. The enumerative method helps to find a close to optimal solution out of all available options and work orders while focussing on improving the state of system.

4.1 Case 1: When Time is sufficient for all fail repairs

The system taken for case 1 has 3 subsystems connected in series and each subsystem has some components attached in parallel with other components of the same subsystem. For the given system we have 3 components in subsystem 1; 2 components in subsystem 2 and 4 components in subsystem 3. The schematic diagram of system is shown in figure 5.

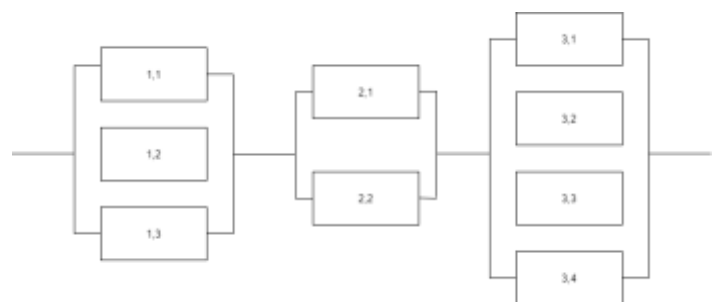


Fig-5: Schematic Diagram of the example system

The input values used are shown in the following table:

Table-2: Reliability, Weibull parameters, time, age and state of component

C	β	α	T_{PR}	T_{MR}	T_{FR}	B_{ij}	Y_{ij}	$R_{ij,k}$
1,1	1.6	250	1.9	1.6	2.1	160	0	0.344809
1,2	2.4	300	1.8	1.5	2	10	1	0.913929
1,3	1.5	300	1.6	1.4	1.8	200	1	0.367879
2,1	2.5	200	1.8	1.6	2	50	0	0.614381
2,2	2.4	175	1.7	1.5	1.8	50	1	0.501194

3,1	2	375	1.8	1.6	2	100	1	0.752432
3,2	1.2	400	1.7	1.5	1.9	70	1	0.698967
3,3	1.4	400	1.5	1.4	1.8	100	1	0.684594
3,4	1.3	400	1.6	1.5	1.7	80	0	0.701775

The enumerative 2 part solution technique shown in this work is then used to create a solution that is at par with optimal and creates the best set of component selection to eliminate maximum number of failed components and bring them to working condition.

The enumerative technique is modifiable according to the reliability requirement needed by the user, for example if the user needs maximum number of his components to be at least 60% reliable, then the user can enter that value at the beginning of the program to segregate and choose a set of components with priority given to all the components that fall under 60% reliability rating.

Table-3: Case 1 Results

Sr.N.	i	j	H _{ij}	R _{ij,k+1} old	R _{ij,k+1} Updated	B _{ij,k} upd
1	1	1	2	0.344809	0.793873	0
2	2	1	2	0.614381	0.837967	0
3	3	4	1	0.701776	0.701776	80
4	1	3	3	0.367879	0.824935	0

Result analysis for case 1: This system is assumed to have returned from a mission k in working state. But few components have failed during the mission and few others have aged and show decreased reliability ratings because of that. Now according to the SPM technique suggested we make the calculations while assuming T_{max} (Time available for maintenance) as 8 units and threshold of replacement reliability (R_{req}) of 65% on all components, this is the parameter which is responsible for the decision making of which components to replace and what components to just minimally repair or leave completely and do nothing on them, thus saving time and other resources if considered. From the output table we can obtain the optimal set of operations and components and the order in which the actions are to be executed, which in this case comes out as [(1,1);(2,1);(3,4);(1,3)] and actions taken as [FR, FR, MR, PR] respectively and in that order. As we can observe from the input table the maximum time needed for maintenance of all failed components is 5.8 units but if we use the suggested technique and pick the most suitable maintenance option, the total time taken for the complete maintenance of failed components is reduced to only 5.6 and the additional time is utilised for preventive maintenance of a working component (1,3) thus imparting an overall better reliability rating. The maximum achievable reliability if all the components in the system were to be maintained according to their

condition and by the method suggested is 0.95304 i.e. **95.304%** and reliability achieved with the method and under time constraint is **90.22%**.

4.2 Case 2: When Time is insufficient for all fail repairs

In the case when all the failed components can't be allotted actions in order of the need the system skips components on the basis of time constraint and goes to the next component that has been ranked in the set.

system is taken as shown in figure 6 has 3 subsystems all connected in series with one another and all subsystems have components connected in parallel within themselves as shown. The system considered has 3 components in subsystem 1, 5 components in subsystem 2 and 4 components in subsystem 3.

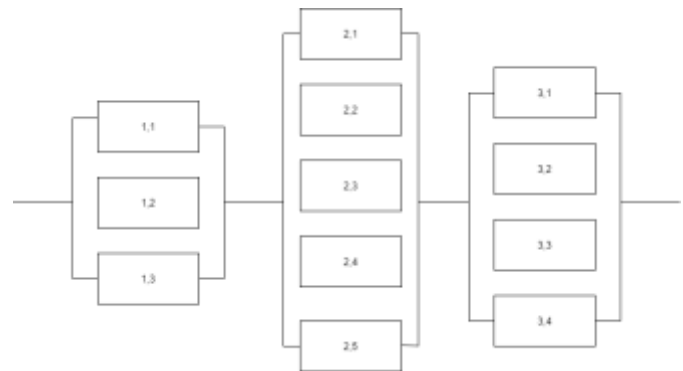


Fig-6: Schematic Diagram of example system 2

The specific parameters values for the system 2 are illustrated in Table (name later) as follows:

Table-4: Reliability, Weibull Parameters, age, time and state of components

C	β	α	T _{PR}	T _{MR}	T _{FR}	B _{ij,k}	S _{ij,k}	Y _{ij}	R _{ij,k}
1,1	1.9	250	1.8	1.4	2	40	140	1	0.712359
1,2	2.4	300	1.9	1.6	2.1	100	200	0	0.686778
1,3	2	300	1.7	1.7	1.9	150	250	1	0.498248
2,1	2.3	350	1.9	1.6	2.1	280	380	0	0.299136
2,2	2.4	340	1.6	1.4	2.1	170	270	0	0.562403
2,3	2.2	280	1.9	1.7	2.1	10	110	1	0.876502
2,4	2.5	310	1.6	1.4	1.9	190	290	1	0.428826
2,5	1.6	400	1.9	1.7	2	270	370	0	0.414117
3,1	1.7	400	1.9	1.6	2	0	180	1	0.777331
3,2	2.3	340	1.7	1.6	1.8	140	240	1	0.639859
3,3	1.8	350	1.9	1.6	2	200	300	0	0.471304
3,4	2.1	270	1.8	1.7	2	160	260	0	0.397621

For this example, we will set a minimum threshold of replacement reliability R_{req} at 65% and time available for maintenance T_{max} is set as 10 units.

Table-5: Results For Case 2

Sr.N.	i	j	H _{ij}	Old R _{ij,k+1}	R _{ij,k+1} updated	B _{ij,k}
1	2	1	2	0.299136	0.945484	0
2	3	4	2	0.397621	0.8832	0
3	2	5	2	0.414117	0.896893	0
4	3	3	2	0.471304	0.900436	0
5	1	2	1	0.686778	0.685297	100

From the output table we can observe that the solution set obtained is [(2,1);(3,4);(2,5);(3,3);(1,2)] and the maintenance actions prescribed are [FR; FR; FR; FR; MR] to be performed in that respective order.

Result Analysis for Case 2: The maximum total time required to maintain and bring all failed components in working condition for the above discussed example is 12.2 units which clearly goes over the maintenance time limit of 10 units. By considering the set obtained by the system of solution used and the maintenance actions suggested, we can get 5 out of 6 failed components in working condition within just 9.7 units of time.

The reliability value addition if only all failed components were repaired and brought to working condition according to the system suggested would have obtained a System Reliability of 95.38%. If Preventive Maintenance was taken into consideration too and no time limit was given then the total system reliability would have been **98.95%**, these reliability ratings can only be achieved at a resource value higher than the 10 unit mark, but since we had to adhere to the time limit given to us, the obtained optimum value of Reliability obtained was limited to **95.35%**, which is as good as all failed components repaired.

5. Conclusion

A Selective Maintenance model has been proposed in this work which is very compliant to user needs and produces results accordingly as per the user requirements while keeping the result set in accordance with the constraints set beforehand. A decision program is developed which considers all possibilities and maintenance options available and the generates a set of components to be repaired along with the recommended maintenance actions associated to it in a priority model. This priority model is used to generate a rank based list of all the components selected in the set previously generated and this is the rank in which the maintenance actions are to be executed. For A system that has to be maintained in a limited time window between 2 consecutive missions this method is very suitable to decide which components to repair and which components to let go

and leave in their original state at which they arrived while also making sure that the allocation of time in a sensible way to maximise time usage as much as possible but the primary objective of the system is to maximize reliability of the entire system for the successful completion of the subsequent mission.

However, the decisions taken by the proposed model are very much prejudiced towards repair of failed components and preventive repair although present is still given a lower priority level and thus can be mostly omitted in some cases like the ones shown in this work. Since preventive maintenance is at a lower priority the overall reliability of the system cannot be improved to its maximum capacity and is only at sub optimal levels.

The Positive points of this work and the proposed priority solution technique proposed in it are that this work is extremely useful for series – parallel systems with low redundancies and dissimilar components throughout. Since the work puts efforts into repairing failed components at the highest priority, it provides extremely useful maintenance sets when the entire system arrives in failed state from a previous mission.

The scope of future works is very obvious as this work focusses on very deterministic values of every aspect involved in the reliability calculation to maintenance priority calculations, however in real life the values of mission duration, maintenance time available and all other aspects discussed in this work are very much stochastic and vary from time to time, thus a model that takes into consideration of those factor and produces priority results can be very useful. Another future work could be including “*improper maintenance technique*” by doing so the maintenance or repair time required on each component can be reduced by impressive proportions and can provide the user with varying degree of maintenance options to improve reliability according to the next mission’s requirement.

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