Plastic Strain Energy Dissipated in Fatigue Crack Propagation

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Abstract: *A comparison between thermal dissipation of fatigue crack propagation in plane stress and plane stress condition is presented. The results show the role of plastic hardening of material as a variable which determines the ratio of plastic dissipation in plane stress and plane strain conditions. The results also show that the plastic dissipation is higher for plane stress compared to plane strain for the same material and same load condition.*

Nomenclature

n'	Cyclic plastic strain hardening exponent	W_p	Cyclic plastic strain energy density (J/m³K)
$I_{n'}$	HRR dimensionless coefficient	ω_0	Plastic region radius at angle 0 (m)
r	Distance to crack tip(cylindrical coordinate) (m)	$\Delta\sigma$	Stress range (MPa)
ΔK_I	First mode stress intensity factor (MPa.m$^{1/2}$)	E	Elastic modulus (GPa)
$\omega(\theta)$	Plastic region radius at angle θ (m)	t	Specimen Thickness (m)
$\tilde{\sigma}_{ij}(n,\theta)$	HRR non-dimensional functions for stress distribution	β	Ratio of Heat dissipated to plastic work
$\tilde{\epsilon}_{ij}(n,\theta)$	HRR non-dimensional functions for strain distribution	ΔP	Normal load range (N)
ν	Poisson ratio	a	Crack length (m)
$\Delta\epsilon$	Strain range	α	Dimensionless constant
k'	Cyclic strength coefficient (MPa)	σ_{eq}	Equivalent stress (MPa)
σ_0'	Cyclic yield stress (MPa)	ε_{eq}^p	Equivalent strain
f	Frequency		

1. Introduction

The study of damage mechanics is an interesting aspect of engineering field in different structure such as composite material [1, 2] honeycomb structure [3] and metals. The study of damage is facilitated through Acoustic Emission [4, 5] and the exciting part of damage is related to crack propagation.

Fatigue, as a damage accumulation in components, has been the subject of study for many years. This is known that the cyclic stress can degrade material up to a point when the small crack nucleate and the final rupture of the part takes place when the crack starts to propagate. This propagation of fatigue crack has essential role in safety of the mechanical parts in rotating machinery where cyclic stress is an inevitable part of the operation condition.

Fatigue crack propagation has been subject of study from different aspects. Conventionally, crack propagation rate, which is the main concern in studying of crack propagation, has been related to stress intensity factor [6, 7] or energy release rate [8], however, in recent years the thermal investigation of fatigue crack propagation has emerges and this puts forward the need for understanding fatigue crack propagation plastic dissipation. The propagation rate of fatigue crack can be found by other aspects including thermography. Recently Hajshirmohammadi and Khonsari [9] found a simple approach for determining the crack propagation rate based on thermography.

In the present paper the approach proposed by Hajshirmohammadi and Khonsari [10] is used to investigate dissipation rate.

The crack front experience concentrated filed of stress when loads are applied to the faces of crack (Fig. 1).

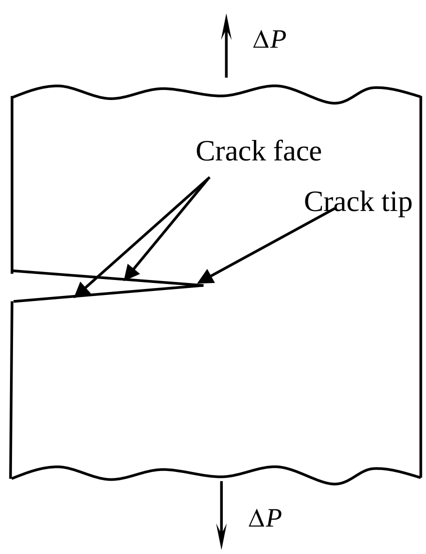

Fig 1. Crack faces loaded in the first mode of crack propagation.

The plastic zone is the area ahead of the crack tip where plastic dissipation takes place. The reason for the plastic dissipation is the excessive values of the stress and strain around the tip. A portion of plastic work which is done around the tip is converted to heat that represents itself as a temperature rise around the crack. It is assumes that the plastic strains and stress are the source of dissipation and a comparison between plane stress/plane strain dissipation in the vicinity of crack tip is given according to plastic stress field given by HRR (Hutchinsone_Rice_Rosengren) [11].

2. Theory and formulation

It is widely accepted to use Ramberg-Osgood relation as an equation to describe plastic strain of material.

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2k'}\right)^{1/n'} \quad (1)$$

where $\Delta\epsilon$ and $\Delta\sigma$ denote the range of strain and stress and n' represents the cyclic strain hardening. E is the elastic modulus and cyclic strengths coefficient is shown by k'.

According to Hutchinson [11] the field of stress and strain ahead of the crack tip can be estimated by the following relations.

$$\Delta\sigma_{ij} = \Delta\sigma_0 \left(\frac{\Delta K_I^2}{\alpha\Delta\sigma_0^2 I_{n'} r}\right)^{\frac{n'}{n'+1}} \tilde{\sigma}_{ij}(n',\theta) \quad (2)$$

$$\Delta\epsilon_{ij} = \frac{\alpha\Delta\sigma_0}{E}\left(\frac{\Delta K_I^2}{\alpha\Delta\sigma_0^2 I_{n'} r}\right)^{\frac{1}{n'+1}} \tilde{\epsilon}_{ij}(n',\theta) \quad (3)$$

In the equations above, $\tilde{\sigma}_{ij}$ and $\tilde{\epsilon}_{ij}$ are functions for angular distribution of stress. The cyclic yield stress is shown by σ_0 and α stands for the dimensionless constant which is given by Eq. 4. and $I_{n'}$ is the integration constant of crack.

$$\alpha = \frac{2E}{(2k')^{\frac{1}{n'}} \Delta\sigma_0^{\frac{n'-1}{n'}}} \quad (4)$$

As illustrated in Fig. 2, the plastic zone has a butterfly shape. Each point in the plastic zone acts as a dissipation source. The total dissipation is the summation of the contribution of all the points in the plastic region. For each point within the plastic zone, the stress and strain can be found by HRR singularity.

Suppose the dissipation is found for the point B shown in Fig. 2. The dissipation W_p is derived using Eq. 5.

$$W_p = \left(\frac{n'-1}{n'+1}\right)\sigma_{eq}\varepsilon_{eq}^p \quad (5)$$

The size of plastic zone is found for plane stress and plane strain condition according to Eq. 6 and Eq. 7 respectively.

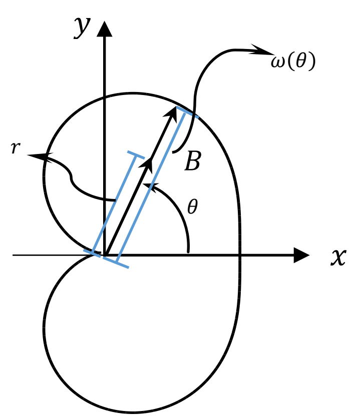

Fig 2. Cyclic plastic zone in front of the plastic zone.

$$\omega(\theta) = \frac{1}{16\pi}\frac{\Delta K_I^2}{{\sigma_0'}^2}\left(1 + \frac{3}{2}sin^2\theta + cos\theta\right) \quad (6)$$

$$\omega(\theta) = \frac{1}{16\pi}\frac{\Delta K_I^2}{{\sigma_0'}^2}\left((1-2v)^2(1 + cos\theta) + \frac{3}{2}sin^2\theta\right) \quad (7)$$

Replacing Eq. 2 and Eq. 3 in Eq. 5, one can derive the relation for plastic dissipation at any point in this region.

$$W_p = f\left(\frac{n'-1}{n'+1}\right)\frac{\Delta K_I^2 \tilde{\sigma}_{eq}(n',\theta)\tilde{\varepsilon}_{eq}(n',\theta)}{EI_{n'}r} \quad (8)$$

in which K_I is the range of stress intensity factor and f is frequency of loading. The angular equivalent distribution of stress and strains are denoted by functions $\tilde{\sigma}_{eq}(n',\theta)$ and $\tilde{\varepsilon}_{eq}(n',\theta)$

respectively. The value for equivalent stress and strain in the case of plain stress is given by the following relations.

$$\tilde{\sigma}_{eq} = (\tilde{\sigma}_r^2 + \tilde{\sigma}_\theta^2 - \tilde{\sigma}_\theta\tilde{\sigma}_r + 3\breve{\sigma}_{r\theta}^2)^{\frac{1}{2}} \quad (9)$$

$$\tilde{\varepsilon}_{eq}^p = \frac{2}{\sqrt{3}}(\tilde{\varepsilon}_r^2 + \tilde{\varepsilon}_\theta^2 + \tilde{\varepsilon}_r\tilde{\varepsilon}_\theta + \tilde{\varepsilon}_{r\theta}^2)^{\frac{1}{2}} \quad (10)$$

For the plain strain case, these value can be found by Eq. 11 and Eq. 12.

$$\tilde{\sigma}_{eq} = \left(\frac{3}{4}(\tilde{\sigma}_r - \tilde{\sigma}_\theta)^2 + 3\breve{\sigma}_{r\theta}^2\right)^{\frac{1}{2}} \quad (11)$$

$$\tilde{\varepsilon}_{eq}^p = \sqrt{\frac{2}{3}}(\tilde{\varepsilon}_r^2 + \tilde{\varepsilon}_\theta^2 + 2\tilde{\varepsilon}_{r\theta})^{\frac{1}{2}} \quad (12)$$

In these relations, $\tilde{\sigma}_r, \tilde{\sigma}_\theta$ and $\tilde{\sigma}_{r\theta}$ represent functions of angular distribution for the case of two dimensional problem.

The total strain energy dissipated from crack tip is the summation of each point generation inside the plastic zone.

$$W_p = \iint_{plastic\ zone} f\beta\left(\frac{n'-1}{n'+1}\right)\frac{\Delta K_I^2 \tilde{\sigma}_{eq}(n',\theta)\tilde{\varepsilon}_{eq}(n',\theta)}{EI_{n'}r}dA \quad (13)$$

The angular function of plastic zone border is:

$$\omega(\theta) = 2r_p\cos(\theta) \quad (14)$$

And the radius of plastic zone circle is given by Eq. 9.

$$r_p = \frac{1}{8\pi}\frac{\Delta K_I^2}{\sigma_0^2} \quad (15)$$

Looking back into Eq. 13, it is seen that the relation can be written in the following form for the polar coordinate system.

$$W_p = \iint_{plastic\ zone} f\beta\left(\frac{n'-1}{n'+1}\right)\frac{\Delta K_I^2 \tilde{\sigma}_{eq}(n',\theta)\tilde{\varepsilon}_{eq}(n',\theta)}{EI_{n'}}drd\theta \quad (16)$$

The parameters in the integration of Eq. 16 are independent of r and θ except for $\tilde{\sigma}_{eq}(n',\theta)$ and $\tilde{\varepsilon}_{eq}(n',\theta)$. This implies that this relation can be written in the following form:

$$W_p = f\beta\left(\frac{n'-1}{n'+1}\right)\frac{\Delta K_I^2 r_p}{EI_{n'}}\int_{\theta=0}^{\theta=2\pi}\tilde{\sigma}_{eq}(n',\theta)\tilde{\varepsilon}_{eq}(n',\theta)d\theta \quad (17)$$

This can be found from Eq. 17 that the $\int_{\theta=0}^{\theta=2\pi}\tilde{\sigma}_{eq}(n',\theta)\tilde{\varepsilon}_{eq}(n',\theta)d\theta$ is a constant for a specific material (when n' is a material parameter). The value of $\int_{\theta=0}^{\theta=2\pi}\tilde{\sigma}_{eq}(n',\theta)\tilde{\varepsilon}_{eq}(n',\theta)d\theta$ is called $\beta_{n'}$. This is understood that the value of $\beta_{n'}$ is different for the same material in plane stress and plane strain condition. To determine the relation between the plastic dissipation in plane stress and plane strain condition, the ratio of $\delta_{n'} = \frac{\beta_{n'}|_{plane\ stress}}{\beta_{n'}|_{plane\ strain}}$.

2.1. Numerical approach

Eq. 13 is used to determine the plastic dissipation where the integration is found using Simpson method. The numerical procedure is carried out by A MATLAB script. The accuracy of integration is set to be 10^{-6}. The singularity problem is solved by subdividing the area of integration into too regions and separation of plastic zone from the rest of area.

3. Results and discussion

Fig. 3 depicts the energy generated from plastic zone ahead of crack for different stress intensity factors. As it is seen, the heat generation from the crack is more when the plane stress condition prevails. This result implies that for the same condition of loading, the energy needed for crack activation is higher when the situation of plane stress prevail. In other words, if a thick plate is under fatigue crack propagation, the plastic dissipation is less in the center of the plate where the situation is closer to plane strain compared to the outer parts close to the surface of plate where loading is close to the plane stress case.

Material	E (GPa)	σ_0 (MPa)	k $(\frac{W}{mK})$	t (mm)
304 Austenitic stainless steel	193	206	16.3	1.85

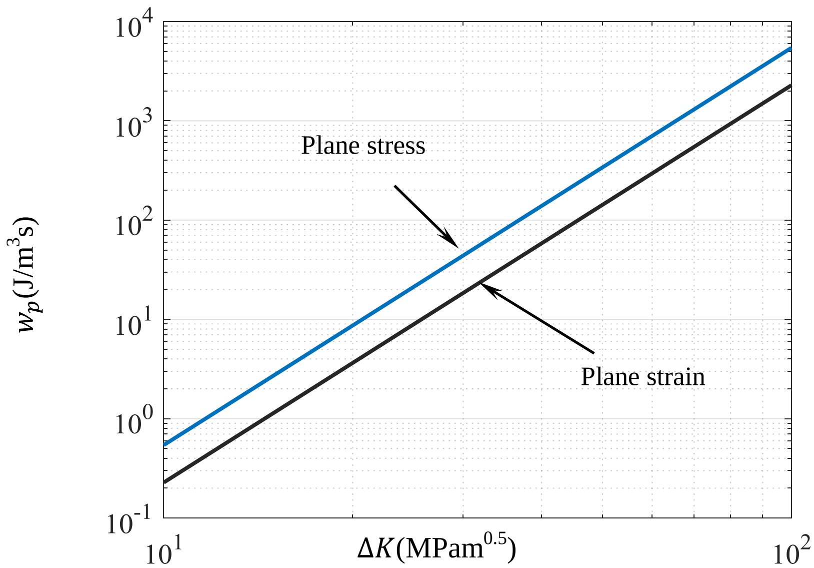

Fig. 3 plastic dissipation as the function of stress intensity factor

Fig. 4 illustrates the variation of $\delta_{n'}$ in respect with n'. This figure shows that for low values of the strain hardening exponent, the ration of $\delta_{n'}$ is less than one which means that the plastic dissipation in plane stress is less than the plane strain plastic dissipation.

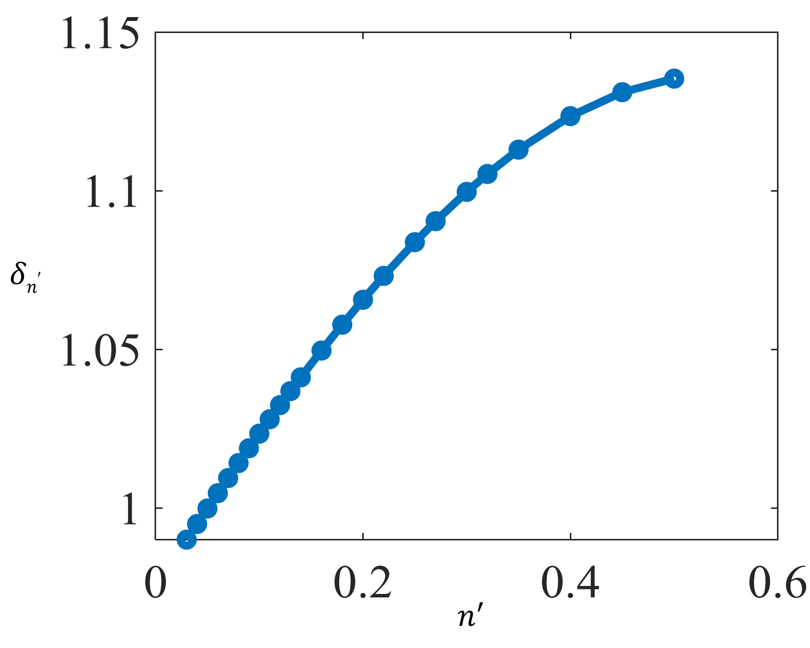

Fig. 4 the ratio of plastic dissipation in plane stress and plane strain condition as a function of cyclic plastic strain

4. Conclusions

A numerical model is developed to find the strain energy dissipation from a crack tip of a moving fatigue crack in mode one. The solution shows that dissipated energy directly increase with the stress intensity factor. This model predicts higher dissipation in plane stress compared to plane strain crack propagation.

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