International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395-0056
Volume: 07 Issue: 07 | July 2020 www.irjet.net p-ISSN: 2395-0072

Characterization of Structural Dynamic Variability Using NPVM through Monte Carlo Simulations

Sajna P[1], Sarah Anil[2]

[1]P. G. Student, Computer Aided Structural Engineering, Dept. of Civil Engineering, Mar Athanasius College of Engineering, Kerala, India
[2]Assistant Professor, Dept. of Civil Engineering, Mar Athanasius College of Engineering, Kerala, India

---***---

Abstract - *FE model is deterministic and the modal frequency response function results are sensitive to the modeling techniques especially at the high frequencies due to the high modal densities. Random uncertainties in finite element models in linear structural dynamics are usually modeled by using parametric models. This means that: (1) the uncertain local parameters occurring in the global mass, damping and stiffness matrices of the finite element model have to be identified; (2) appropriate probabilistic models of these uncertain parameters have to be constructed; and (3) functions mapping the domains of uncertain parameters into the global mass, damping and stiffness matrices have to be constructed. In the low-frequency range, a reduced matrix model can then be constructed using the generalized coordinates associated with the structural modes corresponding to the lowest Eigen frequencies. In this project we propose an approach for constructing a random uncertainties model of the generalized mass, damping and stiffness matrices using NPVM (Non Parametric Variability Method) through MC (Monte Carlo) Simulation . This non parametric model does not require identifying the uncertain local parameters and consequently, obviates construction of functions that map the domains of uncertain local parameters into the generalized mass, damping and stiffness matrices.*

***Key Words*: NPVM (Non Parametric Variability Method), MC (Monte Carlo) Simulation**

1. INTRODUCTION

Finite element analysis is deterministic and results are calculated based on the mean model of a system. The mean model means it does not account for the design geometric tolerances and variations in material properties. The performance of physical system which is inbuilt with variations in geometric tolerances, material properties, manufacturing and assembly processes varies from system to system having the same design. Those variations are unavoidable and it is inherently present in real life physical systems. Along with these variations, the uncertainties in test setup and measurement lead to the gap in correlation level while comparison with the results of mean FE model. FE based vibro and vibro-acoustic responses are more sensitive to the modelling and boundary conditions especially in the mid-high frequency range because of the high modal densities of the system. Also the lack of knowledge and approximation in FE representation of complex systems lead to error in modelling and uncertainties in FE responses [5]. Uncertainties due to geometrical parameters, material properties and boundary conditions are classified under Data Uncertainties. Approximations and simplifications introduced during FE model are classified under Modelling Uncertainties [3]. Data uncertainties can be evaluated by a parametric probabilistic approach. However FE modelling uncertainties cannot be characterized through parametric approach. Non-parametric probabilistic approach which does not explicitly depend on system parameters is a more appropriate method of studying the variability in FE based dynamic responses especially for mid-high frequency range. This method accounts for both the uncertainties in FE (i.e. data and modelling uncertainties) which is a main advantage of this method over the parametric approach. Once the modelling uncertainties are minimized, the results of NPVM method can be used to emphasize the uncertainties in physical system due to its geometric tolerances, manufacturing process and uncertainties in physical testing.

1.1 PARAMETRIC VARIATION METHOD (PVM)

The most straight forward for randomly varying any model is to vary the parameters of the model. For the purposes of this work we considered variations in physical parameters, and did not change model geometry. In addition, we considered only changes that affect stiffness and not mass and only stiffness changes that have a significant effect on the modal properties.

Even with these restrictions there are a large number of possible parameters that can be chosen. The approach taken was to consider the stiffness of every spring property and every material stiffness (Young's modulus) in the model. The sensitivities of the modal frequencies to each of these properties were then calculated, and only properties above a threshold were retained. The number of parameters is a strong function of how the model is organized. For each stiffness parameter, the change in the FEM stiffness matrix is expressed as a change in the modal stiffness matrix. The randomized modal stiffness is then calculated as the nominal plus the sum of terms associated with each uncertain parameter, and the modes are recalculated. Since all these calculations are done on matrices of the size of the number of nominal modes, they are very fast and can easily be implemented in the context of a Monte Carlo analysis.

1.2 NON PARAMETRIC VARIABILITY METHOD (NPVM)

A non-parametric model of random uncertainties, has been introduced for modelling random uncertainties in linear and non-linear elasto dynamics in the low-frequency range. Such an approach, pioneered by C. Soize, does not try to parametrize all model uncertainties, but instead relies on global mass, damping and stiffness variability parameters which directly randomize the mass, damping and stiffness matrices expressed in the nominal modal space. This non-parametric approach differs from the parametric methods for random uncertainties modelling and has been developed in introducing a new ensemble of random matrices constituted of symmetric positive-definite real random matrices

The non-parametric model of random uncertainties in vibration analysis has been introduced to replace the usual parametric model for complex dynamic systems when the number of uncertain local parameters is large and above all, to take into account the model uncertainties which cannot be modelled with the parametric models. NPVM is a probabilistic non-deterministic technique aimed at modeling model uncertainties.

The two main assumptions introduced to construct such a non-parametric model of random uncertainties in linear structural dynamics are: (1) not using the local parameters of the boundary value problem modelling the dynamic system, but using the generalized coordinates directly related to dynamics (non-parametric approach); (2) using the available information which is constituted of the mean reduced model constructed with the n generalized coordinates of the mode-superposition method associated with the elastic modes corresponding to the n lowest Eigen frequencies of the linear dynamic system assumed to be fixed, damped and stable. To satisfy these two main assumptions, the non-parametric probabilistic model of random uncertainties consists in replacing the generalized diagonal mass matrix, the generalized full damping matrix, and the generalized diagonal stiffness matrix of the mean reduced model by the full random matrices. The probability model of each random matrix is constructed using the entropy optimization principle using only the available information.

For instance if there were uncertainties in the generalized mass matrix, the probability distribution should be such that this random generalized mass matrix be positive definite. If not, the probability model would be wrong because the generalized mass matrix of any dynamic system has to be positive definite.

It should be noted that such a non-parametric model of random uncertainties, (1) allows the uncertainties for the parameters of the elasto dynamic model to be taken into account (similarly to the parametric approaches, but using a global approach), (2) but also, allows the model uncertainties to be taken into account, that is to say, modelling the errors which cannot be reached through the model parameters (by definition, any parametric approach cannot model the kind of uncertainties which correspond to non-existing parameters in the boundary value problem under consideration)

Normalization and dispersion parameter of random matrix $[K]$: Since $[K]$ is a positive definite real matrix, there is an upper triangular matrix $[L]$ in Cholesky factorization such that

$$[K]=[L]^T[L]$$

The random matrix $[K]$ can be written as

$$[K]=[L]^T[G][L]$$

In which matrix $[G]$ is a random variable with values such that

$$[G]=E\{[G]\}=[I]$$

In which [I] is the (n×n) identity matrix.

Assuming that a global variability level δ_K characterizes the uncertainty on the modal structural stiffness matrix:

Let $\delta_K > 0$ be the real parameter defined by

$$\delta_K = \sqrt{\frac{E(\| [G] - [G]^2 \|)}{\| [G]^2 \|_F}}$$

It allows the dispersion of the probability model of random matrix [K] to be controlled. $\delta_{K,S}$ can be defined by the analyst for the modal structural mass and damping, as well as modal fluid stiffness, mass and damping matrices.

Monte Carlo numerical simulation of random matrix[G]: The following algebraic representation of positive-definite real random matrix [G] allows a procedure for the Monte Carlo numerical simulation of random matrix [G] to be defined. Random matrix [G]can be written as

$$[G]=[L]^T[L]$$

In which [L] is an upper triangular random matrix.

The statistical analysis is implementing through a Monte-Carlo simulation, which is not intrusive and therefore can be easily implemented in an existing computational framework. The overall algorithm of the nondeterministic approach can therefore be described as follows: • Reduction of the nominal model to modal coordinates • Identification of the modal forces• Loop on Monte-Carlo samples: • Sampling of the modal stiffness, mass, damping matrices • Generation of the response in physical coordinates by modal superposition; • Statistical operations on the sampled responses so as to obtain the response statistics.

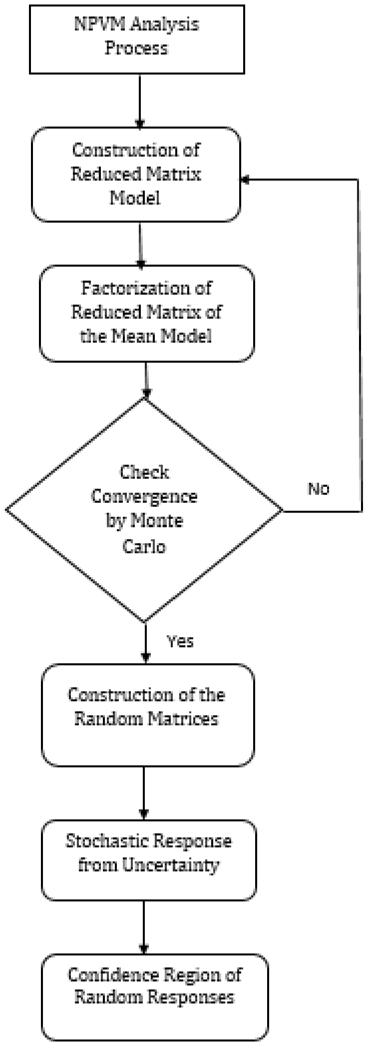

Fig-1: Non-Parametric Variability Modeling and Analysis Process

1.3 MONTE CARLO SIMULATION

Monte Carlo simulation, or probability simulation, is a technique used to understand the impact of risk and uncertainty in financial, project management, cost, and other forecasting models. When you develop a forecasting model-any model that plans ahead for the future – you make certain assumptions. These might be assumptions about the investment return on a portfolio, the cost of a construction project, or how long it will take to complete a certain task. Because these are projections into the future, the best you can do is estimate the expected value. You can't know with certainty what the actual value will be, but based on historical data, or expertise in the field, or past experience, you can draw an estimate. While this estimate is useful for developing a model, it contains some inherent uncertainty and risk, because it's an estimate of an unknown value. In some cases, it's possible to estimate a range of values. In a construction project, you might estimate the time it will take to complete a particular job; based on some expert knowledge, you can also estimate the absolute maximum time it might take, in the worst possible case, and the absolute minimum time, in the best possible case. The same could be done for project costs. In a financial market, you might know the distribution of possible values through the mean and standard deviation of returns. By using a range of possible values, instead of a single guess, you can create a more realistic picture of what might happen in the future. When a model is based on ranges of estimates, the output of the model will also be a range. This is different from a normal forecasting model, in which you start with some fixed estimates – say the time it will take to complete each of three parts of a project – and end up with another value – the total time for the project. If the same model were based on ranges of estimates for each of the three parts of the project, the result would be a range of times it might take to complete the project. When each part has a minimum and maximum estimate, we can use those values to estimate the total minimum and maximum time for the project. When you have a range of values as a result, you are beginning to understand the risk and uncertainty in the model. The key feature of a Monte Carlo simulation is that it can tell you – based on how you create the ranges of estimates – how likely the resulting outcomes are. A random value is selected for each of the tasks, based on the range of estimates. The model is calculated based on this random value. The result of the model is recorded, and the process is repeated. A typical Monte Carlo simulation calculates the model hundreds or thousands of times, each time using different randomly-selected values. When the simulation is complete, we have a large number of results from the model, each based on random input values. These results are used to describe the likelihood, or probability, of reaching various results in the model. Like any forecasting model, the simulation will only be as good as the estimates you make. It's important to remember that the simulation only represents probabilities and not certainty. Nevertheless, Monte Carlo simulation can be a valuable tool when forecasting an unknown future.

1.4 LITERATUTRE REVIEW

A detailed literature survey was conducted for understanding NPVM and Monte Carlo Simulation in structural dynamic problems. The investigations of the available literature is done mainly to understand the extend of developments and findings in the area till date and the same is summed up in the following main categories.

C.Soize [1] introduced new validation point of the non-parametric theory of random uncertainties in vibration analysis. He studied the statistics of the random eigenvalues of random matrices and compared the random matrices in the context of the non-parametric approach. He also gave a new validation for the non-parametric model of random uncertainties in structural dynamics. From all these, he proved that the ''positive-definite'' ensemble of random matrices is well adapted to the low to mid frequency vibration analysis.

Daniel et al. [2] conducted an experimental study to investigate a specific application of uncertainty quantification to the Mars 2020 attitude control system in the face of an uncertain structural model. All the approaches taken here are based on the Monte Carlo method. The method of non-parametrically varying the component matrices proved to be a very simple approach, which captured the variability in measured data very well. The conclusion of this study was that the non-parametric variation method was the best approach for this particular problem.

Christian et al. [3] investigated the sensitivity of low frequencies vibro-acoustic responses to random uncertainties and applied to a current trimmed body model. The numerical implementation is performed using random matrices and a Monte-Carlo simulation. Random matrices are constructed using the available information i.e. the FE model of the studied problem, named mean model. They concluded that all the realizations of the Monte-Carlo simulation remain physically consistent.

Van den et al. [4] presented a framework for the robust vibro-acoustic analysis of structures made of fiber. Two methodologies have been presented that either assume a parametric uncertainty characterization or a non-parametric global variability. The comparison between the non-parametric variability method and the full Monte Carlo simulation in physical coordinates leads to two conclusions.

- First, the non-parametric method tends to predict accurate dispersion around the mean value.
- Due to its computational efficiency, it is a promising approach for the variability assessment, typically in the early design stage.

2. MATERIAL AND MODEL

FREQUENCY RESPONSE OF A PLATE USING NPVM

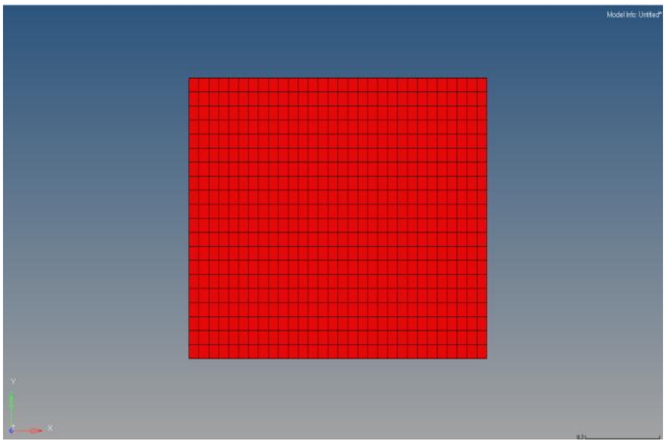

Fig -2: Modelled plate

DETAILS OF ANALYSIS

- Plate dimensions:800mm × 600mm
- Thickness:0.002mm
- E = 2.1×10^{11} N/mm^2 , Poisson's ratio = 0.3
- Variability , $\boldsymbol{\delta}$ = 0.05
- Number of simulations = 101
- Eigenvalue range = 0 to 1000Hz
- Both Eigenvalue analysis and Frequency Response analysis(with MC Simulation) are carried out

MODE SHAPES OF PLATE

MODE 1 (FREQUENCY = 21Hz)

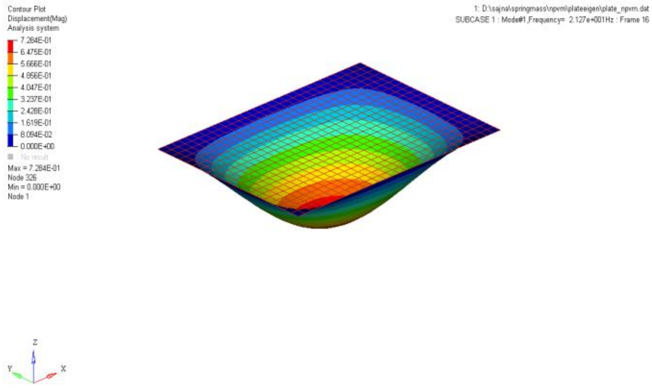

Fig -3: First mode shape

MODE 2 (FREQUENCY = 44Hz)

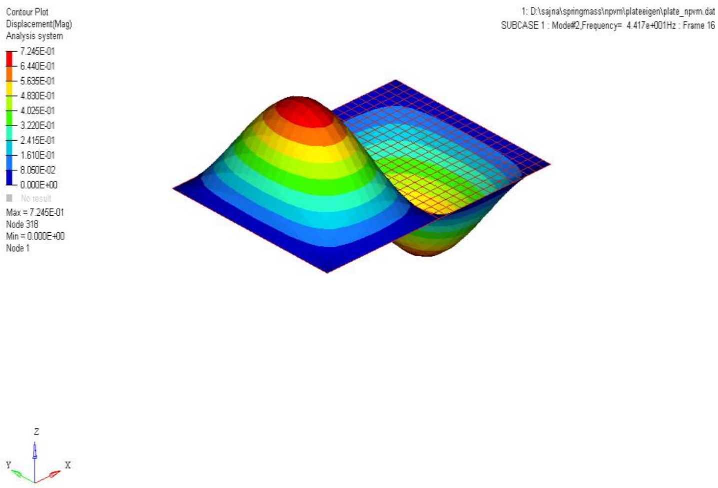

Fig -4: Second mode shape

MODE 3 (FREQUENCY = 62Hz)

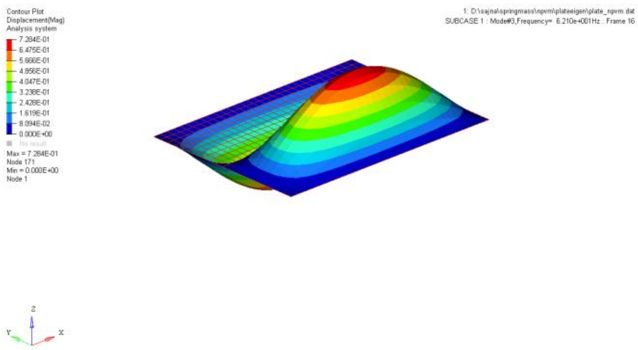

Fig -5: Third mode shape

MODE 4 (FREQUENCY = 83Hz)

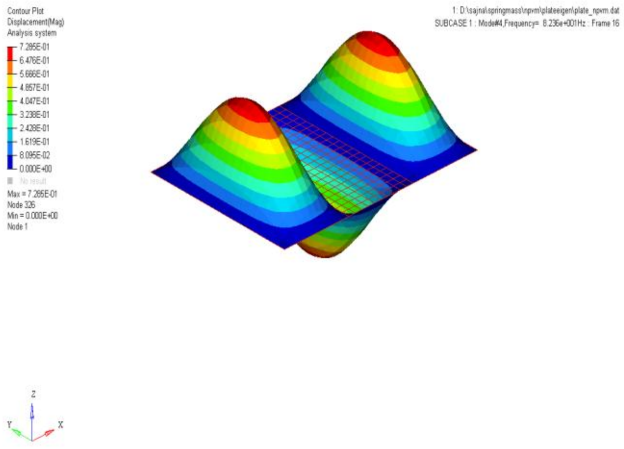

Fig -6: Fourth mode shape

ACCELERATION VS FREQUENCY PLOT FROM NPVM

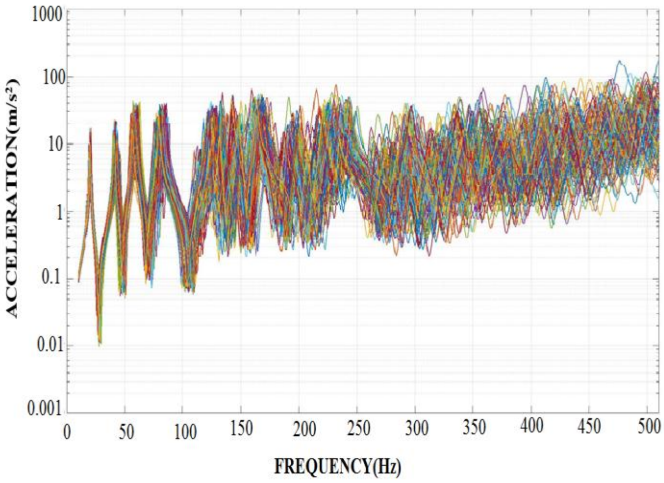

Fig -7: MC Plot

3. CONCLUSION

The application of non-parametric variability method is explained in this paper. From the above plot it is clear that as the frequency increases, spread in the plot increases and there by modal density is increasing. All the plots are peaks at an acceleration range from 10 to 100m/s^2. In higher frequencies, modes are tends to overlap each other. So it is clear from the plots that higher modes are more sensitive to the structural variations.

ACKNOWLEDGEMENT

I would like to express my sincere gratitude and deep appreciation to Mr. Mahind Jayan (Sci/Engr., SE, VSSC/ISRO) for his valuable support and patient guidance. I am very grateful to him for his willingness to give his time so generously throughout the study period.

REFERENCES

[1] C.Soize. Random matrix theory and non-parametric model of random uncertainties in vibration analysis .2012:45:893-916.

[2] Daniel C, Paul A. Uncertainty Quantification For MARS 2020 Powered Descent Closed Loop Stability. 2018:49:59-77.

[3] Durand, L. Gagliardini, Christian, Non-Parametric Modeling Of The Variability Of Vehicle Vibroacoustic Behaviour .2018:46:29–47

[4] Van den .Robust Analysis Of The Acoustic Transmission Properties Of A Structure Made Of Visco-Elastic Composite Material .2011:52:1-12.

[5] Arunachalam. A Sensitivity Study on Inertance Frequency Response Function through Non-Parametric Variability Approach.2017:01:445.

[6] Alok Sinha, Vibrations of Mechanical.2010