# A Study of Adaptive Algorithms using Matlab and Verilog

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**Abstract –** *In the field of signal processing, signals are associated with noise and distortions. This is due to the system undergoing time-varying physical process. These variations can be eliminated by using adaptive filters. These filters adjust their coefficients in an unknown environment to minimize the error signal. This approach of adaptive filters can be achieved by adaptive algorithms. The Least Mean Square algorithm, Normalized Least Mean Square Algorithm, Recursive Mean Square algorithm are used in adaptive filters to achieve noise cancellation. This paper discusses different adaptive algorithms including LMS, NLMS and RLS algorithms and comparing the three on the basis of Matlab and Verilog results. The simulation results shows that RLS has faster convergence rate than LMS and NLMS algorithms.*

***Key Words***: Adaptive filters, Least Mean Square (LMS), Normalized Least Mean Square (NLMS), Recursive Least Square(RLS), MATLAB, VERILOG.

## 1. INTRODUCTION

Their is a requirement of efficient signal processing in the fields of digital communication, radar, sonar, seismology and biomedical engineering. But the audio or video signals are generally corrupted by the noise that makes the signal processing a difficult task. In order to overcome the noise interference linear filters are introduced. The corrupted signals are made to pass through the linear filters in order to compute the signal and eliminate the noise without changing the original signal. These linear filters require memory to compute the large number of signal samples. (1) In order to overcome the disadvantage of linear filter, adaptive filters are introduced. These adaptive filters are capable of operating in any unknown environment and track the time variations of input signal.

Another different between the linear filter and adaptive filter is the use of an additional signal known as reference signal which is a correlated signal of input signal. The error is estimated by subtracting the input signal with the reference signal. This error is diminished by adjusting the filter parameters which in-turn adjust the filter coefficients to adapt to the signal characteristics (2). In this paper the algorithms which are used in adaptive filtering are discussed. These are Least Mean Square Algorithm LMS, Normalized Mean Square algorithm NLMS and Recursive Least Square algorithm RLS. In this paper the noise cancellation of the algorithms is discussed and the algorithms are compared based on the simulation results in MATLAB and ModelSim.

## 2. Noise Cancellation

The noise cancellation is a scheme to improve the SNR signal to noise ratio by retrieving the original signal from the corrupted signal. The input signal x(n), a noise signal and a desired signal d(n).The adaptive filter coefficients adapt to reduce the error terms in between input signal and desired signal. The main aim of the adaptive filters is to converge the input signal towards the desired signal. (3).

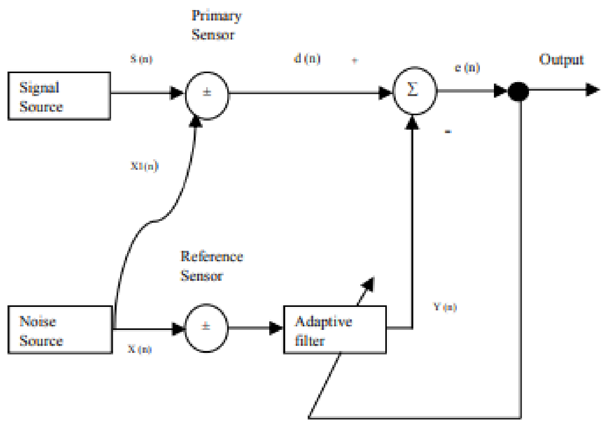


**Fig -1**: Noise Cancellation using Adaptive Filter

## 3. MSE Calculation

MSE is the measures of the average of the squares of the error (4). It is the averaged squared difference between the estimated value and the actual value. The MSE is the second moment of the error, and has the variance of the estimator and its bias. MSE has the same units of measurements as the square of the quantity being estimated.

$MSE=1/n\sum^{n}_{i=1}(y_i\hat{}-y_i)^2$ (1)

Where i y ˆ is the vector of n predictions and i y is the vector of true values. The MSE of an estimator ˆ θ with respect of the unknown parameter θ is defined as

$MSE(\theta)=E[(\theta\hat{}-\theta)^2]$ (2)

### 4.1 LEAST MEAN SQUARE ALGORITHM

LMS algorithm was invented in 1960 by Standford University B. Widrow and his first ph.d student, Ted Hoff. This algorithm males use of gradient descent method to search for minimum error condition, with mean square error as a cost function (5). As discussed elarlier the LMS approach makes use of instantaneous values of the input auto-correlation matrix instead of their actual weights. As a result the convergence is possible in the mean rather than the exact values.

In order to prevent the misleading the convergence in the mean the step size μ has to be chosen carefully. If the step size is taken too large , this might lead to large change in the weights in the random directions because of the change in gradient estimate (6). If μ is chosen too small the convergence of the weights is too large. So the μ should be in a range.

$0 < \mu < 2/\lambda_{max}$ (3)

Where $\lambda_{max}$ is the greatest eigen value of the input auto-correlation matrix R. The convergence speed is maximum when

$\mu = 2/ \lambda_{max} + \lambda_{min}$ (4)

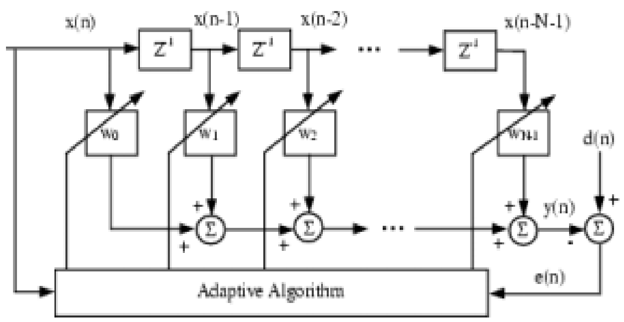


**Fig -2**: Adaptive Filter with adjustable coefficients

Where $\lambda_{min}$ and $\lambda_{max}$ are the minimum and maximum eihen values of the input correlation matrix R. Larger value of λmin close to λmax yields to faster convergence. A small μ value leads to slow convergence and a high μ value leads to fast convergence.

The LMS algorithm is consider to be the simplest algorithm because of its less computational complexity. The LMS algorithm basically consist of two processes- filteration and adaption. In filteration the filter output at $m_{th}$ iteration is given by

$y(m) = w^T (m - 1)x(m)$ (5)

The error signal at $m_{th}$ iteration is

$e(m) = d(m) - y(m)$ (6)

The weight update equation at $(m+1)_{th}$ iteration using weight

at $m_{th}$ iteration is given by

$w(m + 1) = w(m) + \mu\, e(m)x(m)$ (7)

### 4.2. NORMALIZED LEAST MEAN SQUARE ALGORITHM

The NLMS algorithm is considered to be a faster converging algorithm when compared to LMS algorithm. NLMS can be considered as an advanced form of LMS algorithm. The weight control mechanism can be considered as an major advancement. Unlike LMS the NLMS is stable for a wide range of step-size values. The stability of the algorithm is ensured by using the power of the input signal which vary with time for normalizing the step size. In addition to this the product of the step-size, input vector of the tap and real error can be obtained when the tap weights of the filter undergo necessary changes. A K-by-1 tap input vector will contribute to the generation of output signal from the filter (7). Then we take the difference between desired signal and output signal to generate an error signal.

For a $k_{th}$ order adaptive filter, filtering and weight updating action using NLMS algorithm can be characterized as the adaptive filter output at $m_{th}$ iteration is given by

$y(m) = w^T (m - 1)x(m)$ (8)

The error at $m_{th}$ iteration is

$e(m) = d(m) - y(m)$ (9)

The weight update equation at $(m + 1)_{th}$ iteration using

weights at $(m)_{th}$ iteration is given by

$w(m + 1) = w(m) + \mu (m)e(m)x(m)$ (10)

where

$\mu (m) = \beta/r + ||x(m)||^2$ (11)

where β is the adaptation constant

$0 < \beta < 2$ (12)

r is a constant for avoiding the condition of division by zero

in case of weak input signal.

$0 < r < 1$ (13)

Therefore the weight update equation in NLMS algorithm can be modified to below equation by using variable step-size factor μ (m)

$w(m + 1) = w(m) + \beta/r + ||x(m)||^2 e(m)x(m)$ (14)

### 4.3. RECURSIVE LEAST MEAN SQUARE ALGORITHM

Recursive Mean square algorithm was discovered by Gauss.

This algorithm is different from other algorithms in its approach to reduce the mean square error. This algorithm recursively obtains the tap weight coefficients to reduce the least mean square error. In this approach the input signal is considered to be deterministic unlike LMS and NLMS algorithm where the input signal is considered stochastic. The convergence rate of this algorithm is high but has low stability. One way to reduce the instability is to use Kalman gain with infinite precision range. The computational complexity of this algorithm is also high.

The output of the RLS algorithm is generated by convolution sum of the input samples with tap weights (8). The tap weight vector is calculated by adding the previous weight vector to the product of Kalman gain and the error. The optimal weights of the array coefficients to reduce the weighted least square cost function (9).

The cost function is given by

$z(m) = \sum^{m}_{l=1} \alpha(m,l)|e(l)|^2$ (15)

where e(l) is the error at $l_{th}$ time. The forgetting factor α as

written above is within the range

$\alpha(m, l) = \lambda^{m-1}$ l = 1, 2,……. M (16)

The filtering and adapting processes of the algorithm are as follows

The output of the adaptive algorithm is give as

$Y(m)=w^t(m)x(m)$ (17)

Error is calculated as

$E(m)=d(m0)-y(m)$ (18)

Tap weights are updated using preceding set of tap coefficients and a product term of gain and error.

$W(m+1)=w(m)+g(m)e(m)$ (19)

Where g(m) is the gain vector

$g(m)=N(m)x(m)/\lambda + x^T(m)N(m)x(m)$ (20)

where λ is the forgetting factor. The initial value of N(m)

at m=0 is

$N(0)= \delta^{-1} I$ (21)

Where δ is the regulization factor. The inverse correlation matrix is given by

$N(m)= \lambda^{-1}N(m-1)- \lambda^{-1}g(m)x(m)N(m-1)$ (22)

### 5. SIMULATION RESULTS

The algorithms are compared on the basics of mean square error .The three algorithms are compared using MATLAB and Modelsim simulators. Fig.3,4,5 shows the the input signal, desired signal and the output of the respective algorithm. Fig.6,7,8 shows the mse plot of all the algorithms comparing their convergence rates. variation of MSE plot for different step-size factors. It shows that smaller the step-size, rate of convergence is slow and rate of convergence is faster for larger step-size factor.

The algorithms are simulated in modelsim and the verilog implementation is shown in Fig.9,10,11. This simulation results in both MATLAB and Modelsim show that RLS algorithm has the fastest convergence rate. But the computational complexity of the RLS algorithm is high. However, both the MATLAB and Modelsim results prove good characteristics of RLS algorithm for noise cancellation in signal processing.

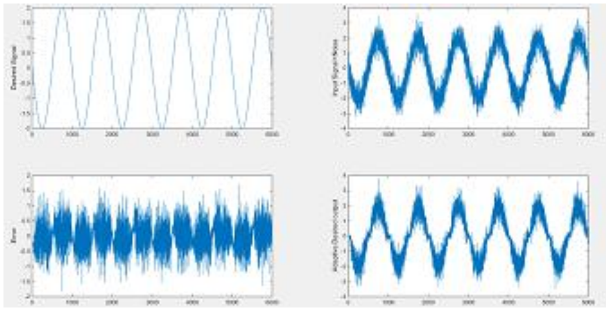

**Fig-3**: Noise cancellation using LMS algorithm

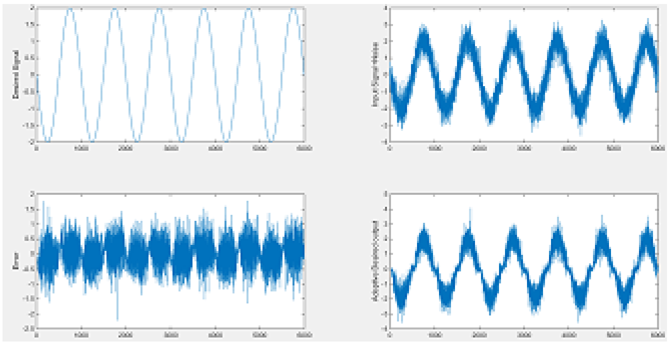

**Fig-4**: Noise cancellation using NLMS algorithm

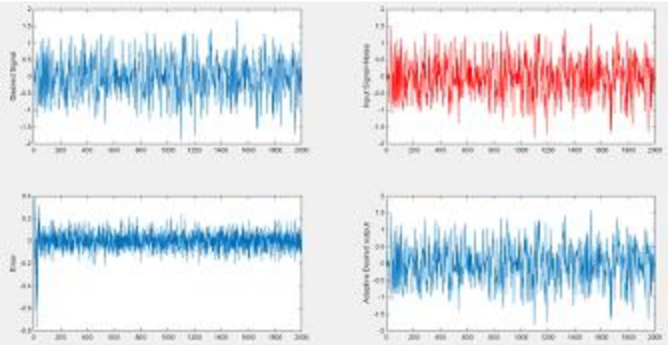

**Fig-5**: Noise cancellation using RLS algorithm

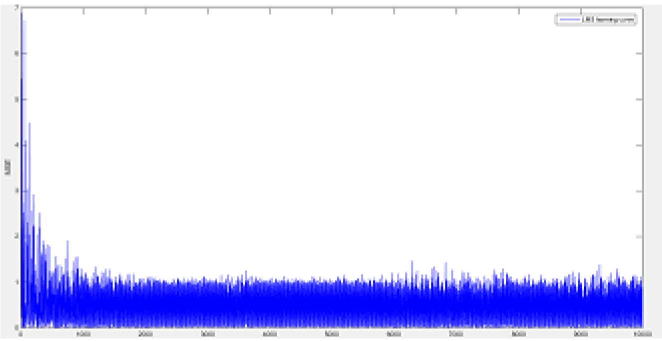

**Fig-6** : Learning curve of LMS algorithm

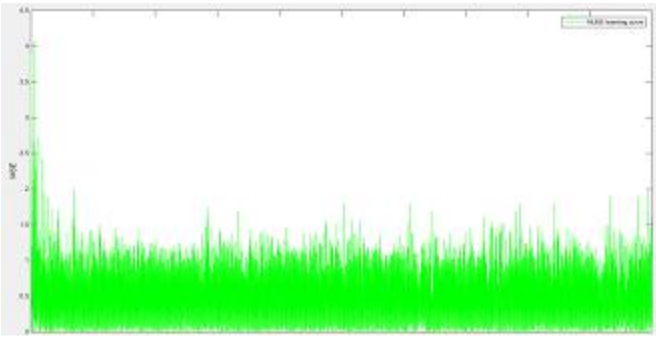

**Fig-7** : learning curve of NLMS algorithm

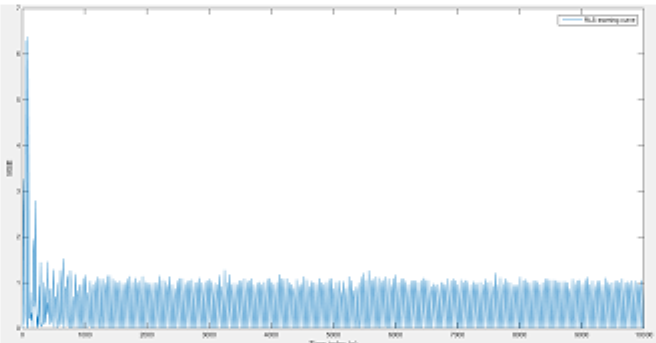

**Fig-8** : Learning curve of RLS algorithm

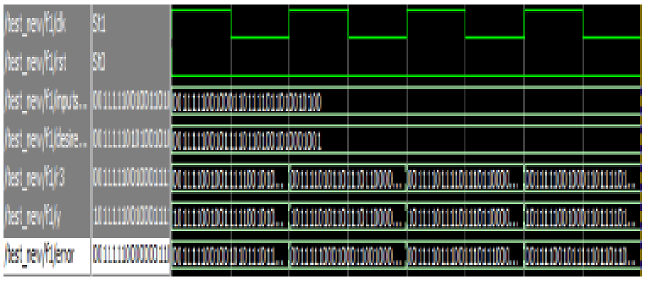

**Fig-9** : Verilog implementation of LMS algorithm

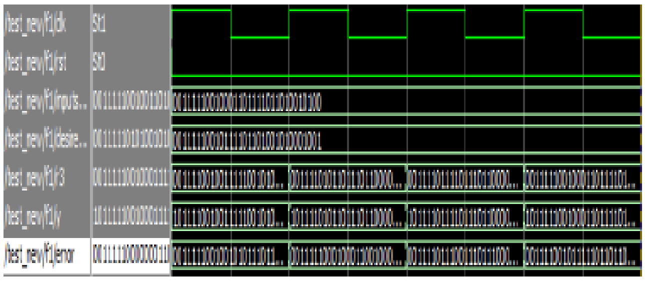

**Fig-10** : Verilog implementation of NLMS algorithm

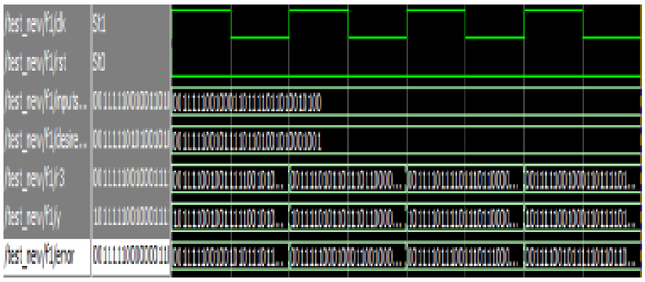

**Fig-11** : Verilog implementation of RLS algorithm

## 6. CONCLUSION

The noise cancellation requires the adaptive filters to generate the control signal. The adaptive updates its coefficients to converge the error in a best possible way. LMS is the simplest and easiest algorithm to implement. RLS algorithm is complex yet has a good convergence rate. NLMS algorithm has a convergence rate more than LMS algorithm. RLS algorithm is an good option when working with time-varying environments when compared to other algorithms.

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